Internet Appendix to
"Consumption Volatility Risk"

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A. Additional Theoretical Results

A.1. Wealth-Consumption Ratio Approximation

We know from Epstein and Zin (1989) that the Euler equation for an arbitrary return $R_{i,t+1}$ can be stated as

$$\mathbb{E}_t\left[\beta^\theta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{Z_{t+1}+1}{Z_t}\right)^{-(1-\theta)}R_{i,t+1}\right]=1, \tag{IA.1}$$

where $\theta=\frac{1-\gamma}{1-1/\psi}$ and $Z_t=P_t/C_t$ denotes the wealth-consumption ratio. Intuitively, the first (stochastic) term in the pricing kernel is consumption growth, C_{t+1}/C_t, and the second one is the growth rate of the wealth-consumption ratio, Z_{t+1}/Z_t.

For the pricing of the return on the consumption claim, Euler equation (IA.1) simplifies to

$$Z_t^\theta=\mathbb{E}_t\left[\beta^\theta\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}(Z_{t+1}+1)^\theta\right]. \tag{IA.2}$$

Based on the law of iterated expectations, equation (IA.2) can be written as

$$Z_t^\theta=\sum_{i=1}^{4}\xi_{t+1|t}(i)Z_{t,i}^\theta, \tag{IA.3}$$

where $\xi_{t+1|t}(i)$ is i-the element of $\xi_{t+1|t}$ and

$$Z_{t,i}^\theta=\mathbb{E}\left[\beta^\theta\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}(Z_{t+1}+1)^\theta\middle| s_{t+1}=i,\xi_{t+1|t}\right]. \tag{IA.4}$$

Equation (IA.3) says that the agent forms a belief-weighted average of the state and belief-conditioned wealth-consumption ratios (IA.4).

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The univariate effects of changing beliefs about the volatility (mean) state while holding the mean (volatility) state constant can be locally approximated. We can show that, given a constant volatility, changes in the log wealth-consumption ratio are

$$\Delta z_{t+1} \approx \Delta b_{\mu,t+1} \left(\frac{1}{\theta} \frac{Z^{\theta}_{\mu=\mu_h,\sigma} - Z^{\theta}_{\mu=\mu_l,\sigma}}{b_{\mu,t} Z^{\theta}_{\mu=\mu_h,\sigma} + (1 - b_{\mu,t})\, Z^{\theta}_{\mu=\mu_l,\sigma}} \right), \tag{IA.5}$$

where $Z_{\mu,\sigma}$ denotes the wealth-consumption ratio when expected consumption growth is μ and consumption volatility is σ. Analogously, given a constant mean, changes in the log wealth-consumption ratio are

$$\Delta z_{t+1} \approx \Delta b_{\sigma,t+1} \left(\frac{1}{\theta} \frac{Z^{\theta}_{\mu,\sigma=\sigma_h} - Z^{\theta}_{\mu,\sigma=\sigma_l}}{b_{\sigma,t} Z^{\theta}_{\mu,\sigma=\sigma_h} + (1 - b_{\sigma,t})\, Z^{\theta}_{\mu,\sigma=\sigma_l}} \right). \tag{IA.6}$$

Equations (IA.5) and (IA.6) illustrate that changes in the log wealth-consumption ratio are locally proportional to changes in beliefs. From an empirical asset pricing perspective, this finding implies that changes in beliefs are priced in the cross-section since they affect the wealth-consumption ratio.

Given equation (IA.3), the local univariate approximations (IA.5) and (IA.6) of the wealth-consumption ratio are derived as follows:

$$\begin{aligned}
\Delta z_{t+1} &= \frac{1}{\theta} \ln \left(\frac{b_{t+1} Z^{\theta}_{t+1,1} + (1 - b_{t+1}) Z^{\theta}_{t+1,2}}{b_t Z^{\theta}_{t,1} + (1 - b_t) Z^{\theta}_{t,2}} \right) \\
&= \frac{1}{\theta} \ln \left(\frac{(b_t + \Delta b_{t+1}) Z^{\theta}_{t+1,1} + (1 - (b_t + \Delta b_{t+1})) Z^{\theta}_{t+1,2}}{b_t Z^{\theta}_{t,1} + (1 - b_t) Z^{\theta}_{t,2}} \right) \\
&= \frac{1}{\theta} \ln \left(\frac{b_t PC^{\theta}_{t+1,1} + (1 - b_t) Z^{\theta}_{t+1,2} + \Delta b_{t+1} (Z^{\theta}_{t+1,1} - PC^{\theta}_{t+1,2})}{b_t Z^{\theta}_{t,1} + (1 - b_t) Z^{\theta}_{t,2}} \right) \\
&= \frac{1}{\theta} \ln \left(1 + \frac{\Delta b_{t+1} (Z^{\theta}_{t+1,1} - Z^{\theta}_{t+1,2})}{b_t Z^{\theta}_{t,1} + (1 - b_t) Z^{\theta}_{t,2}} \right) \\
&\approx \frac{1}{\theta} \Delta b_{t+1} \frac{Z^{\theta}_{1} - Z^{\theta}_{2}}{b_t Z^{\theta}_{1} + (1 - b_t) Z^{\theta}_{2}}.
\end{aligned}$$

A.2. Numerical Solution

Using equation (IA.3), the wealth-consumption ratio, $Z_t = Z(\xi_{t+1|t})$, solves the functional equation

$$Z(\xi_{t+1|t}) = \left(\sum_{i=1}^{4} \xi_{t+1|t}(i) \mathbb{E} \left[\beta^{\theta} (Z(\xi_{t+2|t+1}) + 1)^{\theta} \left(e^{\mu_i + \sigma_i \epsilon_{t+1}} \right)^{1-\gamma} \middle| \, s_{t+1} = i \right] \right)^{1/\theta},$$

where $\xi_{t+1|t}(i)$ is the i-th element of $\xi_{t+1|t}$. We solve this equation as a fixed-point in the wealth-consumption ratio. The grid for the belief state-vector has increments of size 0.025 and the expectation is approximated using Gaus-Hermite quadrature with 21 nodes. Three-dimensional linear interpolation is used between grid points.

