Internet Appendix to
"Idiosyncratic Cash Flows and Systematic Risk"

ILONA BABENKO, OLIVER BOGUTH, and YURI TSERLUKEVICH*

This Internet Appendix supplements the analysis in the main text by extending the model and empirical work in three dimensions. First, we generalize the example with a two-division firm in the main text to allow each of the divisions to depend on both systematic and idiosyncratic shocks. Second, we provide a solution for a modified version of our model with mean-reverting profitability shocks. Finally, we show that our empirical results are robust to alternative estimation.

I. Two-Division Example

In the main text, we show that a positive idiosyncratic shock decreases the systematic risk of a firm if the firm consists of a zero-beta and a unit-beta division. We now show that this intuition extends to the case in which the two divisions are not purely idiosyncratic and systematic, but have different sensitivities to shocks.

Suppose the value of each division is determined by a priced shock y and an unpriced shock x_i, so that the total value of the firm is

$$V(x_i, y) = V_1(x_i, y) + V_2(x_i, y). \tag{IA1}$$

We define $\beta_j^{x_i}$ and β_j^y as the divisions' exposures to idiosyncratic and systematic shocks,

$$\beta_j^{x_i} = \frac{\partial V_j\left(x_i, y\right)}{V_j\left(x_i, y\right)} \frac{x_i}{\partial x_i}, \qquad \beta_j^{y} = \frac{\partial V_j\left(x_i, y\right)}{V_j\left(x_i, y\right)} \frac{y}{\partial y}, \qquad j = 1, 2. \tag{IA2}$$

*Citation format: Babenko, Ilona, Oliver Boguth, and Yuri Tserlukevich, Internet Appendix to "Idiosyncratic Cash Flows and Systematic Risk," *Journal of Finance*, DOI: 10.1111/jofi.12280. Please note: Wiley-Blackwell is not responsible for the content or functionality of any supporting information supplied by the authors. Any queries (other than missing material) should be directed to the authors of the article.

Suppose that the first division has more idiosyncratic risk, whereas the second has more systematic risk, that is,

$$\beta_1^{x_i} > \beta_2^{x_i} \text{ and } \beta_2^y > \beta_1^y. \tag{IA3}$$

For simplicity, we also assume that the division betas are constant. It follows from the definition of firm beta that the change in beta is determined by the relative weight of the more risky division. In particular,

$$\begin{aligned} \frac{\partial \beta}{\partial x_i} &= \frac{\partial}{\partial x_i}\left(\frac{V_1}{V_1+V_2}\beta_1^y + \frac{V_2}{V_1+V_2}\beta_2^y\right) = \frac{\left(\beta_2^y - \beta_1^y\right)}{\left(1+\frac{V_2}{V_1}\right)^2}\frac{\partial}{\partial x_i}\left(\frac{V_2}{V_1}\right) \\ &= \frac{V_1 V_2}{x_i\left(V_1+V_2\right)^2}\left(\beta_2^y - \beta_1^y\right)\left(\beta_2^{x_i} - \beta_1^{x_i}\right) < 0, \end{aligned} \tag{IA4}$$

where the last expression is negative from (IA3).

II. Generalized Setup with Mean-Reverting Cash Flows

In the main text, we assume that cash flow shocks follow geometric Brownian motions. Here we model mean-reverting profitability shocks

$$dx_i = \alpha_x(\overline{x} - x_i)dt + x_i\sigma_x dz_i, \tag{IA5}$$

$$dy = \alpha_y(\overline{y} - y)dt + y\sigma_y dz_y, \tag{IA6}$$

where $\overline{x}$ and $\overline{y}$ are the long-run means, α_x and α_y control the speed of mean reversion, and dz_i and dz_y are increments of uncorrelated standard Wiener processes. The idiosyncratic shocks have identical drifts and volatilities and are uncorrelated across firms.

