

A Novel Approach to Identifying Hedonic Demand Parameters[†]

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Abstract

This note presents a new solution to the classic problem with using hedonic price functions to recover demand curves. Unexpected changes in the composition of a differentiated product can generate instruments that support a simple reduced-form approach to demand estimation.

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1. Introduction

Rosen's (1974) first-stage model of hedonic pricing is among the foremost tools of empirical microeconomics. It is routinely used to estimate the value of small changes in the characteristics of workers, private goods, public goods, and externalities.¹ In comparison, Rosen's vision for a second-stage model of demand remains unfulfilled. The problem is identification. There are two general strategies for identifying demand parameters. One is to make explicit assumptions about the structure of preferences in a single geographic market (e.g. Bajari and Kahn 2005). The other is to collect data from multiple geographic markets, assuming that consumers in each market share a common preference structure (e.g. Palmquist 1984, Bartik 1987; Zabel and Kiel 2000). Neither approach has been widely applied.

The purpose of this note is to present a new solution to Rosen's second stage. We demonstrate that unexpected changes in the composition of a differentiated product can be used to identify demand parameters from data on a single geographic market. Our work provides a new perspective on studies that have sought to use quasi-experiments to improve hedonic modeling. In these studies, researchers have nested first-stage hedonic models within panel data frameworks that aim to estimate marginal values from micro data on prices and characteristics measured before and after shocks to product quality, information, and institutions (e.g. Card 1990, Card and Krueger 1992, Hirsch 1993, Davis 2004, Pope 2008). When these shocks are not marginal, they have the potential to generate instruments that can solve the identification problem with second-stage demand estimation. This is our main point.

Section 2 presents an equilibrium model of a differentiated product market. Section 3 uses the model to explain how the instruments are generated. Section 4 concludes.

2. Hedonic Equilibrium

Price-taking consumers purchase a single unit of a good differentiated by k characteristics, $x = [x_1, \dots, x_k]$. The utility maximization problem is

¹ For example, Kniesner and Viscusi (2005) estimate the impact of job fatality risk on wages, Ashenfelter (2008) estimates the impact of weather on the price of wine, Black (1999) estimates the value of small improvements in public school quality, and Smith and Huang (1995) summarize estimates of the willingness to pay for small improvements in air quality.

$$\max_{x,b} U(x,b;\alpha) \text{ subject to } y = b + P(x;\theta). \quad (1)$$

Each consumer chooses characteristics and the numeraire composite commodity (b) to maximize utility, given her preferences (α), income (y), and the price schedule $P(x;\theta)$, which depends on a parameter vector, θ . The first order conditions are

$$\frac{\partial P(x;\theta)}{\partial x} = \frac{\partial U/\partial x}{\partial U/\partial b} \equiv D(x;\alpha, y). \quad (2)$$

The consumer chooses a good that provides levels of each characteristic at which her marginal willingness to pay (MWTP) for an additional unit equals its marginal implicit price. Assuming marginal utility of income is constant, the second equality in (2) observes that as x varies the marginal rates of substitution define inverse demand curves.

Let $C(m, x; \beta)$ denote a producer's cost function, where m is the number of type- x goods they sell and β is a vector of parameters differentiating producers. In addition to reflecting heterogeneity in production technology, β also reflects heterogeneity in exogenous factors affecting the characteristics of the good supplied. Examples of exogenous factors used in conjunction with hedonic models include unexpected changes in health risk (Davis 2004), weather (Ashenfelter 2008), desegregation (Card and Krueger 1992), and information disclosure laws (Pope 2008).

Price-taking producers are free to vary the number of units they sell as well as the characteristics of each unit. The profit maximization problem is

$$\max_{x,m} \pi = m \cdot P(x;\theta) - C(m, x; \beta), \quad (3)$$

with corresponding first order conditions

$$P(x;\theta) = \frac{\partial C(m, x; \beta)}{\partial m}, \quad \frac{\partial P(x;\theta)}{\partial x} = \left(\frac{1}{m}\right) \frac{\partial C(m, x; \beta)}{\partial x}. \quad (4)$$

Producers choose m to set the offer price of the marginal unit equal to its production costs, and they choose x to set the marginal cost of each characteristic equal to its implicit price.

Equilibrium occurs when the first order conditions are simultaneously satisfied for all consumers and producers. This system of differential equations implicitly defines the equilibrium hedonic price function that clears the market (Rosen 1974). It will be useful to rewrite the price function to highlight its dependence on model primitives,

$$P(x; \theta) \equiv P[x(A, B); \theta(A, B)]. \quad (5)$$

Equilibrium prices and quantities are determined by all of the exogenous variables: $A: F(y, \alpha) \sim A$, a vector of parameters that describes the joint distribution of income and preferences and $B: V(\beta) \sim B$, a parameter vector describing the distribution of producer characteristics.