A.3. Cross-Sectional Asset Pricing Implications

For our empirical exercise, we assume that the log wealth-consumption ratio is approximately affine in the perceived first and second moments of consumption growth,

$$z_t \approx k + A\hat{\mu}_t + B\hat{\sigma}_t.$$

This step provides a more meaningful economic interpretation for mean and volatility states. In Table IA.I, we confirm the quality of this approximation based on simulations of the model. We simulate 1,000 economies for 100 years at the quarterly frequency. In all three panels, the representative agent has an EIS of 1.5 and a rate of time preference of 0.995. The coefficient of relative risk aversion (RRA) increases from 10 (Panel A) to 20 (Panel B) and 30 (Panel C). In the first regression of each panel, we regress the log wealth-consumption ratio, z_t, on the prior probabilities of being in a given state, $\xi_{t+1,t}(i), i = 1, 2, 3$. In the second regression, we regress the log wealth-consumption ratio, z_t, on the perceived first moment, $\hat{\mu}_t$, and second moment, $\hat{\sigma}_t$, of consumption growth. We report the (across-simulation) average regression coefficient and regression R^2.

Equation (IA.3) states that variation in the wealth-consumption ratio depends on the beliefs about four states, three of which are linearly independent. In an exact implementation of the model, the wealth-consumption ratio is thus a nonlinear function of three variables. The first regression of each panel confirms that the log wealth-consumption ratio is approximately affine in the prior probabilities about the state, with the regression R^2 exceeding 99%.

The second regression of each panel confirms that the log wealth-consumption ratio is approximately affine in the perceived first and second moments of consumption growth. This approximation captures most variation in changes in the wealth-consumption ratio, with the regression R^2 exceeding 99%. Intuitively, the third prior probability captures the perceived

Table IA.I
Wealth-Consumption Ratio

We simulate 1,000 economies for 100 years at the quarterly frequency. In all three panels, the representative agent has an EIS of 1.5 and a rate of time preference of 0.995. The RRA increases from 10 (Panel A) to 20 (Panel B) and 30 (Panel C). In the first regression of each panel, we regress the log wealth-consumption ratio, pc_t, on the prior probabilities of being in a given state, $\xi_{t+1,t}(i), i = 1, 2, 3$. In the second regression, we regress the log wealth-consumption ratio, pc_t, on the perceived first moment, $\hat{\mu}_t$, and second moment, $\hat{\sigma}_t$, of consumption growth. We report the average regression coefficient and average R^2.

Const.	$\xi(1)$	$\xi(2)$	$\xi(3)$	$\hat{\mu}$	$\hat{\sigma}$	R^2
Panel A: RRA=10, EIS=1.5						
5.7345	−0.0050	−0.0077	0.0017			0.9973
5.7294				0.0069	−0.0022	0.9958
Panel B: RRA=20, EIS=1.5						
5.7116	−0.0044	−0.0075	0.0022			0.9959
5.7071				0.0068	−0.0027	0.9945
Panel C: RRA=30, EIS=1.5						
5.6899	−0.0037	−0.0073	0.0028			0.9938
5.6861				0.0066	−0.0032	0.9926

comovement between the Markov chains for mean and volatility. However, since these two Markov chains are independent by assumption, the third prior probability is redundant.

To test the model in the cross-section of returns, it is convenient to restate the fundamental asset pricing equation (IA.1) in terms of betas,

$$\begin{aligned}
\mathbb{E}_t[R^e_{i,t+1}] &\approx -\mathrm{Cov}_t(R_{i,t+1}, m_{t+1}) \\
&= \gamma \mathrm{Cov}_t(R_{i,t+1}, \Delta c_{t+1}) + (1-\theta)\mathrm{Cov}_t(R_{i,t+1}, \Delta z_{t+1}) \\
&= \gamma \mathrm{Cov}_t(R_{i,t+1}, \Delta c_{t+1}) + (1-\theta)A\mathrm{Cov}_t(R_{i,t+1}, \Delta\hat{\mu}_{t+1}) + (1-\theta)B\mathrm{Cov}_t(R_{i,t+1}, \Delta\hat{\sigma}_{t+1}) \\
&= \beta^i_{c,t}\lambda_{c,t} + \beta^i_{\mu,t}\lambda_{\mu,t} + \beta^i_{\sigma,t}\lambda_{\sigma,t},
\end{aligned}$$

with

$$\beta_{c,t} = \frac{\mathrm{Cov}_t(R_{i,t+1}, \Delta c_{t+1})}{\mathrm{Var}_t(\Delta c_{t+1})} \quad \beta_{\mu,t} = \frac{\mathrm{Cov}_t(R_{i,t+1}, \Delta\hat{\mu}_{t+1})}{\mathrm{Var}_t(\Delta\hat{\mu}_{t+1})} \quad \beta_{\sigma,t} = \frac{\mathrm{Cov}_t(R_{i,t+1}, \Delta\hat{\sigma}_{t+1})}{\mathrm{Var}_t(\Delta\hat{\sigma}_{t+1})}$$

and

$$\lambda_{c,t} = \gamma \mathrm{Var}_t(\Delta c_{t+1}) \qquad \lambda_{\mu,t} = A(1-\theta)\mathrm{Var}_t(\Delta\hat{\mu}_{t+1}) \qquad \lambda_{\sigma,t} = B(1-\theta)\mathrm{Var}_t(\Delta\hat{\sigma}_{t+1}),$$

where $\beta^i_{c,t}, \beta^i_{\mu,t}, \beta^i_{\sigma,t}$ denote risk loadings of asset i at date t with respect to consumption growth and the conditional first and second moments of consumption growth, and $\lambda_{c,t}, \lambda_{\mu,t}, \lambda_{\sigma,t}$ are the respective market prices of risk.

A.4. Equity Premium

To quantify the equity premium generated by our model, we first have to specify a process for dividend growth. A common approach is to postulate a levered consumption process for dividends such as $D = C^\lambda$. The Markov switching model allows a more general approach by fitting a Markov model for the conditional first and second moments of dividend growth. Specifically, we assume that log dividend growth follows

$$\Delta d_{t+1} = \mu^d_t + \sigma^d_t \epsilon_{t+1} \qquad \epsilon_{t+1} \sim \mathcal{N}(0,1),$$

where $\mu^d_t \in \{\mu^d_l, \mu^d_h\}$ and $\sigma^d_t \in \{\sigma^d_l, \sigma^d_h\}$ follow the same Markov process as consumption. Consequently, we do not reestimate the transition matrix of the Markov process but use the

Table IA.II
Markov Model of Dividend Growth

This table reports parameter estimates of the Markov model for log dividend growth

$$\Delta d_{t+1} = \mu_t^d + \sigma_t^d \epsilon_{t+1}, \qquad \epsilon_t \sim \mathcal{N}(0,1),$$

where $\mu_t^d \in \{\mu_l^d, \mu_h^d\}$ and $\sigma_t^d \in \{\sigma_l^d, \sigma_h^d\}$ follow independent Markov processes with transition matrices P^μ and P^σ, respectively. The consumption and dividend processes follow the same Markov switching process as reported in Table I. We compute quarterly dividends for the period 1955 to 2009 using the value-weighted CRSP index with and without distributions. Standard errors are reported in parentheses.

