A comparison of multi-objective spatial dispersion models for managing critical assets in urban areas

Paul J. Maliszewski*
School of Geographical Sciences and Urban Planning
Arizona State University, AZ 85287, USA
e-mail: paul.maliszewski@asu.edu

Michael J. Kuby
School of Geographical Sciences and Urban Planning
Arizona State University, AZ 85287, USA
e-mail: Mikekuby@asu.edu

Mark W. Horner
Department of Geography
Florida State University, FL, 32306, USA
e-mail: mhorner@fsu.edu

*Correspondence should be sent to Paul J. Maliszewski, School of Geographical Sciences and Urban Planning, Arizona State University, Coor Hall, 975 S. Myrtle Ave., Fifth Floor | P.O. Box 875302, Tempe, AZ 85287-5302 | Phone: 1-850-345-1713

Word Count: 5875

Abstract

A diverse array of spatial optimization models dealing with protection, service, coverage, equity, and risk can potentially aid with the effective placement of critical assets. Protection of assets can be enhanced using the *p*-dispersion model, which locates facilities to maximize the minimum distance between any two. Dispersion, however, is rarely the only objective for a system of facilities, and the *p*-dispersion model is known to be difficult to solve. Therefore, this paper analyzes the trade-offs and computational times of four multi-objective models that combine the *p*-dispersion model with other facility location objectives relevant to siting critical assets, such as the *p*-median, max-cover, *p*-center, and *p*-maxian models. The multi-objective models are tested on a case study of Orlando, Florida. The dispersion/center model produced the most gradual tradeoff curve, while the dispersion/maxian tradeoff curve had the most pronounced "elbow." The center and median multiobjective models were far more computationally demanding than the models using max cover and p-maxian. These findings may inform decision-makers and researchers in deciding what type of multi-objective models to use for planning dispersed networks of critical assets.

Keywords: Critical infrastructure protection; Dispersion; Facility location

1. Introduction

Over the past decade, major disasters in the United States such as the 9/11 attacks, hurricane Katrina, and the H1N1 pandemic have prompted concern about homeland security. One of the most prominent issues in the homeland security community is how to properly manage critical assets (White House, 2001; Critical Infrastructure Protection Program, 2006). Critical assets are the key infrastructure components that are crucial for the continuity of supplies, services, and communications. These assets are critical because their loss would have potentially devastating effects on society (Chopra & Sodhi, 2004). Consequently, the need for developing strategies for effectively managing critical assets and their locations has garnered the attention of policy makers and researchers, especially in the case of possible human sabotage (Parfomak, 2007).

In recent years, many researchers have explored methods for identifying critical infrastructure vulnerabilities and fortifying infrastructure networks (Church, Scaparra, & Middleton, 2004; Snyder, Scaparra, Daskin, & Church, 2006; Taylor, Sekhar, & D'Este, 2006; Murray, Matisziw, & Grubesic, 2008; Nagurney & Qiang, 2008; Li et al., 2009; Akgun, Kandakoglu, & Ozok, 2010). Models have been developed to minimize loss of both supply facilities and population demands in the context of natural disasters (Galindo & Batta, 2010; Rawls & Turnquist, 2010). In resilience-based research such as disaster relief management, objectives commonly involve locating and allocating emergency supplies for critical or vulnerable demands (Sathe & Miller-Hooks, 2005; Horner & Downs, 2010; Widener & Horner, 2011). One strategy for protecting critical assets involves fortifying, or allocating retrofitting resources to vulnerable components of various infrastructures (Snyder, Scaparra, Daskin, & Church, 2006; Qiao et al., 2007; Daskin, 2008; Scaparra & Church, 2008). This paper focuses on an alternative strategy, which aims to protect critical assets by dispersing them from each other (Kim and O'Kelly, 2009). Specifically, the *p*-dispersion model locates *p* critical facilities to maximize the minimum distance separating any pair of facilities (Kuby, 1987). Clustering of like facilities increases vulnerability to system failure (Lovins & Lovins, 1982; Erkut, 1990; Liu, Jung, Heytd, Vittal, & Phadke, 2000; Larson, 2005; Li, Rosenwald, Jung, & Liu, 2005; Goodman, Kirk, & Kirk, 2007). Therefore, dispersing facilities protects them by lessening the chance that a single attack or disaster will disable two neighboring facilities simultaneously.

Planners and managers, however, are unlikely to use the *p*-dispersion model as the sole criteria for planning a network of critical assets because it deals only with the distances between the facilities themselves, and not with distances from facilities to the populations they serve or targets that may threaten them. Critical assets should be available to populations *and* protected from harm. To date, research exploring these trade-offs has been sparse. While *p*-dispersion has been proposed or used as a secondary objective in a multi-objective model (Kim and O'Kelly, 2009), the literature lacks a systematic exploration of the trade-offs between dispersion and conflicting objectives such as coverage, service efficiency, equity, or risk involving distances to other kinds of nodes, especially with respect to man-made disasters such as terrorism and other acts of sabotage where attacks gravitate toward urban centers and the relatively exposed targets that lie therein. In addition, the *p*-dispersion model is well-known to be computationally difficult to solve for medium and large networks, and thus it is also important to understand how fast multi-objective models solve when the *p*-dispersion model is combined with other objectives.

In this paper, we develop and test multi-objective spatial dispersion models to explore and compare the spatial trade-offs and computational performances between critical asset dispersion and other relevant objectives. We integrate the *p*-dispersion problem with the maximal covering problem, the *p*-median problem, the *p*-center problem, and a variant of the *p*-maxian problem, and solve the resulting multiobjective models on a case study network for a major U.S. city (Orlando, Florida). These models have the potential to aid in the management and siting of critical assets such as emergency relief supplies that contain vital goods such as food, water, batteries, first aid, anti-viral drugs and so on. An urban example is suitable because most urban areas do not currently have systems of strategic disaster stockpiles. Given that there are different ways of managing critical asset locations, it is not always clear as to which objectives are most appropriate. Comparing different multi-objective models' resulting trade-off curves and computational efficiencies may inform decision-makers with regard to which models are best suited in practice to combine with facility dispersion.

2. Background on critical asset vulnerability and protection

A large body of work in the critical infrastructure analysis literature has sought to identify critical assets within infrastructure systems that are the most vulnerable or crucial given a loss (White House, 2003; Church et al., 2004; Amin, 2005; Sternberg & Lee, 2006). Network infrastructures such as transportation, energy, and telecommunications systems have received substantial attention because of their interconnected nature. The vulnerabilities of interconnected networks and the impacts resulting from a component outage have been studied extensively (Latora & Marchiori, 2005; Nagurney & Qiang, 2009). Wollmer (1964) was one of the first to develop an optimization model seeking to understand how removing arcs from a network affects system performance. Subsequent research analyzed the removal of arcs for identifying the most critical and vulnerable links of infrastructure systems (Corley & Sha, 1982; Ball, Golden, & Vohra, 1989; Cormican, Morton, & Wood, 1998; Grubesic & Murray, 2006; Taylor et al., 2006; Murray, Matisziw, & Grubesic, 2007; Nagurney & Qiang, 2008; Matisziw & Murray, 2009).

Critical infrastructure vulnerability analysis is not limited to connectivity of linkages. Indeed, some of the most important components within infrastructure systems are nodes (points of connection in space), which also have attracted attention. For example, Nardelli, Proietti, & Widmayer (2003) searched for the node in a network which, when removed, results in a maximal shortest distance between nodes. Locational properties of nodes are important aspects of vulnerability analysis, because an asset's vulnerability is partly a function of its possible exposure to harm, which depends on its location with respect to surrounding hazards and other potential targets. Along these lines, McGill, Ayyub, & Kaminskiy (2007) developed a conceptual framework for analyzing an asset's risk to attack, including that attackers may gravitate to more visible, exposed targets.

Many researchers have also developed methods for protecting critical infrastructures, including allocating resources (e.g. retrofitting, strengthening, equipping for disaster relief, etc.) to critical assets. Generally, there are two schools of thought in allocating resources to critical assets. The first involves allocating fortification resources aimed at protecting critical assets against attack such as optimally allocating hardening resources to critical assets, or determining which assets should receive priority and where resources should be sent (see Salmerón, Wood, & Baldick, 2004; Brown, Carlyle, Salmerón, & Wood, 2006; Church & Scaparra, 2007; Qiao et al., 2007; Scaparra & Church, 2008). To demonstrate, Liu, Fan, & Ordóñez (2009) developed a stochastic network optimization problem for allocating limited retrofitting resources to existing critical transportation infrastructure linkages (i.e., bridges) for protecting against system loss.

The second approach involves allocating emergency response units in support of critical asset resilience (see Sathe & Miller-Hooks, 2005; Berman & Gavious, 2007; Jia, Ordóñez, & Dessouky, 2007). This line of research involves deploying after-the-fact response systems given an attack rather than preemptive fortification strategies. For example, Cheu, Huang, & Huang (2008) developed a model to allocate emergency vehicles to base stations to maximize their coverage of critical transportation infrastructures under varying distance and reliability standards. Also, Ratick, Meacham, & Aoyama (2008) introduced a back-up coverage modeling framework to appropriately position back-ups to support disaster resilience in the case that one emergency facility fails.

Finally, some researchers have taken reliability approaches for siting critical facilities such that a loss in a facility will not have an excessive impact on system supply performance. For example, Snyder and Daskin (2005) introduced the reliable *p*-median problem to situate a stable supply chain network under a probabilistic framework that assumes facilities will randomly fail. This approach seeks to ensure efficient supply under a loss scenario rather than minimizing the loss of critical facilities.