The advantage of using mean-reverting processes over regular geometric Brownian motions (GBM) is that the former are stationary as long as $\alpha_x + \sigma_x^2/2 > 0$ and $\alpha_y +$

$\sigma_y^2/2 > 0$ (Zhao (2009)). This specification is often called the "inhomogeneous" GBM because the diffusion of each process is proportional to the value of shock (i.e., $x_i\sigma_x$ and $y\sigma_y$), but the drift term is not (e.g., Bhattacharya (1978)). Note that the homogeneous GBM is a special case of this process, where $\overline{x} = \overline{y} = 0, \alpha_x = -\mu_x$, and $\alpha_y = -\mu_y$.

The solution for firm value when the cash flow shocks are mean-reverting is based on Bhattacharya (1978) and Zhao (2009), adapted for the case of two independent stochastic processes, two investment options, and infinite horizon. We first determine the risk premium required for the systematic component of cash flows. Specifically, suppose there exists a *traded* security M whose innovations are correlated with innovations in y with an instantaneous correlation coefficient ϕ:

$$dM/M = \alpha_M dt + \sigma_M dz_M, \tag{IA7}$$

$$E(dz_M dz_y) = \phi dt. \tag{IA8}$$

We denote by $\lambda = (\alpha_M - r)/\sigma_M$ the price of risk, so that $\phi\lambda\sigma_y$ measures the risk premium. The following proposition gives total firm value for the case of mean-reverting cash flow shocks.

PROPOSITION IA.1: *Suppose the profitability shocks follow the inhomogeneous Brownian motions (IA5) and (IA6). Denote by ι_x and ι_y indicator functions equal to one if the respective growth option has been exercised. Then the market value of the firm is given by*

$$V_i = V_i^{AX} + V_i^{GX} + \rho_i \left(V^{AY} + V^{GY}\right), \tag{IA9}$$

where the value components $V_i^{AX}, V_i^{GX}, V^{AY},$ and V^{GY} are given by

$$V_i^{AX}(x_i) = \frac{(1+\gamma_x \iota_x)\left(x_i + \frac{\alpha_x \overline{x}}{r}\right)}{r+\alpha_x}, \tag{IA10}$$

$$V_i^{GX}(x_i) = (1-\iota_x) C_x (B_x/x_i)^p U(p, 2p+2-A_x, B_x/x_i), \tag{IA11}$$

$$V^{AY}(y) = \frac{(1+\gamma_y \iota_y)\left(y + \frac{\alpha_y \overline{y}}{r}\right)}{r+\alpha_y+\phi\lambda\sigma_y}, \tag{IA12}$$

$$V^{GY}(y) = (1-\iota_y) C_y (B_y/y)^q U(q, 2q+2-A_y, B_y/y), \tag{IA13}$$

$U(\cdot)$ is Tricomi's confluent hypergeometric function, and $A_x = \frac{-2\alpha_x}{\sigma_x^2}$, $B_x = \frac{2\alpha_x \overline{x}}{\sigma_x^2}$, $A_y = \frac{-2(\alpha_y+\phi\lambda\sigma_y)}{\sigma_y^2}$, $B_y = \frac{2\alpha_y \overline{y}}{\sigma_y^2}$, and $p > 0$ and $q > 0$ solve quadratic equations

$$p^2 + (1-A_x)p - \alpha_x = 0, \tag{IA14}$$

$$q^2 + (1-A_y)q - \alpha_y = 0. \tag{IA15}$$

The constants C_x and C_y as well as the exercise thresholds x^* and y^* are determined by the following value-matching and smooth-pasting conditions:

$$V_i^{GX}(x^*) = \frac{\gamma_x\left(x^* + \frac{\alpha_x \overline{x}}{r}\right)}{r+\alpha_x} - I_x, \tag{IA16}$$

$$\frac{\partial V_i^{GX}(x^*)}{\partial x_i} = \frac{\gamma_x}{r+\alpha_x}, \tag{IA17}$$

$$V^{GY}(y^*) = \frac{\gamma_y\left(y^* + \frac{\alpha_y \overline{y}}{r}\right)}{r+\alpha_y+\phi\lambda\sigma_y} - I_y, \tag{IA18}$$

$$\frac{\partial V^{GY}(y^*)}{\partial y} = \frac{\gamma_y}{r+\alpha_y+\phi\lambda\sigma_y}. \tag{IA19}$$