μ_l	μ_h	σ_l	σ_h
−0.5968	1.3969	1.2983	3.3912
(0.2355)	(0.3164)	(0.1015)	(0.3901)

estimates reported in Table I. We compute quarterly dividends for the period 1955 to 2009 using the value-weighted CRSP index with and without distributions. Parameter estimates are summarized in Table IA.II.

In Table IA.III, we report statistics about the risky and risk-free assets. We simulate 1,000 economies for 100 years at the quarterly frequency. In all three panels, the representative agent has an EIS of 1.5 and rate of time preference of 0.995. The RRA increases from 10 (Panel A) to 20 (Panel B) and 30 (Panel C). We report the average excess return, $\mathbb{E}[R^e]$, the standard deviation of stock returns, $\sigma[R]$, the average risk-free rate, $\mathbb{E}[R^f]$, and the standard deviation of the risk-free rate, $\sigma[R^f]$. In the last two rows of each panel, we also report moments of the Markov switching model without learning, where the agent knows the state of the economy.

In the specification with RRA of 10, the model generates an annual risk premium of 1.3%, stock return volatility of 7.7%, average risk-free rate of 2.8%, and risk-free rate volatility of 0.5%. This poor performance is not surprising since the Markov chain is not very persistent compared to the specification of Bansal and Yaron (2004). For an RRA of 30, the model generates a risk premium of 4.6%.

Table IA.III can also be used to quantify the importance of learning. In the last two rows

Table IA.III
Model Implications

We simulate 1,000 economies for 100 years at the quarterly frequency. In all three panels, the representative agent has an EIS of 1.5 and a rate of time preference of 0.995. The RRA increases from 10 (Panel A) to 20 (Panel B) and 30 (Panel C). We report the average excess return, $\mathbb{E}[R^e]$, the standard deviation of stock returns, $\sigma[R]$, the average risk-free rate, $\mathbb{E}[R^f]$, and the standard deviation of the risk-free rate, $\sigma[R^f]$. In the last row of each panel, we also report moments of the Markov switching model without learning, where the agent knows the state of the economy.

	$\mathbb{E}[R^e]$	$\sigma[R^e]$	$\mathbb{E}[R^f]$	$\sigma[R^f]$
Panel A: RRA=10, EIS=1.5				
Mean	0.0129	0.0772	0.0278	0.0046
10%	0.0038	0.0702	0.0257	0.0039
90%	0.0222	0.0845	0.0298	0.0054
No Learning	0.0096	0.0732	0.0340	0.0024
Power Utility	0.0029	0.0610	0.2404	0.0376
Panel B: RRA=20, EIS=1.5				
Mean	0.0296	0.0758	0.0203	0.0085
10%	0.0208	0.0686	0.0163	0.0071
90%	0.0386	0.0829	0.0239	0.0099
No Learning	0.0190	0.0715	0.0334	0.0024
Power Utility	0.0062	0.1261	0.4696	0.0793
Panel C: RRA=30, EIS=1.5				
Mean	0.0461	0.0739	0.0126	0.0124
10%	0.0367	0.0671	0.0070	0.0104
90%	0.0550	0.0807	0.0179	0.0143
No Learning	0.0275	0.0690	0.0328	0.0023
Power Utility	0.0098	0.1801	0.7080	0.1253

of each panel, we report moments of the Markov switching model without learning, where the agent knows the state of the economy. The difference between the mean excess return generated by the full model and the model without learning is the learning premium. Holding the EIS fixed at 1.5, for an RRA of 10 (Panel A), the learning premium is only 33 basis points; for an RRA of 20 (Panel B), the learning premium increases to 1%; and for an RRA of 30 (Panel C), the learning premium reaches 1.9%. So the fraction of the total excess return coming from learning increases from 26% to 36% to 40%.

A.5. Consumption CAPM

While the theoretical motivation for our additional factors is easily established, it is not obvious whether they are relevant empirically. It is possible that the additional factors are strongly correlated with consumption growth, so that empirically a one-factor model holds, albeit with a coefficient that represents two prices of risk.

To address this question, we repeatedly simulate economies following our estimated consumption dynamics. Not surprisingly, realized consumption growth is strongly linked to beliefs about the mean state, with correlations exceeding 60%. The unconditional correlation between realized consumption growth and beliefs about consumption growth volatility is close to zero. Consequently, consumption growth might subsume some of the risk coming from expected consumption growth in regressions of the consumption CAPM.

To test how well the consumption CAPM works on simulated data of the model, we assume an RRA of 10 and an EIS of 1.5, as in Bansal and Yaron (2004). We gauge the magnitude of mispricing in Table IA.IV, where we simulate 1,000 economies for 100 years and report the cross-simulation average as well as the 10% and 90% cross-simulation quantile of two-stage regressions. In particular, for each simulation we compute the return on wealth, the aggregate dividend return, and the risk-free rate. We then first regress the returns of these three test assets on consumption growth, and in the second stage expected returns on first-stage betas. In the table, we report the second-stage estimates, in particular, the intercept, the market price of consumption growth risk, λ_c, the regression R^2, and the mean absolute pricing error (MAPE).

Table IA.IV
Consumption CAPM

We simulate 1,000 economies for 100 years. We assume that the representative agent has an EIS of 1.5, an RRA of 10, and a rate of time preference of 0.995. In particular, for each simulation we compute the return on wealth, the aggregate dividend return, and the risk-free rate. We then regress the returns of these three test assets on consumption growth, and in the second stage expected returns on first-stage betas. We report the second-stage estimates, specifically the intercept, the market price of consumption growth risk, λ_c, the regression R^2, and the MAPE. In the last column, we report the average conditional pricing error on the aggregate dividend return

	Const.	λ_c	R^2	MAPE	α_d
Mean	0.0076	0.0004	0.6248	0.0005	0.0019
10%	0.0073	−0.0000	0.0622	0.0004	0.0017
90%	0.0079	0.0007	0.9517	0.0006	0.0021

Even though a model with power utility generates an annual risk premium of only 0.29% compared to 1.29% in our full model (see Table IA.III), the second-stage regressions explain around 62% of the cross-sectional variation in average returns. The reason is the correlation between consumption growth and beliefs about the mean growth rate, which falsely attributes some of the premium for long-run risk to the (short-run) consumption growth factor. Importantly for our empirical exercise, the fit is far from perfect and the MAPE is five bp quarterly compared to an average asset return of roughly 80 bp.