The vast majority of studies developing formal mathematical strategies for critical infrastructure protection ignore asset dispersion as a potential strategy for the protection of critical assets. Yet, this is significant given that many researchers have identified dispersion to be an important aspect of increasing the security of critical assets. Geographical concentration is recognized as a contributor to infrastructure vulnerability because the vulnerability of critical assets is exacerbated by their over-concentration in urban areas (Savitch & Ardashev, 2001; Parfomak, 2007). Research has suggested that dense urban areas are more vulnerable to terrorist attacks than rural ones and adversary intentions gravitate toward prominent urban centers to maximize destruction (Savitch & Ardashev, 2001; Glaeser & Shapiro, 2002; McGill et al., 2007). Major cities with clear targets have been a focus for high-profile attacks (Swanstrom, 2002; Baker & Little, 2006). As a result, geographical dispersion has been suggested as a strategy toward more resilient infrastructures (Lovins & Lovins, 1982; Ramberg, 1982; President's Commission on Critical Infrastructure Protection, 1997; Federal Emergency Management Agency, 2005; Larson, 2005; Lane, Chu, & Santoli, 2006). However, dispersion as a formal modeling strategy for protecting critical assets has not been afforded the attention it deserves.

3. Multi-objective spatial modeling for siting critical assets

A diverse array of spatial optimization models can potentially aid with the effective placement of critical assets, which points to the need for multi-objective models that seek solutions for different and often conflicting objectives (Cohon, 1978; Ghosh & Rushton, 1987; Malczewski, 1999; Kuby, Fagan, ReVelle, & Graf, 2005). Multi-objective problems simultaneously optimize a set of objectives and provide a set of alternative solutions instead of a single solution (Marler & Arora, 2004). The most direct way of solving multi-objective problems is by using the weighting method where w_i is the user-specified weight on objective i, with $0 \leq w_i \leq 1$ (Cohon, 1978). This allows for multiple objectives to be combined into a single objective consisting of the weighted linear combination of the separate objectives, and solved for a range of weights. The result of this problem is a set of Pareto-optimal solutions that form a Pareto-optimal trade-off curve (Collette & Siarry, 2003). Pareto-optimal solutions are solutions where no other feasible solution yields an improvement in one objective without causing degradation to another (Cohon, 1978). The Pareto-optimal solution set is optimal in that no other existing solutions can dominate the Pareto-frontier when all objectives are considered (Zitzler & Thiele, 1998). Multi-objective models are useful when a single-objective model is too simplistic given the substantive problem at hand; they provide additional information about trade-offs that single-objective models neglect.

The integration of push and pull objectives into multi-objective models for hazards management has been researched to a far lesser degree (Current, Min, & Schilling, 1990), but is important since location models can entwine different emergency management objectives such as protection and access. Multi-objective models have been developed for siting semi-desirable facilities (e.g., waste disposal sites, which are partly undesirable but partly necessary), mostly for addressing risk to, or impact on, surrounding populations (Melachrinoudis, 1999; Melachrinoudis & Xanthopulos, 2003; Rakas, Teodorović, & Kim, 2004). Ratick and White (1988) developed a risk-sharing model such that the impact of noxious facilities on surrounding populations is equitable and no one population group has more risk exposure than another group. Murray, Church, Gerrard, & Tsui (1998) developed a hybrid location model utilizing an integrated *p*-median and coverage modeling approach for positioning waste disposal sites to maximize the total weighted population not impacted by a site and minimize the total weighted distance of populations to assigned sites. Although these efforts provide solid foundations for impact/risk models, none of these works explicitly considers the impact of potential targets on critical assets, and their access to populations. A recent example of this kind of modeling effort can be found in Maliszewski and Horner (2010), who developed a spatial optimization model that protects critical supplies by maximizing their total weighted distance from all potential targets to each facility while minimizing the total weighted distance of populations assigned to their closest facility. However, their model ignores inter-facility dispersion as an objective.

A fundamental spatial issue with managing critical assets is that concentration of resources in urban areas benefits society through agglomeration economies while simultaneously increasing the vulnerability of those resources due to over-concentration (Eisinger, 2004). This recognition provides a management framework for multi-objective spatial models to be explored in the context of critical asset protection. These assets must be strategically located such that they are proximal to populations in need, taking advantage of the benefits of accessibility and agglomeration economies. Maximizing accessibility has been operationalized with the *p*-median model for minimizing weighted distance from nodes to their closest facilities (Hakimi, 1964; ReVelle & Swain, 1970), the set-covering model for covering all nodes within a specified distance standard with the minimum number of facilities (Toregas, Swain, ReVelle, & Bergman, 1971), the max covering model for maximizing demand coverage within a given distance by a specified number of facilities (Church & ReVelle, 1974), or the *p*-center problem for minimizing the maximum distance from any population node to its closest facility (Hakimi, 1964; Minieka, 1970). These objectives, however, may conflict with the need to protect them by maximizing their distance from each other, which leads to the concept of dispersion, with the goal of creating more geographically resilient systems (Kuby, 1987; Erkut, 1990; Daskin, 1995).

Spacing of facilities can be accomplished in two general forms. First, minimum closeness can be incorporated as a pairwise constraint that requires the distance between any two facilities to be greater or equal to some user-specified critical distance (Church & Murray, 2009). The advantage of this option is that it will not alter the formulation of the objective function, since it is constraining solutions. The drawback is that the distances between facilities are not determined endogenously through a model's output, but are separated by some user-specified distance, which is not necessarily a straightforward decision.

Dispersion can also be modeled as its own objective whereby the minimum inter-facility distance between any two facilities is maximized. This type of objective can be formulated in discrete network space as the *p*-dispersion problem (see Kuby, 1987). It is a maximin objective that seeks to make the "worst-case" as good as possible, that is, it attempts to make the two nearest facilities as far away from each other as possible. The *p*-dispersion problem is often motivated by military operations, where the spatial separation of strategic facilities such as missile silos would help protect them under the event of an attack (Kuby, 1987; Erkut & Neuman, 1989; Erkut, 1990; Daskin, 1995), but it is also applicable to retail franchises and central place theory (Kuby 1989). Many researchers have explored new techniques for solving dispersion problems (Erkut, Ulkusal, & Yenicerioglu, 1994; Ravi, Rosenkrantz, & Tayi, 1994; Drezner & Erkut, 1995; Dimnaku et al., 2005; Pisinger, 2006). The dispersion concept has also been applied to finding dissimilar paths for routing hazardous materials between an origin and a destination (Kuby, Xu, & Xie, 1997, Akgun et al., 2000; Dell'Olmo, 2005; Thyagarajan, Batta, Karwan, & Szczerba, 2005).

Despite many additional works in the facility dispersion literature, only a few published works have considered the *p*-dispersion problem with respect to accessibility objectives or other relevant objectives for siting critical assets. Tamir (2006) present a problem that locates two new facilities such that the minimum of all weighted distances between all points on a graph and the two facilities, and the distance between the two facilities is maximized. Curtin and Church (2006) outlined a family of dispersion models where multiple types of facilities must be dispersed from each other, and where the level of dispersion (the objective function value) is influenced by both the facility types and their locations. Prokopyev, Kong, & Martinez-Torres (2009) presented the equitable dispersion problem, with the goal of maximizing equity among inter-element distances, as opposed to maximizing the minimum inter-element distance. Kuby, Lim, & Upchurch (2005) discuss methods for adding nodes along the arcs of an existing network to serve path flows by dispersing the new nodes from each other and from the network's existing nodes. Kim and O'Kelly (2009) explicitly consider dispersion as a reliability factor and integrate dispersion with another reliability objective to maximize traffic flow in a hub network. Analyzing dispersion with other objectives relevant for placing critical assets would be useful.

4. Methods

This research employs the *p*-dispersion model, which maximizes the distances between like facilities, in a multi-objective model in combination with several desirable-facility objectives. The *p*-dispersion problem is formulated as follows:

Maximize

$$D \qquad (1)$$

subject to

$$\sum_{i=1}^{n} Y_i = p \qquad (2)$$

$$D \leq d_{ij} + M(2 - Y_i - Y_j)\ , \forall i; \forall j > i \qquad (3)$$

$$Y_i = \{1,0\}, \forall_i \qquad (4)$$

where

i, j = index of potential facility sites
n = number of potential facility sites
d_{ij} = shortest path distance between potential sites i and j
M = some large number; such that $M \geq \max_{i,j}\{d_{ij}\}$
p = number of facilities to be located

$$Y_i = \begin{cases} 1 \text{ if facility is located at node } i \\ 0 \text{ if otherwise} \end{cases}$$

Objective (1) maximizes the minimum distance, *D*, between any two facilities that are to be placed. Constraint (2) stipulates that *p* facilities are to be located. Constraint (3) tracks the inter-facility distances between each pair of located facilities and in doing so constrains the overall minimum distance *D* between any two facilities. The constraint is constructed such that if two sites *i* and *j* are selected to open facilities, meaning that

Y_i and Y_j = 1, then the maximum inter-facility distance can be no more than the distance between those two locations. Thus, constraint (3) puts an upper bound on D of d_{ij} + [2M – M – M] if Y_i and Y_j are both sited and equal to one, which simplifies to d_{ij}. If one (or both) of Y_i and Y_j are equal to 0, then the upper bound becomes $d_{ij} + M$ (or $d_{ij} + 2M$), effectively removing d_{ij} from the set of upper bounds on D. There are $n(n–1)/2$ constraints of (3) that are neither condensed nor "integer friendly" (ReVelle, 1993). Constraint (4) is a binary constraint, stating that a facility is either to be placed, *one,* or not, *zero*.

We compare the trade-offs and computational efficiencies of four different bi-objective models, each of which include the above described *p*-dispersion problem. Specifically, we test the *p*-dispersion problem in combination with the *p*-median problem, the max-cover problem, the *p*-center problem, and a variant of the *p*-maxian problem. These are appropriate objectives for managing critical assets for this case study because sites for disaster relief stockpiles need to be accessible but also protected at the same time. The *p*-median has been used for siting emergency supply facilities in many different applications (Oppong & Hodgson, 1994; Alcada-Almeida, Tralhao, Santos, & Coutinho-Rodrigues, 2009). The *p*-center has also been used in locating critical facilities to minimize the worst-case distance from any demand point to its nearest facility (Horner and Widener, 2010). Hence, it seeks to manage accessibility through equity rather than global efficiency across all population demands. Third, we use the max-cover problem because it is a popular method to site service facilities with a specific service range. The policy provision of disaster relief stockpiles may entail locating them within a specific time or distance of population demands, and because the number of facilities is exogenously specified like the *p*-dispersion model, the max-cover problem is a potentially appropriate model to consider.

