

1	2 [000000]	3 [000000]	4 [000000]	5 [000000]
2		3 [100000]	4 [010000]	5 [001000]
3			4 [000100]	5 [000010]
4				5 [000001]

A3.9 References

There is a large literature on the topology of nets and polyhedra. Some references were given in § 7.11.10. The classic references are A. F. Wells' works: *Three-dimensional Nets and Polyhedra* [Wiley, New York (1977)] and *Further Studies of Three-dimensional Nets* [American Crystallographic Association Monograph No. 8 (1979)].

The number and sizes of rings in 4-connected nets has been discussed by C. S. Mariani & L. W. Hobbs, *J. Non-Crystalline Solids* **124**, 242 (1990); L. Stixrude & M. S. T. Bukowinski, *Amer. Mineral.* **75**, 1159 (1990); K. Goetzke & H.-J. Klien, *J. Non-Crystalline Solids* **127**, 215 (1991); M. O'Keeffe, *Zeits. Kristallogr.* **196**, 21 (1991). On the density of three-dimensional nets and its relationship to ring size, see S. T. Hyde, *Acta Crystallogr.* **A50**, 753 (1994).

The topological characterization of linkages of polyhedra has also given rise to quite a large literature. Some recent papers include E. Parthé, *Zeits. Kristallogr.* **189**, 101 (1989); E. Parthé & B. Chabot, *Acta Crystallogr.* **B46**, 7 (1990); N. Engel, *Acta Crystallogr.* **B47**, 217 (1991).

A topological topic, which we don't discuss, but which is nevertheless of considerable interest, is that of *percolation* in nets. A good introduction to this topic is D. Stauffer, *Introduction to Percolation Theory* [Taylor & Francis (1985)].

For applications of topology to molecular chemistry see *Chemical Applications of Topology and Graph Theory* [R. B. King (ed.) Elsevier, Amsterdam (1983)], and *Graph Theory and Topology in Chemistry* [R. B. King & D. Rouvray, (eds.) Elsevier, Amsterdam (1987)].

The literature on periodic minimal surfaces is rapidly expanding. A good introduction is *Crystalline frameworks as hyperbolic films* by S. T. Hyde [in *Defects and Processes in the Solid State: Geoscience Applications*, J. N. Boland & J. D. Fitz Gerald (eds.), Elsevier, (1993)]. Other references (which should be consulted for illustrations) that emphasize crystal-chemical applications are: S. T. Hyde & S. Andersson, *Zeits. Kristallogr.* **174**, 225 and 237 (1986); H. G. von Schnering & R. Nesper, *Angew. Chem. (Int. Ed.)* **26**, 1059 (1987); S. Andersson, S. T. Hyde, K. Larsson & S. Lidin, *Chem. Rev.* **88**, 221 (1988); W. Fischer & E. Koch, *Acta Crystallogr.* **A45**, 726 (1989). E. Koch & W. Fischer, *Acta Crystallogr.* **A46**, 33 (1990). The last two references describes a number of surfaces. A number of applications to chemistry, physics and biology are described in a collection of papers in *J. Phys.* **C7** (1990).

APPENDIX 4

LARGE POLYHEDRA

A4.1 Introduction

In Chapter 5 we discussed some polyhedra with emphasis mainly on those polyhedra with a small number of vertices that commonly occur as coordination figures. Here we discuss some larger polyhedra (with more than 20 vertices) that are of increasing interest in several areas of chemistry and biochemistry.¹ First we review some basic material.

Simple polyhedra are those for which three edges meet at every vertex; clearly for V vertices there are $3V/2$ edges, so the number of vertices is even. *Simplicial* polyhedra have only triangular faces. It should be obvious that they are the duals of simple polyhedra.