In the last column, we report the average conditional pricing error on the aggregate dividend return from

$$\alpha_d = \mathbb{E}^*[R] - \mathbb{E}[R] = \frac{\text{Cov}(M - M^*, R)}{\mathbb{E}[M]},$$

where M is the correct pricing kernel based on Epstein-Zin preferences and M^* is the misspecified pricing kernel based on power utility. This calculation follows Campbell and Cochrane (2000). Conditionally, pricing errors are large, that is, more than half of the true risk premium.

B. Additional Empirical Results

B.1. Alternative Estimation Specifications

To illustrate the robustness of our estimation approach, we also provide results for two alternative specifications, both of which explicitly model service or nondurable consumption as instruments while estimating the dynamics for total consumption growth directly. The following table summarizes the approaches:

Method	Information set
1	Total consumption, Δc
2	Service consumption and service consumption share, Δs & Δv
3	Total consumption and service consumption, Δc & Δs
4	Total consumption and nondurable consumption, Δc & Δn

Method 1 is the standard approach, where the information set only contains total consumption growth. We refer to the use of only the time series of total consumption in the estimation of the Markov process, following equation (1), as the *single series estimation*. Method 2 is our main specification, where the agent observes service consumption growth and changes in the service-consumption share. In methods 3 and 4, the agent not only observes total consumption growth but also service or nondurable consumption growth, respectively. We refer to methods 2 to 4 as *component estimation*.

B.2. Statistical Evidence

The goal of this section is to elaborate on the empirical validity of the inclusion of components of total consumption in the representative agent's information set. We justify our procedure in two steps. First, we assume a joint Markov process for service consumption growth and service consumption share as in equation (11) and show in simulations that all the component methods increase estimation efficiency relative to the single-series approach. Second, we show that the assumptions underlying the component models, namely, simultaneous Markov switching in both components, are supported in the data.

Table IA.V
Markov Chain Parameter Estimates

This table reports parameters and transition probabilities from a Markov model for consumption growth with two conditional mean and volatility states. Method 1 uses only total consumption growth (Δc), method 2 uses service consumption growth and changes in the service consumption share (Δs and Δv), method 3 uses total and service consumption growth (Δc and Δs) and method 4 uses total and nondurable consumption growth (Δc and Δn). The data cover 1964 to 2009.

μ_l	μ_h	σ_l	σ_h	p^{μ}_{ll}	p^{μ}_{hh}	p^{σ}_{ll}	p^{σ}_{hh}
Method 1 (Δc)							
0.360	0.793	0.214	0.489	92.352	87.763	94.565	95.864
(0.040)	(0.062)	(0.029)	(0.059)	(3.566)	(6.131)	(4.173)	(5.367)
Method 2 (Δs and Δv)							
0.350	0.706	0.204	0.477	92.052	92.791	94.103	97.277
(0.033)	(0.054)	(0.020)	(0.036)	(4.857)	(5.109)	(3.307)	(1.663)
Method 3 (Δc and Δs)							
0.351	0.706	0.206	0.475	92.179	92.922	94.106	97.297
(0.034)	(0.054)	(0.019)	(0.035)	(4.716)	(4.959)	(3.320)	(1.648)
Method 4 (Δc and Δn)							
0.338	0.716	0.208	0.473	90.631	90.363	91.834	95.438
(0.033)	(0.045)	(0.023)	(0.035)	(4.719)	(4.743)	(4.135)	(2.752)

Table IA.VI
Correlations of Beliefs

This table reports the correlations between beliefs over mean states (Panel A) and volatility states (Panel B) from a Markov model for consumption growth with two conditional mean and volatility states. Method 1 uses only total consumption growth (Δc), method 2 uses service consumption growth and changes in the service consumption share (Δs and Δv), method 3 uses total and service consumption growth (Δc and Δs), and method 4 uses total and nondurable consumption growth (Δc and Δn). The data cover 1964 to 2009.

Method	1	2	3	4
Panel A: Beliefs about the Mean				
1	1.0000			
2	0.8426	1.0000		
3	0.8413	0.9999	1.0000	
4	0.8879	0.9846	0.9831	1.0000
Panel A: Beliefs about the Volatility				
1	1.0000			
2	0.7928	1.0000		
3	0.7860	0.9995	1.0000	
4	0.8078	0.9423	0.9373	1.0000

To this end, we first estimate methods 1 to 4 on post-war real quarterly consumption data. The results for the dynamics of total consumption growth are presented in Table IA.V. We obtain standard errors for method 2 from the delta method. Two observations are important. First, most parameter estimates are very similar across methods. Second, all but the standard error for p_{ll}^{μ} are reduced by using component estimation methods 2 to 4 relative to single-series method 1. In Table IA.VI, the pairwise correlation of filtered beliefs about the mean state (Panel A) and volatility state (Panel B) further indicate that all methods yield comparable results. Beliefs based on the single-series estimation have correlations between 0.79 and 0.89 with the beliefs estimated using components. For the three component methods, all pairwise correlations exceed 93%.

Figure IA.1 shows the filtered beliefs of the consumption volatility state of the four methods. The high correlations are easily observable. At the same time, the single-series estimation yields considerably noisier beliefs than the component methods. This is especially true in the