The motivation for the fourth objective, the *p*-maxian, is protection of the facilities rather than service of demands. The variant of the *p*-maxian we consider in this paper is another type of protection objective that maximizes total weighted distance among all facility locations and all potential targets, which could be appropriate for extremely sensitive critical assets such as the SNS. It does not stipulate assignments as the standard *p*-maxian problem does (Church and Garfinkel, 1978; Drezner and Wesolowsky, 1985). Pairing the *p*-maxian with *p*-dispersion changes up the mix of objectives, allowing for interesting analysis and options. Although the *p*-dispersion could be paired with many other types of facility location models, we chose these basic models because of their simplicity and popularity.

The *p*-median problem is formulated as:

Minimize

$$\sum_{i=1}^{n}\sum_{j=1}^{m} f_j d_{ij} X_{ij} \tag{5}$$

subject to

$$\sum_{i=1}^{n} X_{ij} = 1, \forall_{j} \tag{6}$$

$$Y_i - X_{ij} \geq 0, \forall_{i,j} \tag{7}$$

$$X_{ij} = \{1,0\}, \forall_{i,j} \tag{8}$$

plus the constraints in (2) and (4), where

f_j = measure of population demand
j = index of demand and repellant nodes
m = number of all demand and repellant nodes

$$X_{ij} = \begin{cases} 1 \text{ if demands at node } j \text{ are served by facility at node } i \\ 0 \text{ if otherwise} \end{cases}$$

In this case study, *i* and *j* are similarly defined as are *n* and *m* since candidate sites and population demands are represented by the same set of point locations, as is often the case in location models. The objective (5) minimizes the sum of weighted distances from population demands to their closest facility. Constraints in (6) stipulate that each population demand node on the network is allocated to be served by exactly one facility. Constraints in (7) require population demand nodes to be allocated to a node only where a facility is opened up. Constraints in (8) are binary constraints, stating assignments are made, *one,* or otherwise not, *zero.* The decision variables include the facility location and the allocation of a facility's resources to population demands. The allocation of a facility's resources to population demands, X_{ij}, is ascribed a *one* if the demands at node *j* are served by a facility at node *i*; *zero* if otherwise.

The max covering problem, which maximizes the amount of demand covered by a specified number of facilities under some distance or time standard is structured as:

Maximize

$$\sum_{j=1}^{m} f_j C_j \tag{9}$$

subject to

$$\sum_{i=1}^{n} a_{ij} Y_i - C_j \geq 0, \forall_{j} \tag{10}$$

$$C_j = \{1,0\}, \forall_{j} \tag{11}$$

plus constraints in (2) and (4), where

$$a_{ij} = \begin{cases} 1 \text{ if demands at node } j \text{ can be covered by facility at node } i \\ 0 \text{ if otherwise} \end{cases}$$

$$C_j = \begin{cases} 1 \text{ if demands at } j \text{ are covered by at least one facility} \\ 0 \text{ if otherwise} \end{cases}$$

Objective (9) maximizes the amount of population demand covered. Constraints (10) define whether demands are covered by a located facility. Constraints (11) are integer requirements, stipulating that demands are covered or not.

We also explore the trade-offs of the *p*-dispersion with the *p*-center problem, which minimizes the maximum distance from any demand node to its closest facility given a specified number of facilities (Hakimi, 1964; Minieka, 1970):

Minimize

$$T \tag{12}$$

subject to

$$\sum_{j=1}^{m} d_{ij} X_{ij} - T \leq 0, \forall i \tag{13}$$

and the *p* facilities constraint (2), the integrality constraints (4), the *p*-median allocation constraints (6-8). The objective (12) minimizes the maximum distance *T* from each demand node to its closest open facility. Constraints (13) prevent *T* from being smaller than the distance between a demand node and the facility to which it is assigned. This formulation of the *p*-center problem is from (Daskin, 1995), and while it does not guarantee that every demand node is assigned to its closest open facility, it does guarantee that every demand node is assigned to an open facility no farther away than *T*. Finally, we consider pairing the *p*-dispersion with the *p*-maxian variant developed by Maliszewski and Horner (2010). The problem is useful as a protection objective because it considers the total weighted distances between facility locations and *all* potential targets. Although the problem's objective is different from the aforementioned models, it is relevant as a supplementary objective to dispersion and worth exploring because it is another type of objective to protect facilities. This variant of the *p*-maxian is formulated as:

Maximize

$$\sum_{i=1}^{n} \sum_{j=1}^{m} v_j d_{ij} Y_i \tag{14}$$

subject to constraints (2) and (4), where v_j = number of potential targets. The second objective (14) is similar to the *p*-maxian problem in that the objective seeks to maximize total weighted distances. The difference between this model and the *p*-maxian is that it considers total weighted distance from *all* potential targets to *each* facility, thus maximizing system-wide protection.

5. Multi-objective models

Each of the aforementioned models is relevant for siting critical assets because they all involve objectives of access or protection in different ways. Given that these models are relevant in siting critical assets, this research will explore the trade-offs and computational efficiencies between the *p*-dispersion/*p*-median, *p*-dispersion/max-cover, *p*-dispersion/*p*-center, and the *p*-dispersion/*p*-maxian (variant). Each of the multi-objective models explored are constructed using the weighting method of multi-objective programming (Cohon, 1978).

The following bi-objective function integrates the *p*-median and the *p*-dispersion:

Minimize

$$Z_1 = w\sum_{i=1}^{n}\sum_{j=1}^{m} f_j d_{ij} X_{ij} - (1-w)D \tag{15}$$

subject to constraints (2-4) and (6-8). The following bi-objective function includes the max covering problem and the *p*-dispersion:

Maximize

$$Z_2 = w\sum_{j=1}^{m} f_j C_j + (1-w)D \tag{16}$$

subject to constraints (2-4) and (10-11). The next bi-objective model includes the *p*-dispersion and the *p*-center problems:

Minimize

$$Z_3 = wT - (1-w)D \tag{17}$$

subject to constraints (2-4), (6-8), and (13). One reason this a particularly interesting multiobjective model to study is the weak duality relationship between the *p*-dispersion and (*p*-1)-center problems (Meir and Moon, 1975; Shier 1977). Specifically, assuming a general network and that all nodes are both demand nodes and candidate sites, one half of the optimal maximin separation distance for the dispersion problem for *p* facilities (½*D*) provides a lower bound for the optimal minimax distance from a demand node to its closest facility for the center problem for *p*-1 facilities (Kuby 1987). On a continuous tree network, this weak duality relationship becomes a strong one, where ½*D* for the *p*-dispersion problem equals *T* for the *p*-center problem.

The final model is also bi-objective and includes a variant of the *p*-maxian and the *p*-dispersion:

Maximize

$$Z_4 = w\sum_{i=1}^{n}\sum_{j=1}^{m} v_j d_{ij} Y_j + (1-w)D \tag{18}$$

subject to constraints (2-4). To help establish some guidance as to how these models perform when combined, trade-off curves and computational times will be comparatively analyzed, both visually and numerically.

6. Study area and data

The study area for this research is the city of Orlando, Florida (Fig. 1). Orlando is a prime opportunity for a case study because of its recognized vulnerability and participation in the Cities Readiness Initiative (Caruson, MacManus, Kohen, & Watson, 2005; Centers for Disease Control and Prevention, 2010). The municipal boundary of Orlando is nearly symmetrical, thereby delineating a clear study area to be investigated.

The data set for Orlando consists of a network of 268 nodes representing census tract centroids. It contains information on demands (i.e., populations) and repellants (i.e., potential targets that "repel" critical facilities away from them for protection) associated with each node as well as both Euclidean and road network shortest paths between each pair of nodes. Population estimates were taken from the 2000 U.S. Census geocoded based on place of residence. The data used in this paper is the same data used in Malszewski (2008) and Maliszewski and Horner (2010). Please refer to the former for more details about the data.

There are several established indicators that are useful for determining potential targets, including the type of building, location, type of construction, number of occupants, and economic life (FEMA, 2005). In addition to those factors listed, the size of the building as an indicator of criticality was included. Based on the guidelines established by FEMA (2005), a comprehensive database of potential targets was filtered from the InfoUSA 2007 database. Potential targets were defined as an establishment with a critical standard industrial classification (SIC) code, number of occupants greater than 25, dollar assets and income greater than $1 million, and square footage greater than 10,000. An establishment is defined as a potential target that meets each of the aforementioned thresholds. Thus v_j represents the total number of potential targets. The potential targets are highly clustered in and around the urban center of Orlando (see Fig. 1).

In this research, network distances measure the separation between demands and potential facility sites assuming that goods are distributed from critical facilities to populations over road networks. Euclidean distances measure the separation between potential targets and potential facility sites assuming that an attack on critical facilities may spill over directly onto nearby entities though shock waves or plumes of smoke, chemicals and other debris.

Since critical asset protection is primarily an urban issue, many of the outlying census tracts on the outskirts of the city were eliminated from the analysis due to being unsuitable sites. The resulting number of census tracts is 143, all of which are within the official city boundary of Orlando and provide suitable areas for critical asset location analysis (Fig. 1).