Figure 1 provides a stylized picture of how the equilibrium price function reveals the distribution of marginal values for each characteristic. It relates the marginal price function for x_1 to demand curves for two consumers and supply curves for two producers. Evaluating $\partial P(\cdot)/\partial x_1$ at a consumer's chosen level of x_1 will return their MWTP. Combining this information with x_1 identifies exactly one point on their demand curves.

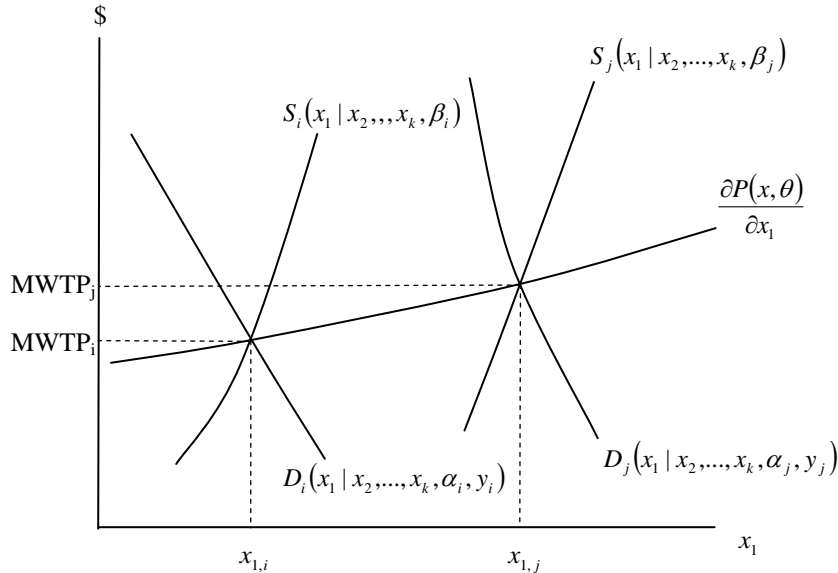


Fig 1. Demand curves for two consumers, supply curves for two producers, and the equilibrium marginal price function for x_1 .

3. Using Market Shocks to Identify Demand Parameters

To see the difficulty with demand estimation, consider the demand curve for x_1 ,

$$D_1(x_1; x_2, \dots, x_k, \alpha, y) = f(x_1, \kappa; x_2, \dots, x_k, w, y) + \nu. \quad (6)$$

The expression to the left of the equality is the true demand curve and the expression to the right is an econometric approximation, where κ is the parameter vector to be estimated, w is a set of observable consumer demographics that are correlated with tastes, and ν is the residual unobserved taste heterogeneity. Each consumer's choices for x_1, \dots, x_k will reflect their tastes. Therefore x_1, \dots, x_k are correlated with ν so that κ is not identified. This is the classic identification problem characterized by Epple (1987) and Bartik (1987).

Now consider an unexpected shock to B that influences the equilibrium distribution of x_1 . In general, a shock to x will change the shape of the price function in (5). A change in the price function alters the budget constraint for any α, y -type consumer in (1) which, in turn, changes their optimal choices for x_1, \dots, x_k . Therefore, as long as re-equilibration of the market does not change the distribution of unobserved tastes in the consumer population, a valid set of instruments can be developed by multiplying an indicator for the post-shock observation period by the intercept and w .

Intuition can be seen from a simple example. Suppose supply is predetermined, utility is quadratic, and heterogeneous preferences and characteristics are normally distributed. These assumptions conveniently provide a closed-form expression for the equilibrium price function (Tinbergen 1959, Epple 1987).² Specifically, let utility be parameterized as

$$U = -(x - \alpha)' \frac{\Omega}{2} (x - \alpha) + b, \quad (7)$$

where Ω is a positive definite diagonal scaling matrix. When x and α are both normally distributed such that $x \sim N(\mu_x, \Sigma_x)$ and $\alpha \sim N(\mu_\alpha, \Sigma_\alpha)$, the price function can be expressed

² Tinbergen (1959) developed this linear-quadratic-normal model to consider the properties of equilibria in labor markets with heterogeneous workers. Epple (1987) and Ekeland, Heckman, and Nesheim (2004) use the model to illustrate other features of hedonic equilibria.

as

$$P(x) = \Psi'x + x' \frac{\Gamma}{2} x, \quad \text{where } \Psi = \Omega(\mu_\alpha - \Sigma_\alpha^{0.5} \Sigma_x^{0.5} \mu_x) \quad \text{and} \quad \Gamma = -\Omega(I - \Sigma_\alpha^{0.5} \Sigma_x^{0.5}). \quad (8)$$

The reduced-form parameters of the price function (Ψ, Γ) are functions of the structural parameters describing the distributions of consumer preferences $(\mu_\alpha, \Sigma_\alpha)$ and product characteristics (μ_x, Σ_x) .