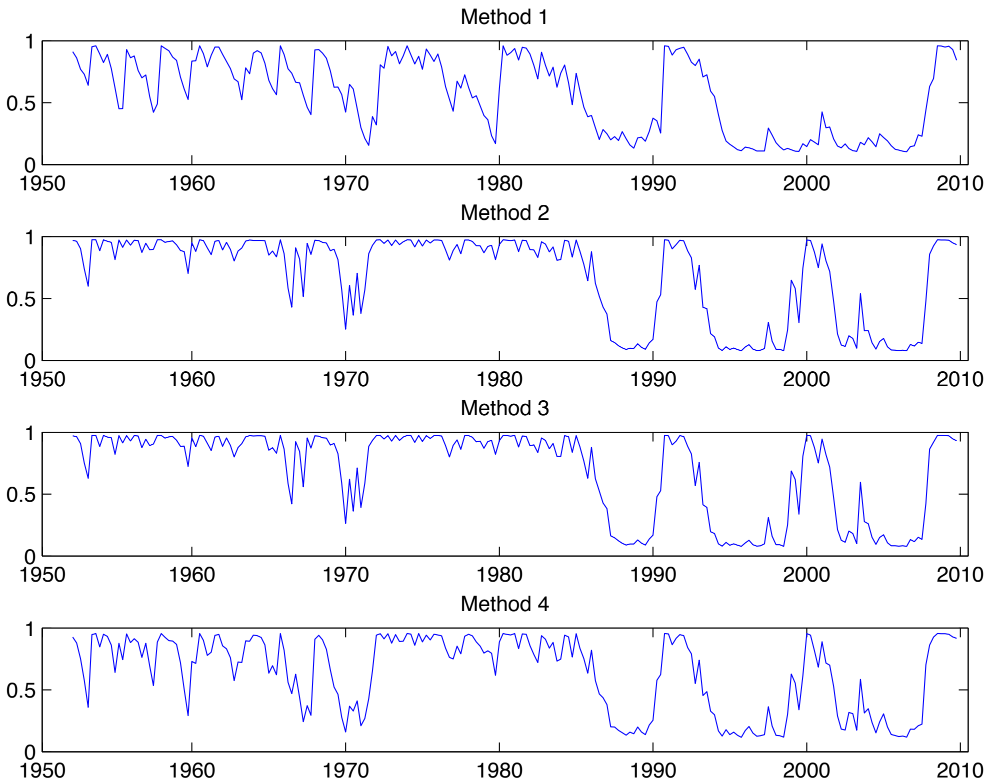

Figure IA.1. Bayesian beliefs about the volatility state. This figure displays the estimated Bayesian belief processes for being in the high volatility state for the four different estimation methods for consumption dynamics. The estimation procedure follows Hamilton (1994). We use quarterly per capita real consumption expenditure for non-durable goods and services for the years 1952.Q1 to 2009.Q4.

early part of the sample, and we thus expect lower power for asset pricing tests in the early sample using the single-series method.

To illustrate the increased estimation efficiency of the component methods, we start by assuming a joint Markov process for service consumption growth and the service consumption share as in equation (11). Under this assumption, it is intuitive that aggregating the two components and using only the time series of total consumption growth in the estimation of the Markov process in equation (1) constitutes an *inefficient* use of the data. We confirm in simulations that using the information contained in the two consumption components is valuable and results in more precise estimates.

Table IA.VII shows the results from 2,000 simulations. Each simulation represents an economy of 231 quarters as in the data. The economy is described by equation (11) with population parameters based on the empirical point estimates from our main specification (method 2 in Table IA.V). We report the mean and standard deviation of the estimates as well as the median standard error across simulations. Method 1 uses only total consumption growth (Δc), method 2 service consumption growth and changes in the service consumption share (Δs and Δv), method 3 total and service consumption growth (Δc and Δs), and method 4 total and nondurable consumption growth (Δc and Δn).

Comparing the mean estimates across methods confirms that the point estimates are largely unaffected by the estimation approach. The minor variations are well within what should be expected statistically. In contrast, standard deviations and standard errors are dramatically reduced in component estimation methods 2 to 4 compared to single-series method 1. At the same time, there are no systematic differences across the three component methods in terms of estimation precision.

While this simulation evidence is intuitive, it does not address the broader concern of whether the consumption components (service and nondurable expenditures) are indeed jointly Markov switching. We now show that (*i*) means and volatilities of both service and nondurable consumption growth are well described by a Markov model, (*ii*) means and volatilities of the two series switch simultaneously, and (*iii*) the component estimation does result in a good

Table IA.VII
Simulation Evidence of the Joint Markov Model for Consumption Growth

This table presents simulation evidence of the joint Markov model for service consumption growth and service consumption share as in equation (11). We simulate 2,000 economies for 231 quarters as in the data. We report the mean and standard deviation of estimates as well as the median standard error across simulations. Method 1 uses only total consumption growth (Δc), method 2 uses service consumption growth and changes in the service consumption share (Δs and Δv), method 3 uses total and service consumption growth (Δc and Δs), and method 4 uses total and nondurable consumption growth (Δc and Δn).

	μ_l	μ_h	σ_l	σ_h	p^{μ}_{ll}	p^{μ}_{hh}	p^{σ}_{ll}	p^{σ}_{hh}
Population	0.350	0.706	0.204	0.477	92.052	92.791	94.103	97.277
Method 1 (Δc)								
Mean Est	0.335	0.721	0.212	0.519	88.943	89.864	91.298	94.386
Std Est	0.080	0.077	0.051	0.054	9.227	8.859	9.986	8.693
Median SE	0.045	0.043	0.027	0.040	5.198	4.848	4.019	2.558
Method 2 (Δs and Δv)								
Mean Est	0.349	0.711	0.221	0.512	90.745	91.475	92.983	96.127
Std Est	0.048	0.044	0.034	0.036	5.694	5.063	5.183	4.404
Median SE	0.037	0.035	0.019	0.032	3.837	3.627	3.303	1.920
Method 3 (Δc and Δs)								
Mean Est	0.350	0.709	0.224	0.509	90.735	91.426	92.682	95.997
Std Est	0.053	0.047	0.037	0.035	5.987	5.669	7.211	4.737
Median SE	0.036	0.034	0.019	0.031	3.816	3.605	3.315	1.920
Method 4 (Δc and Δn)								
Mean Est	0.349	0.712	0.221	0.513	90.726	91.428	92.609	96.043
Std Est	0.049	0.046	0.039	0.037	5.821	5.276	6.030	3.464
Median SE	0.036	0.035	0.019	0.032	3.813	3.640	3.354	1.977

overall fit of the data.

To test for joint switching of the conditional moments of service and nondurable consumption growth, we assume the joint Markov dynamics:

$$\Delta s_{t+1} = \mu_t^s + \sigma_t^s \epsilon_{t+1}^s \qquad \Delta n_{t+1} = \mu_t^n + \sigma_t^n \epsilon_{t+1}^n,$$

where Δs denotes log service consumption growth and Δn log nondurable consumption growth. For $i \in \{s, n\}$, μ_t^i denotes the conditional expectation, σ_t^i denotes the conditional standard deviation, and ϵ_{t+1}^i is standard normal with $\mathrm{Cov}_t(\epsilon_{t+1}^s, \epsilon_{t+1}^n) = \varrho_{sn}$.

Our empirical model has four states for each series, two for the conditional mean and two for the conditional volatility. A test for joint switching of both moments across the two consumption components would thus require the estimation of a 16×16 Markov transition matrix for the unrestricted case. This is not feasible given the relatively short time series for consumption data. Consequently, we reduce the dimensionality of the problem and first test for joint switching in the conditional mean in nondurable and service consumption growth, holding the volatility state constant:

$$\Delta s_{t+1} = \mu_t^s + \sigma^s \epsilon_{t+1}^s \qquad \Delta n_{t+1} = \mu_t^n + \sigma^n \epsilon_{t+1}^n.$$

In Panel A of Table IA.VIII, we present parameter estimates from the unrestricted model that allows the mean states of both components to switch freely. In other words, one series can be in state 1 while the other one is in state 2. The estimates in the transition matrix already suggest that the states "12" and "21" are not very persistent. We estimate the restricted model in which the conditional means of the two series switch jointly in Panel B. In Panel C, we perform a likelihood ratio test on the parameter restrictions. Under the null hypothesis, the test statistic of 5.28 is χ^2-distributed with 10 degrees of freedom. The 10% critical value is 16.00, suggesting that the restriction of joint switching does not statistically reduce the overall model fit.