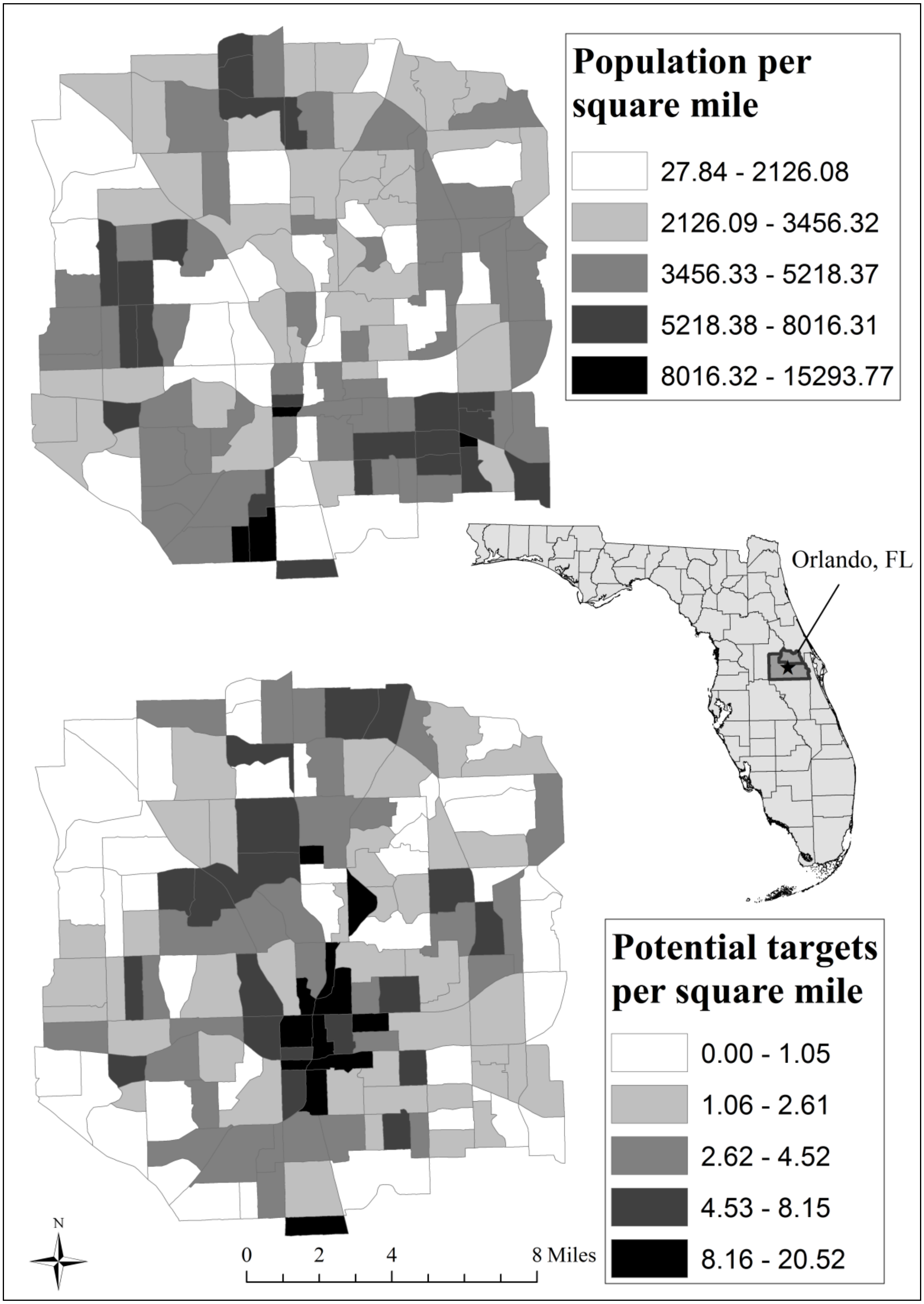


Fig. 1. Maps of population density and potential target density in Orlando, Florida. Color symbologies are classified in natural breaks.

7. Computational results

Each model (15-18) was solved in ILOG CPLEX 11.0 using the branch-and-bound algorithm on a 1.8 GHZ processor running Microsoft Windows Vista with 2 GBs of RAM. The weights, *w*, varied in increments of tenths, except for the extreme weights where 0.01 was used to ensure there was at least some weight to assign demands to facility locations. We also set $p = 5$. A five-mile coverage standard was used for the max-cover problem. In terms of computational tractability, the *p*-dispersion, *p*-median, max-cover, and *p*-center problems are all NP-hard (see Kariv & Hakimi, 1979; Megiddo, Zemel, & Hakimi, 1983; Garey & Johnson, 1979, respectively). The *p*-dispersion problem in particular is computationally intensive because the constraints in equation (3) are prone to be fractional and large instances of it are difficult to solve. As mentioned previously, there are $n(n\text{-}1)/2$ constraints, which are not "integer friendly." Given the difficulty of solving large problem instances of the *p*-dispersion with branch-and-bound, a method was used to set a lower-bound for D such that $D \geq D_{LB}$, where D from the initial or previous run is set as D_{LB} to prune unnecessary branches (see Kuby, 1987). Table 1 shows the computational performance including time (in seconds) and iterations to solve each of the multi-objective models. Fig. 3 shows the trade-off curves connecting non-dominated solution values of the resulting objective function's Pareto-optimal solutions. Fig. 4 maps the resulting facility locations.

Some interesting results from the simulations are worth discussing. The *p*-median, max-cover, and *p*-maxian problems solved relatively faster with no weight on the *p*-dispersion objective. However, the *p*-center problem was far more difficult to solve in comparison with the other problems with zero weight assigned to the *p*-dispersion objective. The time needed to find Pareto-optimal solutions appeared to increase exponentially for the *p*-maxian/*p*-dispersion and *p*-median/*p*-dispersion models as the weight is increased on the *p*-dispersion objective, resulting in a non-linear pattern as shown in Fig. 3. Clearly, the difficulty of solving the *p*-dispersion adds to the computational complexity of solving the *p*-median, *p*-maxian, max-cover, and *p*-center problems. In terms of computation time, the *p*-median/*p*-dispersion performed worse than the max covering/*p*-dispersion and the *p*-maxian variant/*p*-dispersion, but outperformed the *p*-center/*p*-dispersion model. This is because the *p*-center problem is also difficult to solve for large problem instances (Current, Daskin, & Schilling, 2002), and is certainly exacerbated by its combination with the *p*-dispersion problem in this case study.

Fig. 3 shows the trade-offs between each multi-objective model. Each model provided conspicuous trade-off gains of one objective against the other in terms of the degree to which the trade-off curve is more L-shaped than straight (i.e., the degree of "elbow" in the curve). This is important because it depicts the gains in one objective with respect to losses in another objective as the weights shift. In other words, Pareto-optimal curves with more pronounced "elbows" tend to dominate a larger amount of inferior solution space than curves with less pronounced elbows, and come closer to a win-win solution. And if this is true, we can expect to find a gain-to-loss ratio greater than unity near the "elbow." This has important implications for infrastructure systems design that need to be both protected (via dispersion) and accessible. For $p = 5$, the *p*-maxian/*p*-dispersion model performed the best in terms of the degree to which the trade-off curves dominate the inferior solution space for each model, perhaps because of the spatial preferences of this *p*-maxian variant and *p*-dispersion objectives in relation with the other models. Specifically, looking at the maps in Fig. 4, the *p*-maxian variant usually places facilities at sites along the area's boundary, often in cliques, due to the repelling orientation of the maximization objective in the *p*-maxian variant, which sums the distances of all repellants to each facility.

After the *p*-maxian/*p*-dispersion model, the *p*-center/*p*-dispersion problem and *p*-median/*p*-dispersion models rank next in the degree to which their trade-off curves are L-shaped. The max-cover/*p*-dispersion problem provided the most linear trade-off curve, with the weakest gain to loss ratio.

The number of non-inferior solutions generated also change with each problem. The *p*-center offered 8 unique solutions—the most out of all of the problems. The *p*-median problem provided 5 unique solutions, the max-cover problem provided 4, and the *p*-maxian variant provided only 3, two of which are end-point solutions. The importance of the number of unique solutions depends on the preference of the decision maker. That is, some decision makers might prefer multiple options, while another might prefer to make a quick decision, in which a single option might be ideal.

Table 1. Multi-objective dispersion model function values, locations, and times.

(a) *p*-median

p	*w*	*p*-median	*D*	Time (sec.)	Iterations
5	0.01	99.09	8.14	2,118.28	362,244
5	0.1	80.68	7.09	3,285.09	654,589
5	0.2	80.68	7.09	11,508.77	3,504,755
5	0.3	78.49	6.30	62,834.13	13,582,795
5	0.4	78.10	6.04	10,652.51	5,301,355
5	0.5	77.53	5.66	3,358.30	1,286,295
5	0.6	77.53	5.66	3,049.36	343,058
5	0.7	77.53	5.66	678.89	47,439
5	0.8	77.53	5.66	294.34	17,469
5	0.9	77.53	5.66	132.47	8,059
5	1	77.53	5.66	5.19	4,370

(b) Max-cover (5-miles)

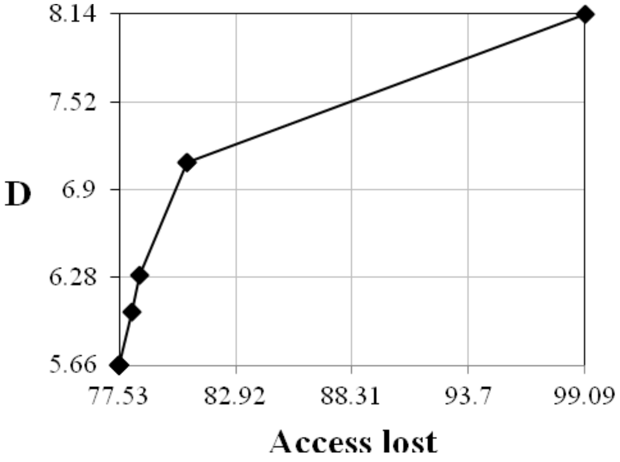
p	*w*	Max-cover	*D*	Time (sec.)	Iterations
5	0.01	23.81	8.14	62.50	14,546
5	0.1	23.81	8.14	599.30	76,332
5	0.2	23.81	8.14	820.56	143,201
5	0.3	25.05	7.68	1,185.25	202,615
5	0.4	25.32	7.53	811.13	161,166
5	0.5	25.32	7.53	559.91	116,327
5	0.6	25.32	7.53	693.37	130,578
5	0.7	25.32	7.53	671.48	168,335
5	0.8	25.32	7.53	463.19	94,651
5	0.9	25.40	7.29	350.49	70,965
5	1	25.40	7.29	0.17	31