Proof: The total value of the firm is equal to the sum of the perpetuity of cash flows from assets in place and the values of the two growth options. By following a standard "instantaneous hedging" argument, we can find firm value $V(x_i, y)$ as a solution to the partial differential equation

$$rV_i = x_i + \rho_i y + \frac{\partial V_i}{\partial x_i}\alpha_x(\overline{x} - x_i) + \frac{\partial V_i}{\partial y}\left(\alpha_y(\overline{y} - y) - y\rho\lambda\sigma_y\right) + \frac{1}{2}\frac{\partial^2 V_i}{\partial x_i^2}\sigma_x^2 x_i^2 + \frac{1}{2}\frac{\partial^2 V_i}{\partial y^2}\sigma_y^2 y^2. \tag{IA20}$$

Because this equation is separable in components x_i and y, the solution for $V(x_i, y)$ can also be presented as a sum of the two components $V(x_i)$ and $\rho_i V(y)$, which are solutions to the ordinary differential equations

$$rV(x_i) = x_i + \alpha_x(\overline{x} - x_i)\frac{\partial V}{\partial x_i} + \frac{1}{2}\sigma_x^2 x_i^2 \frac{\partial^2 V}{\partial x_i^2}, \tag{IA21}$$

$$rV(y) = y + \left(\alpha_y(\overline{y} - y) - y\phi\lambda\sigma_y\right)\frac{\partial V}{\partial y} + \frac{1}{2}\sigma_y^2 y^2 \frac{\partial^2 V}{\partial y^2}. \tag{IA22}$$

Bhattacharya (1978, p. 1324) solves a similar one-dimensional problem for a firm without investment options and finite maturity. His solution can be easily adapted to value the cash flow perpetuity in our model, with one priced and one unpriced shock, and with infinite maturity:

$$V_i^{CF} = \frac{x_i + \frac{\alpha_x \overline{x}}{r}}{r + \alpha_x} + \frac{\rho_i\left(y + \frac{\alpha_y \overline{y}}{r}\right)}{r + \alpha_y + \phi\lambda\sigma_y}. \tag{IA23}$$

Next, the values of the two options, V_i^{GX} and V^{GY}, are determined separately as the values of American call options with the appropriate boundary conditions (see Zhao (2009) and Robel (2001)) and are given by (IA10) and (IA13). □

We use the results in the above proposition to investigate how priced risk changes with idiosyncratic profit shocks. The firm's beta with respect to the systematic shock is

$$\beta_i = \frac{\rho_i y \frac{\partial}{\partial y}\left(V^{AY} + V^{GY}\right)}{V_i^{AX} + V_i^{GX} + \rho_i\left(V^{AY} + V^{GY}\right)}. \tag{IA24}$$

To demonstrate the size anomaly, we sign the sensitivities of market value and beta with respect to the idiosyncratic shock as follows:

$$\frac{\partial V_i}{\partial x_i} = \frac{\partial}{\partial x_i}\left(V_i^{AX} + V_i^{GX}\right) > 0, \tag{IA25}$$

$$\frac{\partial \beta_i}{\partial x_i} = -\frac{\rho_i y \frac{\partial}{\partial y}\left(V^{AY} + V^{GY}\right)\frac{\partial}{\partial x_i}\left(V_i^{AX} + V_i^{GX}\right)}{\left(V_i^{AX} + V_i^{GX} + \rho_i\left(V^{AY} + V^{GY}\right)\right)^2} < 0. \tag{IA26}$$

As in the model in the main text, idiosyncratic shocks have opposite effects on firm value and beta, giving rise to size-related anomalies.