In Table IA.IX, we test for joint switching in the conditional volatility assuming a constant mean:

$$\Delta s_{t+1} = \mu^s + \sigma_t^s \epsilon_{t+1}^s \qquad \Delta n_{t+1} = \mu^n + \sigma_t^n \epsilon_{t+1}^n.$$

Table IA.VIII
Test for Joint Switching in the Mean

This table reports parameter estimates and transition probabilities for a joint Markov model for service (Δs) and nondurable log consumption growth (Δn) with two states for each conditional mean:

$$\Delta s_{t+1} = \mu_t^s + \sigma^s \epsilon_{t+1}^s \qquad \Delta n_{t+1} = \mu_t^n + \sigma^n \epsilon_{t+1}^n,$$

where for $i \in \{s, n\}$, μ_t^i denotes the conditional expectation, σ^i denotes the constant standard deviation, and ϵ_{t+1}^i is standard normal with $\mathrm{Cov}_t(\epsilon_{t+1}^s, \epsilon_{t+1}^n) = \varrho_{sn}$. In the unrestricted model (Panel A), the states for the expected growth rates of the two consumption components can switch separately, while the restriction in Panel B enforces states to switch jointly. Panel C reports estimated values of the likelihood function and performs a likelihood ratio test on the restrictions. The data cover 1964 to 2009.

Panel A: Unrestricted Model

Parameter Estimates

μ_1^s	μ_2^s	μ_1^n	μ_2^n	σ^s	σ^n	ϱ_{sn}
−0.08	0.69	−0.40	0.51	0.38	0.67	0.30
(0.09)	(0.03)	(0.25)	(0.06)	(0.02)	(0.04)	(0.07)

Transition Matrix

	11	22	12	21
11	0.74	0.00	0.00	0.26
	(0.36)	(0.24)	(0.34)	
22	0.04	0.96	0.00	0.00
	(0.02)	(0.02)	(0.00)	
12	0.57	0.00	0.00	0.43
	(0.82)	(0.28)	(0.40)	
21	0.00	0.56	0.21	0.23
	(0.10)	(0.28)	(0.20)	

Panel B: Joint Switching Model

Parameter Estimates

μ_1^s	μ_2^s	μ_1^n	μ_2^n	σ^s	σ^n	ϱ_{sn}
0.28	0.83	0.08	0.57	0.37	0.71	0.31
(0.05)	(0.04)	(0.08)	(0.07)	(0.02)	(0.03)	(0.07)

Transition Matrix

	11	22
11	0.91	0.09
	(0.04)	
22	0.08	0.92
		(0.03)

Panel C: Test on Parameter Restrictions

$Unrest.LL$	$Rest.LL$	$\chi^2 - Stat$	10%$-Crit.$	$p - Value$
1764.18	1761.54	5.28	16.00	0.87

Table IA.IX
Test for Joint Switching in the Volatility

This table reports parameter estimates and transition probabilities for a joint Markov model for service (Δs) and nondurable log consumption growth (Δn) with two states for each conditional volatility:

$$\Delta s_{t+1} = \mu^s + \sigma_t^s \epsilon_{t+1}^s \qquad \Delta n_{t+1} = \mu^n + \sigma_t^n \epsilon_{t+1}^n,$$

where for $i \in \{s, n\}$, μ^i denotes the constant expectation, σ_t^i denotes the conditional standard deviation, and ϵ_{t+1}^i is standard normal with $\text{Cov}_t(\epsilon_{t+1}^s, \epsilon_{t+1}^n) = \varrho_{sn}$. In the unrestricted model (Panel A), the states for growth rate volatility of the two consumption components can switch separately, while the restriction in Panel B enforces states to switch jointly. Panel C reports estimated values of the likelihood function and performs a likelihood ratio test on the restrictions. The data cover 1964 to 2009.

Panel A: Unrestricted Model

Parameter Estimates

μ^s	μ^n	σ_1^s	σ_2^s	σ_1^n	σ_2^n	ϱ_{sn}
0.56	0.38	0.27	0.51	0.38	0.94	0.39
(0.03)	(0.04)	(0.02)	(0.03)	(0.01)	(0.07)	(0.06)

Transition Matrix

	11	22	12	21
11	0.92	0.00	0.08	0.00
	(0.04)	(0.17)	(0.06)	
22	0.04	0.68	0.00	0.28
	(0.04)	(0.15)	(0.09)	
12	0.00	0.00	0.23	0.77
	(0.07)	(0.98)	(0.15)	
21	0.00	0.88	0.00	0.12
	(0.03)	(0.31)	(0.23)	

Panel B: Joint Switching Model

Parameter Estimates

μ^s	μ^n	σ_1^s	σ_2^s	σ_1^n	σ_2^n	ϱ_{sn}
0.56	0.38	0.28	0.52	0.42	0.86	0.38
(0.03)	(0.04)	(0.04)	(0.03)	(0.05)	(0.06)	(0.06)

Transition Matrix

	11	22
11	0.91	0.09
	(0.05)	
22	0.04	0.96
		(0.03)

Panel C: Test on Parameter Restrictions

$Unrest.LL$	$Rest.LL$	$\chi^2 - Stat$	10%$-Crit.$	$p - Value$
1759.26	1756.08	6.38	16.00	0.78

We estimate the unrestricted model in Panel A, the restricted model in Panel B, and perform a likelihood ratio test on the parameter restrictions in Panel C. Similar to the results of Table IA.VIII, the p-value of 0.78 suggests that the restricted model of joint volatility switching cannot be rejected.

B.3. Robustness of Cross-Sectional Asset Pricing

The previous section shows that the component-based consumption specifications dominate the single-series approach econometrically. In this section, we turn our attention to the asset pricing implications. The aim is to show that the pricing of consumption volatility risk is generally robust to different specifications of the consumption process. To this end, we repeat the exercises of Tables II and IV in the paper using different specifications for the Markov chain for consumption growth.

Table IA.X shows estimated prices of risk from the second-pass regressions of average excess returns on risk loadings estimated in the first pass as well as the second-pass R^2 and MAPE. The test assets are the 25 size-value portfolios (Panel A), 40 industry-value portfolios (Panel B), and 25 net share issuance-size portfolios (Panel C). Consumption dynamic estimates are based only on total consumption growth (method 1).