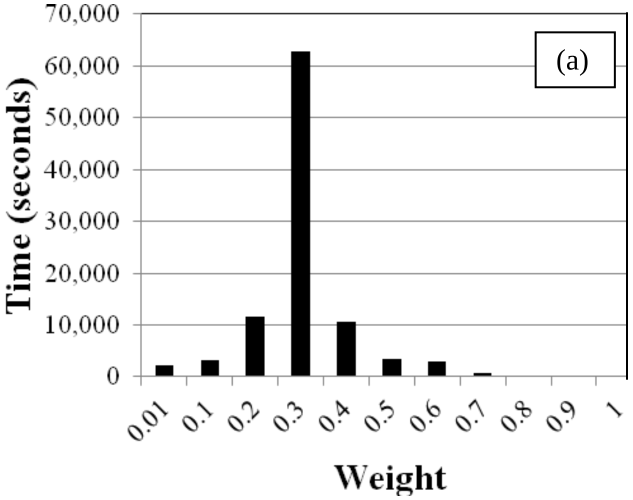
(c) *p*-center

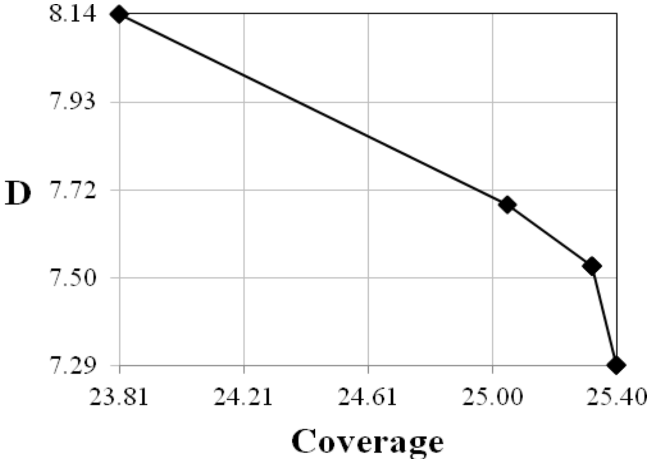
p	*w*	*p*-center	*D*	Time (sec.)	Iterations
5	0.01	6.74	8.14	1,802.37	475,222
5	0.1	6.74	8.14	1,806.94	663,281
5	0.2	5.93	7.98	2,935.60	1,365,515
5	0.3	5.22	7.68	1,981.24	802,039
5	0.4	5.22	7.68	45,632.74	28,471,826
5	0.5	4.59	7.12	63,105.85	31,981,350
5	0.6	4.59	7.12	82,291.06	41,584,852
5	0.7	4.25	6.42	54,472.51	36,491,021
5	0.8	4.08	6.10	9,246.74	8,426,883
5	0.9	4.08	6.10	1,995.99	3,788,136
5	0.99	4.07	5.82	99.97	34,463
5	1	4.07	5.11	11.12	16,678

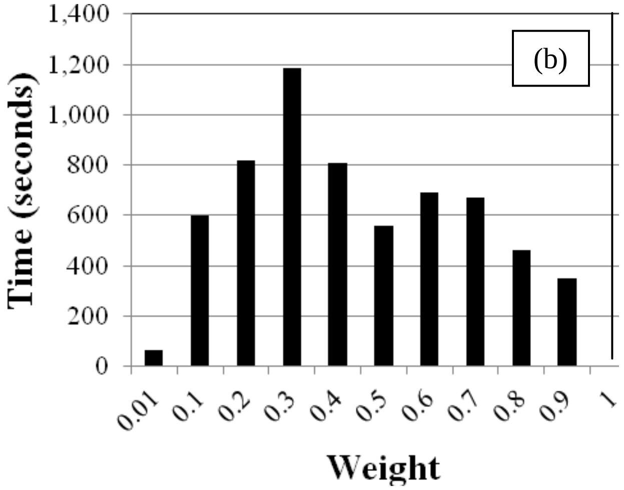
(d) *p*-maxian

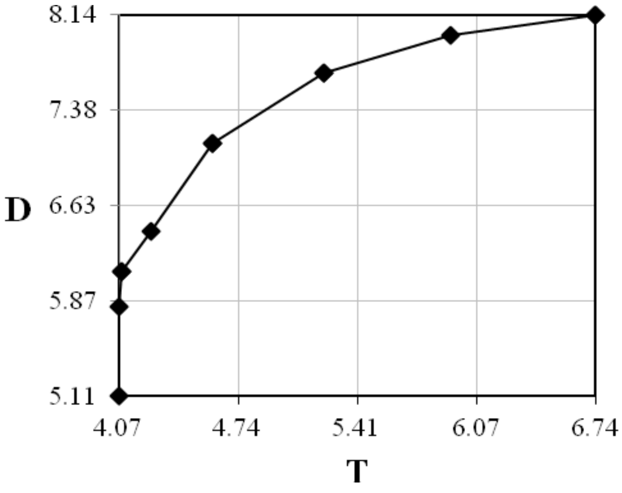
p	*w*	*p*-maxian	*D*	Time (sec.)	Iterations
5	0.01	342.79	8.14	5,194.22	807,258
5	0.1	362.37	6.81	3,107.83	517,731
5	0.2	362.37	6.81	406.67	53,613
5	0.3	362.37	6.81	158.75	26,808
5	0.4	362.37	6.81	46.80	9,602
5	0.5	362.37	6.81	128.75	24,102
5	0.6	367.29	0.81	84.11	12,757
5	0.7	367.29	0.81	40.83	7,214
5	0.8	367.29	0.81	12.59	5,048
5	0.9	367.29	0.81	4.88	3,639
5	1	367.29	0.81	7.17	4,040

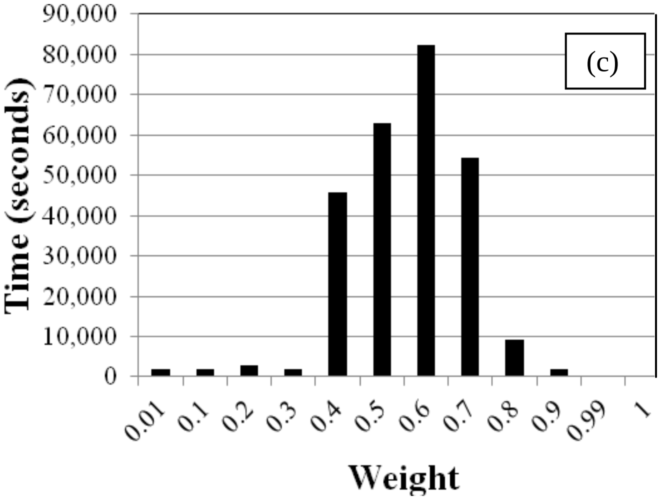

8.14
7.52
6.9
6.28
5.66
D
77.53
82.92
88.31
93.7
99.09
Access lost


(a)
70,000
60,000
50,000
40,000
30,000
20,000
10,000
0
Time (seconds)
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Weight


8.14
7.93
7.72
7.50
7.29
D
23.81
24.21
24.61
25.00
25.40
Coverage


(b)
1,400
1,200
1,000
800
600
400
200
0
Time (seconds)
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Weight


8.14
7.38
6.63
5.87
5.11
D
4.07
4.74
5.41
6.07
6.74
T


(c)
90,000
80,000
70,000
60,000
50,000
40,000
30,000
20,000
10,000
0
Time (seconds)
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0.99
1
Weight

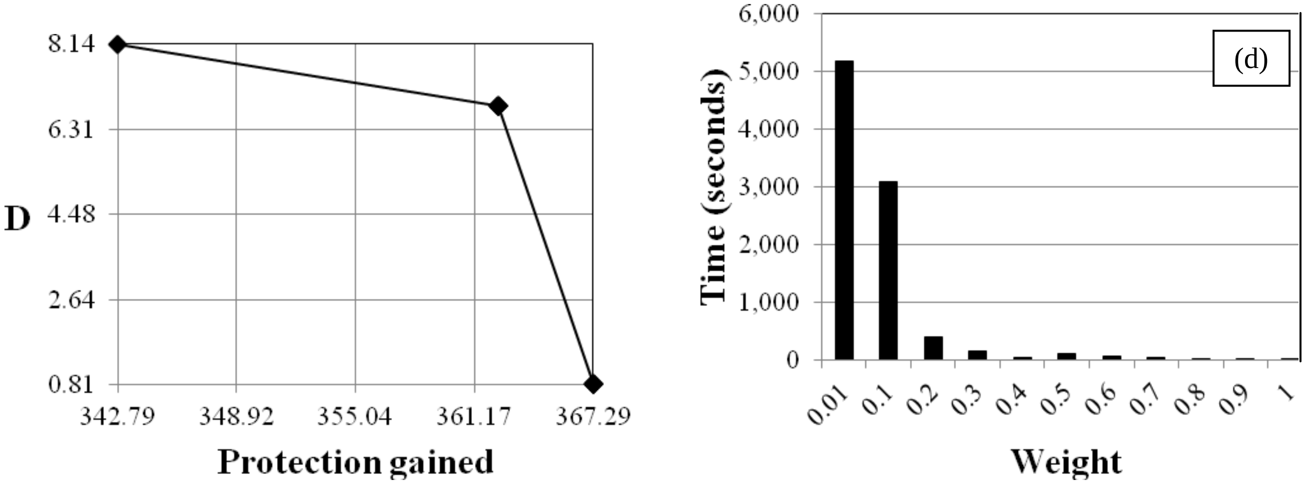


Fig. 2. Trade-off curves and solution times for (a) *p*-median/*p*-dispersion; (b) Max-cover/*p*-dispersion; (c) *p*-center/*p*-dispersion; (d) *p*-maxian/*p*-dispersion. Weight refers to the *w* on the alternative objective, with 1-*w* on the *p*-dispersion objective.