Now consider a shock to x_1 . Before the shock, $MWTP_1 = \Psi_1 + \Gamma_1 x$. After the shock, $MWTP_2 = \Psi_2 + \Gamma_2 x$. It follows from (8) that, in general, $\Psi_1 \neq \Psi_2$ and $\Gamma_1 \neq \Gamma_2$. Thus, holding preferences fixed, an unexpected shock to a product characteristic changes MWTP for each consumer type, tracing out two points on their demand curves. We demonstrate this numerically:

$$x = [x_1, x_2, x_3], \quad \mu_\alpha = [20 \ 50 \ 25], \quad \mu_x = [5 \ 10 \ 0], \quad (9)$$

$$\Sigma_\alpha = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad \Omega = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad \text{and} \quad \Sigma_x = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

With these parameter values, all three characteristics are normal goods. After using the distributions in (9) to evaluate the price function in (8), we shock x_1 using $\Delta x_1 \sim N(3, 0.25)$. Then we evaluate the new price function.

Figure 2 displays marginal price functions for x_1 before and after the shock, as well as demand curves for two consumers. Because demand is downward sloping, a positive shock increases the price of the good but decreases each consumer's MWTP for a further improvement in x_1 . In aggregate, these decreases cause the marginal price function to shift. The shift traces out two points on the demand curve for each consumer "type" in the figure. This simple example illustrates how the sorting process that underlies adjustment between different equilibria can identify the demand for characteristics of a differentiated product, using data from a single geographic market.

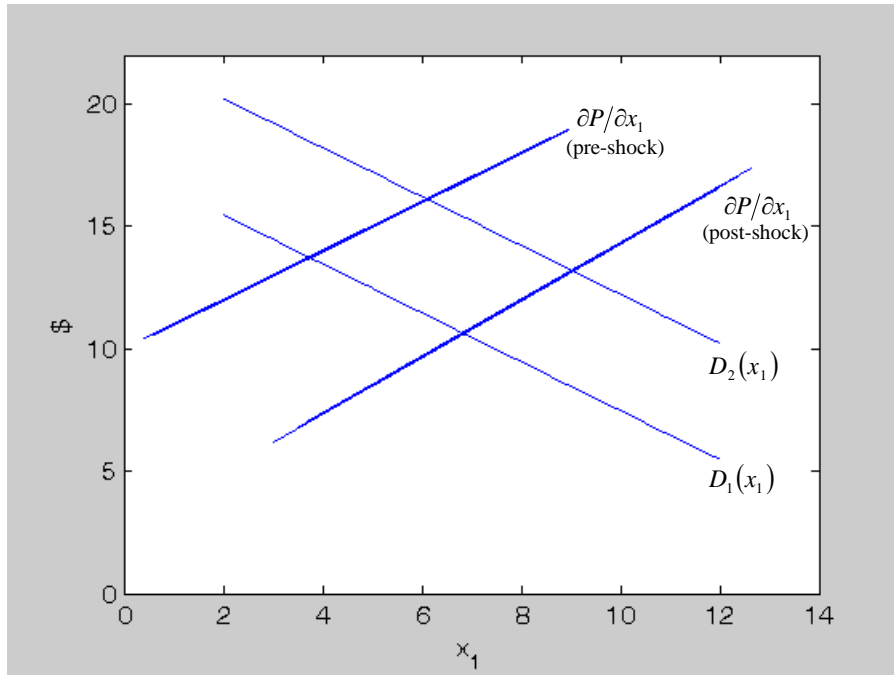


Fig 2. Numerical example of equilibria before and after a shock to x_1 . D_1 and D_2 are demand curves for two consumers, $\partial P/\partial x_1$ is the marginal price function.

Our identification strategy requires repeated cross section data describing consumers and their choices, straddling a market shock.³ Econometric analysis would proceed in two stages: (i) estimate single-period price functions before and after the shock, and then (ii) use the shock to develop instruments for demand estimation. The first stage exploits the nonlinearity of the marginal price function (Ekeland, Heckman, and Nesheim 2004). The second stage exploits the mechanics of the hedonic model to recover demand curves.

4. Conclusion

We have suggested a novel approach to hedonic demand estimation. The identification derives from the ways in which heterogeneous consumers adjust their behavior to unexpected changes in the composition of the differentiated product. Their collective adjustments can produce shifts and rotations in the gradient of the price function that effectively trace out market de-

³ Data on repeated cross-sections of consumers are now widely available from commercial vendors in the form of scanner data and real estate assessment databases.

mand curves. Limiting the analysis to a single geographic market enhances the validity of the estimates by improving the comparability of the “before” and “after” groups (Meyer 1995).

Rosen (2002) first called our attention to the fact that changes in the gradient of the price function provide additional information about preferences and technology, beyond what is revealed by a single equilibrium. He suggested using this information to adjust price indices for advances in technology that decrease the cost of living. We have outlined a counterpart to his proposal. Changes in the hedonic gradient may also serve to identify the demand for characteristics of goods and services that improve the quality of life.

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