This table corresponds to Table II in the paper, which uses service consumption growth and changes in the service consumption share (Δs and Δv). The coefficients on consumption volatility exposure are all strongly negative, albeit not always statistically significant when the whole sample (1964 to 2009) is used. In accordance with the observation that the single-series estimation yields very noisy volatility estimates in the early sample, we repeat the analysis on a 35-year subsample starting in 1975. The coefficients are generally similar on the subsample, but now all are statistically significant.

Volatility risk pricing across the different component estimation methods (2 to 4) is summarized in Table IA.XI. This table repeats the analysis from Table II in the paper (method 2), and shows that the results using methods 3 (Δc and Δs) and 4 (Δc and Δn) are very similar.

Table IA.XII shows average equally weighted (EW) and value-weighted (VW) monthly

Table IA.X
Volatility Risk Pricing Based on Single-Series Estimation

This table reports market prices of risk from cross-sectional regressions of average excess returns on estimated factor loadings. The factors considered are consumption growth, Δc_t, and changes in beliefs about the first moment, $\Delta\hat{\mu}_t$, and second moment, $\Delta\hat{\sigma}_t$, of consumption growth. Consumption dynamics are estimated using the single-series approach (method 1). In Panel A the test assets are the 25 size-value portfolios, in Panel B 40 industry-value portfolios, and in Panel C 25 net share issuance-size portfolios. For each specification, we report the estimated prices of risk, the R^2, and the MAPE. t-statistics are corrected for estimation error in the first stage as proposed by Shanken (1992) and Newey and West (1987) adjusted using four lags. The sample period starts in 1964 or 1975, and ends in 2009.

	Δc (t-stat)	$\Delta\hat{\mu}$ (t-stat)	$\Delta\hat{\sigma}$ (t-stat)	R^2	MAPE
Panel A: 25 Value-Size Portfolios					
1964–2009	0.21	0.02	−0.13	80.51	1.11
	(0.14)	(0.07)	(−1.31)		
1975–2009	0.30	0.01	−0.07	73.52	1.36
	(0.52)	(0.13)	(−2.15)		
Panel B: 40 Industry-Value Portfolios					
1964–2009	0.42	−0.03	−0.07	58.35	1.96
	(0.75)	(−0.67)	(−2.20)		
1975–2009	0.61	−0.03	−0.06	61.48	2.11
	(1.19)	(−0.45)	(−2.36)		
Panel C: 25 NSI-Size Portfolios					
1964–2009	0.09	0.13	−0.09	54.60	2.67
	(0.07)	(0.56)	(−1.58)		
1975–2009	0.22	0.03	−0.09	54.63	2.99
	(0.36)	(0.27)	(−1.90)		

Table IA.XI
Volatility Risk Pricing Based on Component Estimation

This table reports market prices of risk from cross-sectional regressions of average excess returns on estimated factor loadings. The factors considered are consumption growth, Δc_t, and changes in beliefs about the first moment, $\Delta\hat{\mu}_t$, and second moment, $\Delta\hat{\sigma}_t$, of consumption growth. Consumption dynamics are estimated using the component approaches based on Δs and Δv (method 2), Δc and Δs (method 3), and Δc and Δn (method 4). In Panel A the test assets are the 25 size-value portfolios, in Panel B 40 industry-value portfolios, and in Panel C 25 net share issuance-size portfolios. For each specification, we report the estimated prices of risk, the R^2, and the MAPE. t-statistics are corrected for estimation error in the first stage as proposed by Shanken (1992) and Newey and West (1987) adjusted using four lags. The data cover 1964 to 2009.

Method	Δc (t-stat)	$\Delta\hat{\mu}$ (t-stat)	$\Delta\hat{\sigma}$ (t-stat)	R^2	MAPE
Panel A: 25 Value-Size Portfolios					
2	1.24	0.12	−0.06	80.55	1.16
	(0.71)	(0.55)	(−2.23)		
3	1.36	0.10	−0.06	79.94	1.19
	(0.72)	(0.58)	(−2.14)		
4	0.74	0.03	−0.09	79.38	1.13
	(0.58)	(0.18)	(−2.22)		
Panel B: 40 Industry-Value Portfolios					
2	0.70	−0.03	−0.04	50.94	2.10
	(1.54)	(−0.56)	(−2.04)		
3	0.71	−0.02	−0.04	50.58	2.12
	(1.57)	(−0.47)	(−2.07)		
4	0.56	−0.03	−0.05	59.37	1.96
	(1.22)	(−0.70)	(−2.12)		
Panel C: 25 NSI-Size Portfolios					
2	0.68	0.10	−0.09	42.92	3.10
	(0.50)	(0.41)	(−2.14)		
3	0.72	0.08	−0.09	37.89	3.27
	(0.55)	(0.46)	(−2.36)		
4	0.18	0.07	−0.12	34.98	3.16
	(0.12)	(0.29)	(−1.15)		

Table IA.XII
Portfolios Formed on Consumption Volatility Exposure

This table reports average equally weighted (EW) and value-weighted (VW) monthly returns in percent of portfolios based on estimated consumption volatility loadings. The loadings are obtained from 10-year rolling time-series regressions of individual excess returns on log consumption growth, changes in the perceived conditional mean, and changes in the perceived conditional volatility of consumption growth using quarterly data. We then form five portfolios based on the estimated consumption volatility exposure and hold the investments for one year. The column "High−Low" shows returns of a zero investment portfolio that is long in the high exposure portfolio and short in the low exposure portfolio. t-statistics are based on Newey and West (1987) adjusted standard errors using 12 lags. Panel A repeats the results from the paper using the component estimation (Panel C of Table IV). Panel B shows the portfolio returns if consumption dynamics are estimated using the single-series approach (method 1). For both Panels A and B, the sample period is January 1964 to December 2010. Panel C repeats the analysis in Panel B, but using a subsample from January 1975 to December 2010.

Panel A: Component Estimation (1964–2010)						
	Low		Med		High	High−Low
EW	1.50	1.29	1.14	1.12	1.00	−0.50
	(5.79)	(5.86)	(5.48)	(5.58)	(4.68)	(−4.30)
VW	1.27	0.99	0.88	0.85	0.70	−0.57
	(5.29)	(4.96)	(4.87)	(4.45)	(3.36)	(−3.63)

Panel B: Single-Series Estimation (1964–2010)						
	Low		Med		High	High−Low
EW	1.35	1.24	1.14	1.13	1.13	−0.22
	(5.23)	(5.90)	(5.75)	(5.89)	(5.06)	(−2.07)
VW	1.15	0.92	0.90	0.87	0.94	−0.21
	(4.83)	(4.36)	(4.89)	(4.84)	(4.61)	(−1.38)

Panel C: Single-Series Estimation (1975–2010)						
	Low		Med		High	High−Low
EW	1.66	1.50	1.36	1.35	1.27	−0.39
	(5.62)	(6.12)	(5.85)	(6.15)	(4.93)	(−3.22)
VW	1.46	1.13	1.09	1.07	1.04	−0.42
	(5.45)	(4.42)	(5.16)	(5.22)	(4.34)	(−2.30)

returns in percent of quintile portfolios based on estimated consumption volatility loadings. This corresponds to Panel C of Table IV in the paper, which is also repeated in Panel A. The sorts in Panels B and C are based on consumption dynamics estimated using the single-series approach (method 1), and differ in their sample length: Panel B presents results for the entire sample, while Panel C uses the shortened sample starting in 1975.