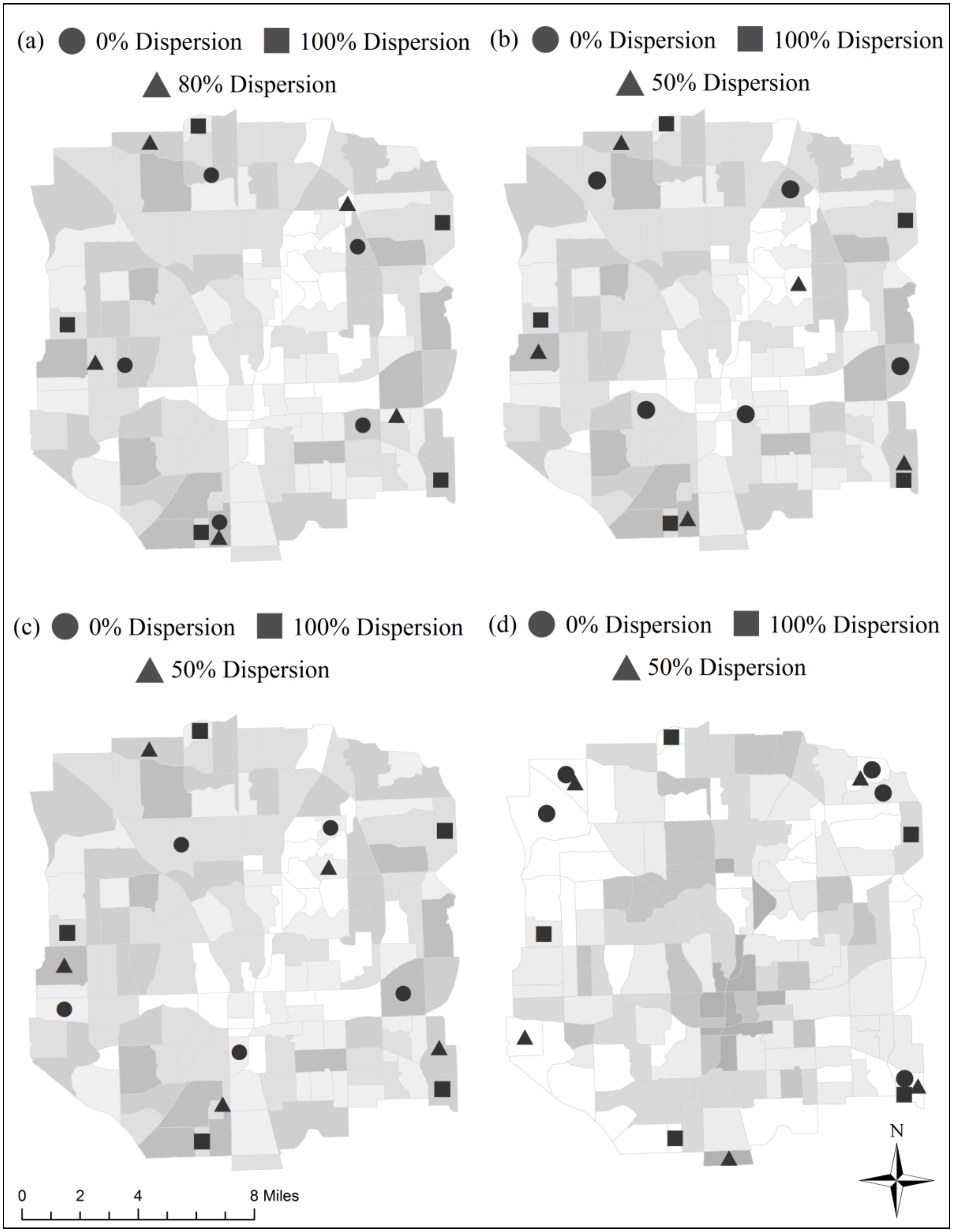


Fig. 3. Maps of facility location results for $p = 5$ for (a) p-median/p-dispersion; (b) Max-cover/p-dispersion with a cover distance of 5 miles; (c) p-center/p-dispersion; (d) p-maxian/p-dispersion. Maps (a-c) show population classified in natural breaks while map (d) shows potential target density classified in natural breaks.

8. Discussion and conclusions

The spatial management of critical assets has received substantial attention as a result of concerns regarding issues in homeland security and disaster preparedness. In the context of spatially managing assets for critical infrastructure protection, the use of multi-objective modeling can provide conspicuous trade-off gains among conflicting spatial objectives. However, this is also at the expense of considerable amounts of time finding Pareto-optimal solutions. Given that many facility location problems are NP-hard, multi-objective facility location problems will tend to be at least NP-hard. Yet, multi-objective problems have become increasingly important as a result of the extensive development of single-objective models, the recognition that single-objective models are perhaps too simplistic for actual siting situations, and because computers are fast enough to deal with the computational burden of solving them. Although computational times in this application were not a burden, real world problems can be much larger (i.e., more candidate sites), which will increase the computational burden or even make the problems intractable. Therefore, it is important for policy makers or analysts working for policy makers to understand more about the computational times of potentially useful multi-objective spatial models for critical asset location management.

It was found that some multi-objective dispersion problems provide more L-shaped trade-offs than others. This is useful for modelers who have an array of models to choose from, but are not sure which to use. In terms of computational performance, the *p*-center was the slowest. In terms of trade-off gains between dispersion and competing objectives, the max-cover had the most linear tradeoff curve. The trade-off curves for the *p*-median the *p*-center problems provided greater domination of inferior solution space than the max-cover. However, the max-cover generally solved faster than the *p*-median and *p*-center. The *p*-maxian variant performed the best in terms of the trade-off curve, and also solved relatively fast. The trade-off performance appears to be associated with the relationship between the distributions of facilities at extreme weights. Since the *p*-dispersion seeks to maximize inter-facility distances, it appears the trade-off curve will increase in domination of inferior solution space as more facilities are added and the resulting distance between facilities (with total weight on the opposing objective) reduces and the facilities are generally closer to the edge of the graph. In other words, we hypothesize that solutions with dispersed facilities that are not near the edge of the graph will have a less pronounced elbow in the trade-off curve than a group of facilities that relatively clustered and close to the edge of the graph.

As discussed in Section 5 above, the *p*-dispersion and (*p*-1) center problems are known to have a weak duality relationship (Meir and Moon, 1975; Shier, 1977). Intuitively, it is easy to see the relationship between the two problems using an example for $p=2$ on a tree network where facilities can be located anywhere on the continuous length of any arc. The solution to the 2-dispersion problem is to locate the two facilities at the farthest-apart ends of any two branches, and *D* will be the distance between them. For the 1-center problem, the optimal solution is to put the facility exactly equidistant between those same two branch ends—in a sense at an exact *opposite* type of location—and *T* will equal exactly $\frac{1}{2}D$. On a general network, and with location only at discrete nodes, the strong duality becomes a weak one, and $\frac{1}{2}D$ becomes a lower bound for *T*. In our case study here, not only are we studying a discrete location problem on a general network, but we using the same number of facilities for both problems, rather than one less for the center problem. With one more facility to locate in the center problem, the maximin distance of any demand node from its closest facility would be the same or lower, and the weak duality relationship would not *necessarily* continue to hold. Surprisingly, as Table 1 shows, the lowest *T* in any of the 11 runs is exactly half (4.07) of the maximum *D* (8.14). These values of *T* and *D* are of course achieved at the opposite ends of the tradeoff curve, and the closest *T* ever gets to *½D in the same solution* is 64.5% of *D*. On average, *T* is 71.2% of the value of *D*. Looking at Figure 4, the tradeoff between maximizing *D* and minimizing *T* for the same *p* value is fairly smooth and rounded; the companion diagram of computation times shows that the extreme solutions are found quite fast, while the solutions with more equal weights require large amounts of time.

The trade-off curve shape is significant for critical infrastructure management because, from a spatial perspective, protection and other objectives such as accessibility are in conflict. If gains in one objective (e.g. maximize protection) are permitted with fractional losses in another (e.g. maximize accessibility), compromise solutions may increase the protection of critical stockpiles while still choosing locations that provide decent access to population demands.

Although there is some variation in the geographic distribution of sited facilities for the various combinations of model objectives and weighting parameters, there is a general tendency for many to be placed in the region's peripheral areas. As the weight on the dispersion objective is reduced, there is a noticeable movement for the facilities in the *p*-center/*p*-dispersion problem from the edge of the network and towards the middle, some movement for the max cover problem, little movement for the *p*-median, and virtually none for the *p*-maxian variant. Even when no weight is given to the dispersion objective, the *p*-maxian facilities remain on the edges, as do most of the *p*-median facilities, and even some of the max cover and *p*-center facilities. This ordering makes sense, given that, on one extreme, the *p*-center/*p*-dispersion problem deals purely with unweighted distances, and on the other extreme the *p*-maxian/*p*-dispersion problem combines two distance-maximization objectives. And yet, it remains somewhat surprising that no critical facilities are ever located in the inner third of Orlando no matter what the objectives or their weights.

Thus, one possible takeaway for planning purposes is the idea that less centralized locations are generally favored for critical asset projection. While this result is most likely influenced by (a) the small number of facilities being located and (b) the low population density and (c) high number of targets in the central area, these three conditions may typically hold true for locating critical assets in American cities generally. Based on our results, we might suggest that American cities engaging in critical infrastructure planning without actually constructing formal optimization models consider this locational finding in their deliberations when evaluating multiple siting alternatives.

Overall, this paper compared and assessed the computational efficiencies of four different multi-objective models for spatially managing critical assets. Specifically, this paper has illustrated the trade-offs of inter-facility dispersion against access via the *p*-median and *p*-center problems, coverage via the max covering problem, and protection via a variant of the *p*-maxian problem. Although multi-objective facility location models in the context of siting critical assets are computationally intensive, multi-objective models are nonetheless feasible with a reasonable number of candidate facility sites. This research has illustrated the potential use of several multi-objective spatial models for the management of critical assets whereby dispersion among critical assets is an important goal in addition to others and has highlighted both the trade-offs of different conflicting goals as well as their associated computational (in)efficiencies. Analyzing competing objectives such as the ones illustrated in this paper provide opportunities and directions for both future research as well as strategic aspects of management that can be applied to the design of future critical infrastructure systems. These competing objectives are well recognized to be important in government applications and academia. According to our literature review, research on, and modeling of, critical infrastructure systems involving competing spatial objectives for their design is sparse and not well understood.