III. Robustness of Empirical Results

Our empirical estimation of idiosyncratic cash flow volatility uses growth rates of quarterly firm cash flows, CF_{it}/CF_{it-4}. Because the growth rate is undefined if a firm's cash flows are negative, we exclude those observations. Since firms with high cash flow volatility are more likely to report negative earnings, our procedure tends to eliminate highly volatile firms from the sample. However, there is no obvious reason to believe that this selection biases our return predictability results. To show that our results are robust, we reestimate the volatility of idiosyncratic cash flows using an alternative method proposed by Irvine and Pontiff (2009). Their approach relies on differences in earnings per share rather than on ratios and can therefore accommodate negative cash flow realizations.

We extract idiosyncratic cash flow shocks by estimating

$$\Delta E_{it} = \alpha + \beta_1 \Delta E_{it-1} + \beta_2 \Delta E_{it-2} + \beta_3 \Delta E_{it-3} + e_{it}, \tag{IA27}$$

where $\Delta E_{it} = E_{it} - E_{it-4}$ is the difference between earnings per share in the current quarter and earnings per share in the same quarter of the previous year. Following Irvine and Pontiff (2009), we winsorize earnings-to-price ratios at the 1st and 99th percentiles before converting them to earnings per share. The regressions are conducted at the industry level (Fama and French 49 industries) using the full sample of data. Therefore, this method cannot be used to construct a trading strategy. The residuals from these regressions are then scaled by the price per share at the end of the previous quarter. We compute the systematic cash flow innovation in each quarter as the equal-weighted average of the individual firm shocks.

Using this alternative cash flow innovation measure, we repeat the procedure used in Tables V and VI in the main text. In particular, we compute idiosyncratic cash flow volatility using 40-quarter rolling window regressions of individual firm cash flow shocks on the systematic cash flow shock, and use NYSE breakpoints to sort stocks into five portfolios based on idiosyncratic cash flow volatility. Each portfolio is then subdivided into five groups by either size or book-to-market.

The results, reported in Tables IA.I and IA.II, are similar to the results in Tables V and VI in the main text. Small stocks outperform large stocks in all volatility groups, but much more so among stocks for which idiosyncratic cash flows are more important. A value-weighted portfolio that buys the size anomaly in high cash flow volatility stocks and shorts the size-anomaly in low cash flow volatility stocks (the difference-in-difference) has an average return of 0.81% per month. This return is statistically significant, economically large, and robust to risk adjustment using the Fama and French (1993) three-factor model and equal weighting. Likewise, the average difference-in-difference return for the value anomaly (Table IA.II) is 0.60% per month for the value-weighted portfolios and 0.98% per month for the equal-weighted portfolios. Overall, our results are robust to using this alternative methodology to estimate idiosyncratic volatility.

Table IA.I
Irvine and Pontiff (2009) Idiosyncratic Cash Flow Volatility and the Size Anomaly

This table reports average monthly returns of value-weighted (Panel A) and equal-weighted (Panel B) portfolios sorted first by the volatility of idiosyncratic cash flows and then by market capitalization, as well as the difference between high and low idiosyncratic cash flow volatility and small and big market capitalization. Idiosyncratic cash flow volatility is the residual standard deviation from rolling 40-quarter regressions of firm cash flow innovations onto aggregate cash flow shocks, computed following Irvine and Pontiff (2009). We also provide alphas from the CAPM and Fama-French three-factor models, and average log market capitalization in Panel C. t-statistics are based on Newey and West (1987) adjusted standard errors using 12 lags. The sample period is January 1973 to December 2013.