In all specifications, average returns decrease in consumption volatility exposure. The magnitude, however, varies somewhat across methods. The estimated difference between the high and low consumption volatility portfolios is about 50 bp per month in the component estimation. The single-series approach over the whole sample yields 21 to 22 bp per month, with an insignificant estimate for value-weighted portfolios. In the shortened sample, return differences are around 40 bp and highly significant. This overall evidence is consistent with large observed noise in the early period of the single-series consumption volatility estimation. If consumption dynamics are measured with less noise (component estimation), or if the early part of the sample is excluded, the results are much stronger.

B.4. Relation to Coskewness Risk

Going back to Kraus and Litzenberger (1976), a coskewness premium rewards agents with a preference for skewed portfolio returns when asset payoffs are nonlinear in the market return. This premium can arise in a setting with normally distributed market returns that are i.i.d. over time. In contrast, a volatility risk premium requires recursive preferences and *persistent* shocks to aggregate uncertainty.

Since coskewness risk is estimated as covariance with squared market returns, it seems reasonable to assume that it is related to volatility exposure. In Table IA.XIII, we test for a possible relation between our consumption volatility risk factor and coskewness. In particular, we regress the CVR factor on the Fama-French and momentum factors augmented by the squared market return, similar to Table VII, Panel C in the paper. It is important to note that the intercepts in this case cannot be interpreted as performance measures as the square of market returns is not a portfolio return but a payoff. We first observe that the estimated coefficients for coskewness are statistically insignificant. Moreover, CVR loads negatively on

Table IA.XIII
Consumption Volatility Risk and Coskewness

This table extends Panel C of Table VII and reports coefficients from time-series regressions of the CVR portfolio on market, size, value, and momentum factors as well as the squared market return. The CVR portfolio is the return of holding a long position in the value-weighted quintile of stocks with high volatility risk ($\beta^i_{\sigma,t}$) and a short position in low volatility risk, as reported in Panel C of Table IV. t-statistics are based on Newey and West (1987) adjusted standard errors using 12 lags. The sample period is January 1964 to December 2010 with a total of 564 monthly observations.

Model	α	β_{MKT}	β_{SMB}	β_{HML}	β_{MOM}	β_{MKT^2}
CoSkew	−0.46	−0.01				−0.00
	(−2.73)	(−0.14)				(−1.04)
CoSkew 3-F	−0.29	−0.03	−0.14	−0.23		−0.01
	(−1.77)	(−0.59)	(−1.34)	(−2.35)		(−1.54)
CoSkew 4-F	−0.10	−0.06	−0.14	−0.28	−0.14	−0.01
	(−0.61)	(−1.47)	(−1.21)	(−2.98)	(−1.98)	(−1.89)

coskewness. Since the market price of coskewness risk is negative (as suggested by Harvey and Siddique (2000)), controlling for coskewness strengthens CVR.

B.5. Seasonal Adjustment

The consumption data used in this paper are quarterly, per capita, real consumption of nondurable goods and services, and seasonally adjusted at annual rates. Ferson and Harvey (1992) investigate the asset pricing implications of consumption growth rates obtained from data that are seasonally adjusted with the X-12-ARIMA filter developed by the U.S. Census Bureau. Ex ante, the impact of the filter on latent volatility regimes is not obvious.

To measure the impact of the X-12-ARIMA filter, we simulate 1,000 time series of 200 quarterly log consumption growth rates generated by the Markov model estimated in Table I. We then perturb every fourth-quarter data point by +5% and every first-quarter data point by -5%, which approximately generates the seasonality observed in the discontinued series for seasonally unadjusted quarterly consumption data. We transform the seasonally perturbed

Table IA.XIV
Effect of X-12-ARIMA Filter

We simulate 1,000 time series of 200 quarterly log consumption growth rates generated from the Markov model estimated in Table I. We then perturb every fourth-quarter data point by +5% and every first-quarter data point by -5%, which approximately generates the seasonality observed in the discontinued series for seasonally unadjusted quarterly consumption data. We transform the seasonally perturbed series into consumption levels and apply the X-12-ARIMA filter to get seasonally adjusted consumption data. We then estimate the four-state Markov model of Section I on both the original and the seasonally adjusted data.

Panel A: Summary Statistics for Estimates of Undisturbed and X-12 Data								
	μ_l	μ_h	σ_l	σ_h	p_μ^{ll}	p_μ^{hh}	p_σ^{ll}	p_σ^{hh}
Mean	0.36	0.79	0.20	0.49	92.26	93.83	92.39	93.77
Mean X-12	0.37	0.80	0.19	0.47	91.94	93.52	90.36	89.96
Median	0.36	0.79	0.20	0.49	94.11	95.67	94.76	95.95
Median X-12	0.36	0.79	0.18	0.46	93.38	94.90	93.64	93.47
SD	0.08	0.06	0.04	0.06	8.18	7.98	9.07	6.91
SD X-12	0.08	0.05	0.04	0.10	6.38	5.09	12.40	11.70
5th Percent	0.28	0.71	0.15	0.41	77.67	85.49	78.57	79.93
5th Percent X-12	0.27	0.72	0.13	0.38	79.36	84.83	66.38	67.92
95th Percent	0.47	0.87	0.27	0.57	98.30	98.91	98.75	99.19
95th Percent X-12	0.48	0.88	0.24	0.57	97.89	98.66	98.60	99.01

Panel B: Median Correlations of Beliefs from Undisturbed and X-12 Data				
	ρ_μ	0.98	ρ_σ	0.90

series into consumption levels and apply the X-12-ARIMA filter to get seasonally adjusted consumption data. Last, we estimate the four-state Markov model of Section I on both the original and the seasonally adjusted data.

Table IA.XIV shows summary statistics of the estimated Markov chain parameters. We observe that the seasonal adjustment has negligible influence on the estimated states and state-transitions. Moreover, the median correlation between beliefs over states estimated from the original and the filtered data is very high (0.98 for the mean state, 0.90 for the standard deviation state). We conclude that the Markov model is robust to the X-12-ARIMA filter.

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