Unsurprisingly then, there are a number of opportunities for future research that others may undertake based on opportunities that arise from the work completed here. One possible area for additional research would be to look in more detail at ways of differentiating critical infrastructure facilities, perhaps relative to specific types of targets and/or supplies. The notion that facilities' functions/capacities may not be homogenous is well documented in the literature; in the sense that various sizes/types of facilities may be needed (Rawls & Turnquist, 2010; Horner & Downs, 2010) and/or that there is a nested hierarchy in terms of facilities' capabilities with respect to one another (Oppong & Hodgson, 1994; Daskin, 1995; Widener & Horner, 2011). A second possibility for new research would be to build in uncertainty regarding road network availability in the event of a disaster. In this paper, the modeling implicitly assumes that road networks will be available to move supplies to potential demands. If this assumption is changed, it could affect the geographical distribution of facilities, and learning if this would happen in a predictable way would be informative to broader planning efforts. It is also worth exploring how these results would change with a different number of facilities, different cover distances, and different distributions of populations and targets.

References

Akgun, I., Kandakoglu, A., & Ozok, A. F. (2010). Fuzzy integrated vulnerability assessment model for critical facilities in combating the terrorism. *Expert Systems with Applications, 37*, 3561-3573.

Akgun, V., Erkut, E., & Batta, R. (2000). On finding dissimilar paths. *European Journal of Operational Research, 121* (2), 232-246.

Alcada-Almeida, L., Tralhao, L., Santos, L., Coutinho-Rodrigues, J. (2009). A multiobjective approach to locate emergency shelters and identify evacuation routes in urban areas. *Geographical Analysis, 41* (1), 9-29

Amin, M. (2005). Energy infrastructure defense systems. *Proceedings of the IEEE, 93* (5), 861-875.

Baker, G. H., & Little, R. G. (2006). Enhancing homeland security: Development of a course on critical infrastructure systems. *Journal of Homeland Security and Emergency Management, 3* (4). DOI: 10.2202/1547-7355.1263.

Ball, M. O., Golden, B. L., & Vohra, R. V. (1989). Finding the most vital arcs in a network. *Operations Research Letters, 8*, 73-76.

Berman, O., & Gavious, A. (2007). Location of terror response facilities: A game between state and terrorist. *European Journal of Operations Research, 117* (2), 1113-1133.

Brown, G., Carlyle, M., Salmerón, J., & Wood, K. (2006). Defending critical infrastructure. *Interfaces, 36* (6), 530-544.

Caruson, K., MacManus, S. A., Kohen, M., & Watson, T. A. (2005). Homeland security preparedness: The rebirth of regionalism. *Publius—The Journal of Federalism, 35*, 143-68.

Centers for Disease Control and Prevention. (2010). Cities Readiness Initiative. http://www.bt.cdc.gov/cri/ (last accessed 9 June 2011).

Cheu, R. L., Huang, Y., & Huang, B. (2008). Allocating emergency service vehicles to serve critical transportation infrastructures. *Journal of Intelligent Transportation Systems, 12* (1), 38-49.

Chopra, S., & Sodhi, M. S. (2004). Managing risk to avoid supply chain breakdown. *MIT Sloan Management Review, 46* (1), 53-61.

Church, R. L., & Garfinkel, R. (1978). Locating an obnoxious facility on a network. *Transportation Science, 12*, 107-118.

Church, R. L., & Murray, A. T. (2009). *Business site selection, location analysis, and GIS*. Hoboken, NJ: Wiley.

Church, R. L., & ReVelle, C. (1974). The maximal covering location problem. *Papers of the Regional Science Association, 32*, 101-118.

Church, R. L., & Scaparra, M. P. (2007). Protecting critical assets: The *r*-interdiction median problem with fortification. *Geographical Analysis, 39* (2), 129-146.

Church, R. L., Scaparra, M. P., & Middleton, R. S. (2004). Identifying critical infrastructure: The median and covering facility interdiction problems. *Annals of the Association of American Geographers, 94* (3), 491-502.

Cohon, J. L. (1978). *Multiobjective programming and planning*. New York: Academic Press.

Collette, Y., & Siarry, P. (2003). *Multiobjective optimization: Principles and case studies.* Berlin: Springer-Verlag.

Corley, H. W., & Sha, D. Y. (1982). Most vital links and nodes in weighted networks. *Operations Research Letters, 1*, 157-160.

Cormican, K. J., Morton, D. P., & Wood, R. K. (1998). Stochastic network interdiction. *Operations Research, 46* (2), 184-197.

Critical Infrastructure Protection Program. (2006). *What is CIP?* http://cipp.gmu.edu/cip/ (last accessed on 26 May 2010).

Current, J., Daskin, M. S., & Schilling, D. (2002). Discrete network location models. In Z. Drezner, & H. W. Hamacher (Eds.), *Facility location: Applications and theory* (pp. 83-120). New York: Springer-Verlag.

Current, J., Min, H., & Schilling, D. (1990). Multiobjective analysis of facility location decisions. *European Journal of Operations Research, 49*, 295-307.

Curtin, K. M., & Church, R. L. (2006). A family of location models for multiple-type discrete dispersion. *Geographical Analysis, 38*, 248-270.

Daskin, M. (1995). *Network and discrete location: Models, algorithms, and applications*. New York: John Wiley and Sons, Inc.

Daskin, M. S. (2008). What you should know about location modeling. *Naval Research Logistics, 55*, 283-294.

Dell'Olmo, P. (2005). On finding dissimilar Pareto-Optimal paths. *European Journal of Operational Research, 162* (1), 70-82.

Dimnaku, A., Kincaid, R., & Trosset, M. W. (2005). Approximate solutions of continuous dispersion problems. *Annals of Operations Research, 136* (1), 65-80.

Drezner, Z., & Erkut, R. (1995). Solving the continuous *p*-dispersion problem using nonlinear-programming. *Journal of Operational Research Society, 46* (4), 516-520.

Drezner, Z., & Wesolowsky, G. O. (1985). Location of multiple obnoxious facilities. *Transportation Science, 19* (3), 193-202.

Eisinger, P. (2004). The American city in the age of terror: A preliminary assessment of the effects of September 11. *Urban Affairs Review, 40* (1), 115-130.

Erkut, E. (1990). The discrete *p*-dispersion problem. *European Journal of Operational Research, 46* (1), 48-60.

Erkut, E., & Neuman, S. (1989). Analytical models for locating undesirable facilities. *European Journal of Operational Research, 40* (3), 275-291.

Erkut, E., Ulkusal, Y., & Yenicerioglu, O. (1994). A comparison of *p*-dispersion heuristics. *Computers & Operations Research, 21* (10), 1103-1113.

Federal Emergency Management Agency. (2005). Risk assessment: A how-to guide to mitigate potential terrorist attacks against buildings. http://www.fema.gov/plan/prevent/rms/ (last accessed 26 May 2010).

Galindo, G., & Batta, R. (2010). Prepositioning of supply points in preparation for foreseeable disasters. *INFORMS*, Austin, TX.

Garey, M. R., & Johnson, D. S. (1979). *Computers and intractability: A guide to the theory of NP completeness*. New York: W. H. Freeman and Co.

Ghosh, A., & Rushton, G. (1987). *Spatial analysis and location-allocation models*. New York: Van Nostrand Reinhold Company.

Glaeser, E. L., & Shapiro, J. M. (2002). Cities and warfare: The impact of terrorism on urban form. *Journal of Urban Economics*, *51*, 205-224.

Goodman, S. E., Kirk, J. C., & Kirk, M. H. (2007). Cyberspace as a medium for terrorists. *Technological Forecasting and Social Change, 74* (2), 193-210.

Grubesic, T. H., & Murray, A. T. (2006). Vital nodes, interconnected infrastructures, and the geographies of network survivability. *Annals of the Association of American Geographers, 96* (1), 64-83.

Hakimi, S. L. (1964). Optimum locations of switching centers and the absolute centers and medians of a graph. *Operations Research*, *12*, 450-459.

Horner, M. W., & Downs, J. A. (2010). Optimizing hurricane disaster relief goods distribution: Model development and applications with respect to planning strategies. *Disasters, 34* (3), 821-844.

Horner, M. W., & Widener, M. J. (2010). How do socio-economic characteristics interact with equity and efficiency considerations? An analysis of disaster relief goods provision. In B. Xiang, & A. Yao (Eds.), *Geospatial analysis in urban environments* (pp. 393-414). New York: Springer.

Horner, M. W., & Widener, M. J. (2011). The effects of transportation network failure on people's accessibility to hurricane disaster relief goods: A modeling approach and application to a Florida case study. *Natural Hazards,* 59 (3), 1619-1634.

Jia, H., Ordóñez, F., & Dessouky, M. M. (2007). Solution approaches for facility location of medical supplies for large-scale emergencies. *Computers & Industrial Engineering, 52* (2), 257-276.

Kariv, O., & Hakimi, S. L. (1979). An algorithmic approach to network location problems II: The *p*-medians. *SIAM Journal of Applied Mathematics, 37,* 513-538.

Kim, H., & O'Kelly, M. E. (2009). Reliable *p*-hub location problems in telecommunications networks. *Geographical Analysis, 41*, 283-306.

Kuby, M. (1987). Programming models for facility dispersion: The *p*-dispersion and *p*-defense problems. *Geographical Analysis, 19,* 315-329.

Kuby, M. (1989). A location-allocation model of Losch's central place theory: Testing on a uniform lattice network. *Geographical Analysis, 21,* 316–338.

Kuby, M., Xu, Z. Y., & Xie, X. D. (1997). A minimax method for finding the *k* best 'differentiated' paths. *Geographical Analysis, 29* (4), 298-313.