	Volatility of Idiosyncratic Cash Flows					
	Low	2	3	4	High	H−L
	Panel A: Value-Weighted Returns					
Small	1.03	1.29	1.28	1.46	1.83	0.80
	(5.68)	(6.36)	(4.99)	(4.74)	(4.74)	(2.42)
2	1.17	1.23	1.27	1.54	1.52	0.35
	(6.29)	(4.98)	(4.86)	(5.62)	(4.32)	(1.20)
3	1.09	1.09	1.25	1.49	1.40	0.32
	(5.98)	(5.22)	(5.95)	(5.65)	(4.13)	(1.13)
4	1.04	1.05	1.07	1.05	1.26	0.21
	(5.27)	(5.39)	(5.21)	(4.38)	(4.03)	(0.91)
Big	0.90	1.01	1.05	1.07	0.89	-0.01
	(4.71)	(5.42)	(5.03)	(4.35)	(3.26)	(-0.06)
S-B	0.13	0.27	0.23	0.39	0.94	0.81
	(0.84)	(1.73)	(1.11)	(1.62)	(3.28)	(2.65)
α_{S-B}^{CAPM}	0.20	0.34	0.29	0.44	0.97	0.78
	(1.30)	(2.16)	(1.36)	(1.80)	(3.41)	(2.66)
α_{S-B}^{FF3}	-0.00	0.11	0.13	0.22	0.85	0.85
	(-0.03)	(0.78)	(0.65)	(1.05)	(3.27)	(2.85)

Table IA.I continued.

	Volatility of Idiosyncratic Cash Flows					
	Low	2	3	4	High	H−L
			Panel B: Equal-Weighted Returns			
Small	1.04	1.37	1.28	1.81	2.00	0.96
	(5.78)	(6.39)	(5.39)	(5.82)	(5.17)	(3.01)
2	1.22	1.33	1.36	1.51	1.64	0.42
	(6.56)	(5.58)	(5.62)	(5.76)	(4.62)	(1.43)
3	1.12	1.17	1.33	1.47	1.43	0.30
	(6.05)	(5.80)	(6.40)	(5.93)	(4.25)	(1.16)
4	1.05	1.11	1.20	1.17	1.30	0.25
	(5.23)	(5.56)	(5.73)	(4.93)	(4.05)	(1.06)
Big	0.93	1.03	1.13	1.25	1.08	0.15
	(5.03)	(5.40)	(5.21)	(5.15)	(3.98)	(0.84)
S-B	0.11	0.34	0.15	0.56	0.91	0.80
	(0.77)	(2.19)	(0.86)	(2.25)	(3.54)	(2.83)
α_{S-B}^{CAPM}	0.20	0.42	0.25	0.64	1.03	0.83
	(1.45)	(2.66)	(1.36)	(2.49)	(3.74)	(2.96)
α_{S-B}^{FF3}	0.03	0.23	0.12	0.46	0.96	0.93
	(0.35)	(1.71)	(0.72)	(1.98)	(3.82)	(3.29)
			Panel C: Average Market Capitalization			
Small	11.71	11.24	10.68	10.04	9.22	-2.50
2	12.97	12.61	12.06	11.52	10.57	-2.39
3	13.75	13.45	13.11	12.64	11.59	-2.16
4	14.53	14.24	14.05	13.69	12.68	-1.85
Big	15.87	15.63	15.69	15.39	14.47	-1.40
S-B	-4.16	-4.39	-5.01	-5.34	-5.25	-1.10

Table IA.II
Irvine and Pontiff (2009) Idiosyncratic Cash Flow Volatility and the Value Anomaly

This table reports average monthly returns of value-weighted (Panel A) and equal-weighted (Panel B) portfolios sorted first by the volatility of idiosyncratic cash flows and then by book-to-market ratios, as well as the difference between high and low idiosyncratic cash flow volatility and high and low book-to-market ratios. Idiosyncratic cash flow volatility is the residual standard deviation from rolling 40-quarter regressions of firm cash flow innovations onto aggregate cash flow shocks, computed following Irvine and Pontiff (2009). We also provide alphas from the CAPM and the Fama-French three-factor model, and average log market capitalization (book-to-market ratio) in Panel C. t-statistics are based on Newey and West (1987) adjusted standard errors using 12 lags. The sample period is January 1973 to December 2013.