Kuby, M. J., Fagan, W. F., ReVelle, C. S., & Graf, W. L. (2005). A multiobjective optimization model for dam removal: An example trading off salmon passage with hydropower and water storage in the Willamette basin. *Advances in Water Resources, 28* (8), 845-855.

Kuby, M., Lim, S., & Upchurch, C. (2005). Dispersion of nodes added to a network. *Geographical Analysis, 37* (4), 383-409.

Lane, K. S., Chu, S. Y., & Santoli, J. M. (2006). The United States pediatric vaccine stockpile program. *Clinical Infectious Diseases, 42* (3), 125-129.

Larson, R. C. (2005). Decision models for emergency response planning. In D. G. Kamien (ed.), *The McGraw-Hill Handbook of Homeland Security*. New York: McGraw-Hill.

Latora, V., & Marchiori, M. (2005). Vulnerability and protection of infrastructure networks. *Physical Review E,* 71.

Li, H., Apostolakis, G. E., Gifun, J., VanShalkwyk, W., Leite, S., & Barber, D. (2009). Ranking the risks from multiple hazards within a small community. *Risk Analysis, 29* (3), 438-456.

Li, H., Rosenwald, G. W., Jung, J., & Liu, C. (2005). Strategic power infrastructure defense. *Proceedings of the IEEE, 93* (5), 918-933.

Liu, C., Fan, Y., & Ordóñez, F. (2009). A two-stage stochastic programming model for transportation network protection. *Computers & Operations Research, 36*, 1582-1590.

Liu, C., Jung, J., Heytd, G. T., Vittal, V., & Phadke, A. G. (2000). The strategic power infrastructure defense (SPID) system. *IEEE Control Systems Magazine, 20* (4), 40-52.

Lovins, A. B., & Lovins, L. H. (1982). *Brittle power: Energy strategy for national security*. Andover, MA: Brick House Publishing Company.

Malczewski, J. (1999). *GIS and multicriteria decision analysis*. New York: Wiley.

Maliszewski, P. J. (2008). *Modeling Critical Vaccine Supply Location: Protecting Critical Infrastructure and Population in Central Florida*. Masters Thesis, Florida State University, Tallahassee.

Maliszewski, P. J., & Horner, M. W. (2010). A spatial modeling framework for siting critical supply infrastructures. *Professional Geographer, 62* (3), 426-441.

Marler, R. T., & Arora, J. S. (2004). Survey of multi-objective optimization methods for engineering. *Structural and Multidisciplinary Optimization, 26*, 369-395.

Matisziw, T. C., & Murray, A. T. (2009). Modeling *s*–*t* path availability to support disaster vulnerability assessment of network infrastructure. *Computers & Operations Research, 36*, 16-26.

McGill, W. L., Ayyub, B. M., & Kaminskiy, M. (2007). Risk analysis for critical asset protection. *Risk Analysis, 27* (5), 1265-1281.

Megiddo, N., Zemel, E., & Hakimi, S. L. (1983). The maximal coverage location problem. *SIAM Journal of Algebraic and Discrete Methods, 4*, 253-261.

Meir, A. & Moon, J. W. (1975). Relations between packing and covering numbers of a tree. *Pacific Journal of Mathematics, 61*, (1), 225-233.

Melachrinoudis, E. (1999). Bi-criteria location of a semi-obnoxious facility. *Computers & Industrial Engineering, 37* (3), 581-593.

Melachrinoudis, E., & Xanthopulos, Z. (2003). Semi-obnoxious single facility location in Euclidean space. *Computers and Operations Research, 30* (14), 2191-2209.

Minieka, E. (1970). The *m*-center problem. *Siam Review, 12*, 138-139.

Murray, A. T., Church, R. L., Gerrard, R. A., & Tsui, W. S. (1998). Impact models for siting undesirable facilities. *Papers in Regional Science, 77* (1), 19-36.

Murray, A. T., Matisziw, T. C., & Grubesic, T. H. (2007). Critical network infrastructure analysis: Interdiction and system flow. *Journal of Geographical Systems, 9*, 103-117.

——— (2008). A methodological overview of network vulnerability analysis. *Growth and Change, 39* (4), 573-592.

Nagurney, A., & Qiang, Q. (2008). A network efficiency measure with application to critical infrastructure networks. *Journal of Global Optimization, 40*, 261-275.

Nagurney, A., & Qiang, Q. (2009). *Fragile networks: Identifying vulnerabilities and synergies in an uncertain world.* Hoboken, NJ: John Wiley & Sons.

Nardelli, E., Proietti, G., & Widmayer, P. (2003). Finding the most vital node of a shortest path. *Theoretical Computer Science, 296*, 167-177.

Oppong, J. R., Hodgson, M. J. (1994). Spatial accessibility to health-care facilities in Suhum district, Ghana. *Professional Geographer, 46* (2), 199-209.

Parfomak, P. W. (2007). *Vulnerability of concentrated critical infrastructure: Background and policy options*. CRS Report for Congress. http://www.fas.org/sgp/crs/homesec/RL33206.pdf (last accessed 21 June 2011).

Pisinger, D. (2006). Upper bounds and exact algorithms for *p*-dispersion problems. *Computers & Operations Research, 33* (5), 1380-1398.

President's Commission on Critical Infrastructure Protection. (1997). *Critical foundations: Protecting America's infrastructures*. Washington, DC: The White House.

Prokopyev, O. A., Kong, N., & Martinez-Torres, D. L. (2009). The equitable dispersion problem. *European Journal of Operational Research, 197* (1), 59-67.

Qiao, J., Jeong, D., Lawley, M., Richard, J. P. P., Abraham, D. M., & Yih, Y. (2007). Allocating security resources to a water supply. *IIE Transactions, 39*, 95-109.

Rakas, J., Teodorović, D., & Kim, T. (2004). Multi-objective modeling for determining location of undesirable facilities. *Transportation Research Part D, 9* (2), 125-138.

Ramberg, B. (1982). Attacks on nuclear reactors: The implications of Israel's strike on Osiraq. *Political Science Quarterly, 97* (4), 653-669.

Ratick, S. J., & White, A. L. (1988). A risk-sharing model for locating noxious facilities. *Environment and Planning B-Planning & Design*, *15* (2), 165-179.

Ratick, S., Meacham, B., & Aoyama, Y. (2008). Locating back-up facilities to enhance supply chain disaster resilience. *Growth and Change*, *39* (4), 642-666.

Ravi, S. S., Rosenkrantz, D. J., & Tayi, G. K. (1994). Heuristic and special case algorithms for dispersion problems. *Operations Research, 42* (2), 299-310.

Rawls, C. G., & Turnquist, M. A. (2010). Pre-positioning of emergency supplies for disaster response. *Transportation Research Part B, 44*, 521-534.

ReVelle, C. (1993). Facility siting and integer friendly programming. *European Journal of Operational Research, 65* (2), 147-158.

ReVelle, C. S., & Swain, R. (1970). Central facilities location. *Geographical Analysis, 2*, 30-42.

Salmerón, J., Wood, K., & Baldick, R. (2004). Analysis of electric grid security under terrorist threat. *Institute of Electrical and Electronic Engineers Transactions on Power Systems, 19* (2), 905-912.

Sathe, A., & Miller-Hooks, E. (2005). Optimizing location and relocation of response units in guarding critical facilities. *Transportation Research Record, 1923*, 127-136.

Savitch, H. V., & Ardashev, G. (2001). Does terror have an urban future? *Urban Studies, 38* (13), 2515-2533.

Scaparra, M. P., & Church, R. L. (2008). A bilevel mixed-integer program for critical infrastructure protection planning. *Computers & Operations Research*, *35*, 1905-1923.

Shier, D. R, (1977). A min-max theorem for *p*-center problems on a tree. *Transportation Science, 11* (3), 243-252.

Snyder, L. V., & Daskin, M. S. (2005). Reliability models for facility location: The expected failure cost case. *Transportation Science, 39* (3), 400-416.

Snyder, L. V., Scaparra, M. P., Daskin, M. S., & Church, R. L. (2006). Planning for disruptions in supply chain networks. *INFORMS Tutorials in Operations Research*, 234-257.

Sternberg, E., & Lee, G. C. (2006). Meeting the challenge of facility protection for homeland security. *Journal of Homeland Security and Emergency Management, 3* (1). DOI: 10.2202/1547-7355.1153.

Swanstrom, T. (2002). Are fear and urbanism at war? *Urban Affairs Review, 38* (1), 135-140.

Tamir, A. (2006). Locating two obnoxious facilities using the weighted maximin criterion. *Operations Research Letters, 34* (1), 97-105.

Taylor, M. A. P., Sekhar, S. V. C., & D'Este, G. M. (2006). Application of accessibility based methods for vulnerability analysis of strategic road networks. *Networks & Spatial Economics, 6* (3-4), 267-291.

Thyagarajan, K., Batta, R., Karwan, M. H., & Szczerba, R. J. (2005). Planning dissimilar paths for military units. *Military Operations Research, 10* (1), 25-42.

Toregas, C., Swain, R., ReVelle, C., & Bergman, L. (1971). The location of emergency service facilities. *Operations Research, 19* (6), 1363-1373.

White House, The. (2001). Executive order on critical infrastructure protection. http://www.whitehouse.gov/news/releases/2001/10/20011016-12.html (last accessed 25 April 2007).

White House, The. (2003). *The national strategy for the physical protection of critical infrastructures and key assets.* http://www.dhs.gov/files/publications/publication_0017.shtm (last accessed 26 May 2010).

Widener, M. J., & Horner, M. W. (2011). A hierarchical approach to modeling hurricane disaster relief goods distribution. *Journal of Transport Geography, 19* (4), 821-828.

Wollmer, R. (1964). Removing arcs from a network. *Operations Research, 12* (6), 934-940.

Zitzler, E., & Thiele, L. (1998). Multiobjective optimization using evolutionary algorithms: A comparative case study. *Parallel Problem Solving from Nature PPSN V, 1498*, 292-301.