	Volatility of Idiosyncratic Cash Flows					
	Low	2	3	4	High	H−L
	Panel A: Value-Weighted Returns					
Low	1.04	1.01	1.07	0.95	1.03	-0.01
	(4.50)	(4.33)	(4.26)	(3.69)	(3.29)	(-0.05)
2	0.93	1.03	1.06	1.06	0.78	-0.14
	(4.68)	(4.89)	(4.85)	(3.84)	(2.45)	(-0.72)
3	1.03	1.00	1.07	1.09	1.01	-0.02
	(5.91)	(4.42)	(4.74)	(3.98)	(3.97)	(-0.10)
4	0.91	1.15	1.13	1.11	1.15	0.24
	(4.71)	(5.80)	(5.12)	(4.58)	(3.27)	(0.77)
High	0.88	1.11	1.21	1.53	1.47	0.59
	(4.00)	(5.95)	(6.19)	(5.55)	(4.19)	(1.79)
H-L	-0.16	0.10	0.14	0.57	0.44	0.60
	(-0.74)	(0.46)	(0.57)	(2.50)	(1.38)	(1.70)
α_{H-L}^{CAPM}	0.02	0.30	0.31	0.67	0.50	0.48
	(0.10)	(1.34)	(1.30)	(2.87)	(1.55)	(1.40)
α_{H-L}^{FF3}	-0.48	-0.18	-0.04	0.05	-0.23	0.25
	(-2.73)	(-0.91)	(-0.16)	(0.24)	(-0.69)	(0.66)

Table IA.II continued.

	Volatility of Idiosyncratic Cash Flows					
	Low	2	3	4	High	H−L
		Panel B: Equal-Weighted Returns				
Low	1.18	1.16	1.17	1.30	1.15	-0.03
	(5.22)	(4.38)	(4.43)	(4.45)	(3.19)	(-0.13)
2	0.99	1.19	1.19	1.44	1.21	0.22
	(4.94)	(5.63)	(5.16)	(5.24)	(3.86)	(1.16)
3	1.08	1.14	1.17	1.44	1.30	0.23
	(5.94)	(5.75)	(5.16)	(6.00)	(4.40)	(1.00)
4	1.10	1.22	1.29	1.47	1.81	0.70
	(6.13)	(6.22)	(6.28)	(6.04)	(5.67)	(2.45)
High	1.03	1.28	1.49	1.54	1.98	0.95
	(5.22)	(6.26)	(6.97)	(6.10)	(4.88)	(2.69)
H-L	-0.15	0.12	0.32	0.24	0.84	0.98
	(-0.73)	(0.55)	(1.33)	(1.04)	(2.72)	(3.35)
α_{H-L}^{CAPM}	0.05	0.31	0.53	0.40	0.97	0.92
	(0.28)	(1.44)	(2.22)	(1.66)	(3.26)	(3.20)
α_{H-L}^{FF3}	-0.32	-0.09	0.15	-0.09	0.44	0.75
	(-2.21)	(-0.47)	(0.77)	(-0.39)	(1.59)	(2.64)
		Panel C: Average Book-to-Market Ratios				
Low	-1.53	-1.24	-1.18	-1.13	-1.16	0.37
2	-0.87	-0.57	-0.50	-0.42	-0.36	0.51
3	-0.52	-0.27	-0.20	-0.13	0.01	0.53
4	-0.23	-0.05	0.03	0.13	0.36	0.59
High	0.09	0.25	0.39	0.59	0.96	0.87
H-L	1.62	1.50	1.57	1.72	2.13	0.51

REFERENCES

Bhattacharya, Sudipto, 1978, Project valuation with mean-reverting cash flow streams, *Journal of Finance* 33, 1317–1331.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Irvine, Paul J., and Jeffrey Pontiff, 2009, Idiosyncratic return volatility, cash flows, and product market competition, *Review of Financial Studies* 22, 1149–1177.

Newey, Whitney K., and Kenneth D. West, 1987, A simple, positive semidefinite, heteroskedasticity and autocorrelation consistent covariance-matrix, *Econometrica* 55, 703–708.

Robel, Gregory F., 2001, Real options and mean-reverting prices, Technical report, Phantom Works Mathematics and Engineering Analyisis, The Boeing Company, 5th Annual International Real Options Conference, University of California, Los Angeles.

Zhao, Bo, 2009, Inhomogeneous geometric brownian motions, Working paper, City University of London, England.