Polyhedra with F faces, all of which are either m -gons or $(m+1)$ -gons, where $m = [6 - 12/F]$,² are sometimes called *medial*. Their duals are simplicial polyhedra with m - and $(m+1)$ -connected vertices. For $m \leq 4$ these latter are topologically equivalent to the deltahedra of § 5.1.6 (i.e. they, and only they, can be realized with equilateral triangles as faces). The interest in this appendix is mainly with the case of simple medial polyhedra with $m = 5$, i.e. those simple polyhedra with pentagon and hexagon faces (and their duals). For convenience we refer to these polyhedra as 5-6 polyhedra in what follows. In general, polyhedra cannot be realized with all faces as plane regular polygons although they are often "almost" regular polygons.³

Non-crystallographic symmetries are commonly encountered; as well as icosahedral symmetry, 5-fold and $\bar{1}0$ and $\bar{1}2$ axes often occur. Now the (probably more familiar) Schoenflies symmetry symbols are more appropriately used. Commonly encountered non-crystallographic symmetries in Hermann-Mauguin notation are $D_{6d} = \bar{1}2m2$, $D_{5h} = \bar{1}0m2$ and $D_{5d} = \bar{5}m$. We use the number of vertices to identify the polyhedron as this is generally more useful in chemistry (i.e. it is the number of atoms making up the polyhedron); the notation V_N refers to a polyhedron with N vertices.

A4.2 5-6 Polyhedra

For 5-6 polyhedra there are exactly 12 pentagon faces and for V vertices there are $E =$

¹See e.g. T. G. Schmalz et al., *J. Amer. Chem. Soc.* **110**, 1113 (1988) for chemical applications and D. L. D. Caspar & A. Klug, *Cold Spring Harbor Symp. Quant. Biol.* **27**, 1 (1962) for biological applications.

²Here brackets indicate rounding down to the nearest integer.

³The dodecahedron and the truncated icosahedron are the only 5-6 polyhedra that can be constructed from regular plane polygons. See V. A. Zalgaller, *Convex Polyhedra with Regular Faces* [Consultants Bureau, New York (1969)].

$3V/2$ edges and $F = V/2 + 2$ faces.

The first member of this family is the pentagonal dodecahedron (V_{20}), already described in Chapter 5. This and some of the next members are encountered in the structures of the clathrate hydrates and clathrasils (see § 7.6 for V_{24} and V_{28} which are also illustrated in Figs. A4.1 and A4.2). Note that V_{22} is topologically impossible. For $V > 26$ there is more than one isomer: two for V_{28} (Fig. A4.2) and three for V_{30} (Fig. A4.3) and the number of isomers grows very rapidly with V so we only list some of the simpler cases in Table A4.1. The first four of the entries in the table are the duals of the Frank-Kasper polyhedra (§ 5.1.7, p. 143). It follows that these are the only 5-6 polyhedra in which hexagonal faces are completely surrounded by pentagons ("isolated" hexagons).

Table A4.1 Some smaller 5-6 polyhedra

V		symmetry
20	pentagonal dodecahedron	$I_h = m\bar{3}5$
24	14-hedron of Type I hydrates	$D_{6d} = \bar{1}2m2$
26	15-hedron	$D_{3h} = \bar{6}m2$
28	16-hedron of Type II hydrates	$T_d = \bar{4}3m$
28	isomer of above	$D_2 = 222$
30	pentagonal barrel	$D_{5h} = \bar{1}0m2$
30	isomer of above (9 kinds of vertex)	$C_{2v} = mm2$
30	isomer of above (10 kinds of vertex)	$C_{2v} = mm2$
36	hexagonal barrel	$D_{6h} = 6/mmm$
36	tennis ball	$D_{2d} = \bar{4}m2$
36	isomer of above	$D_{3h} = \bar{6}m2$

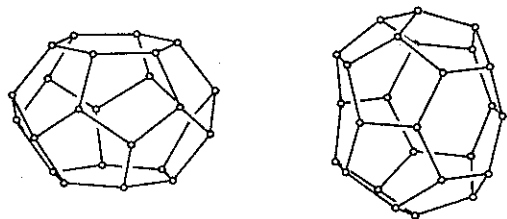


Fig. A4.1. The V_{24} (left) and V_{26} (right) polyhedra of Table A4.1.

The isomers of V_{28} are illustrated in Fig. A4.2 and those of V_{30} are illustrated in Fig. A4.3. It might be noted that the latter are easily inter-converted by rotation of two vertices and their connecting edge in the manner shown in Fig. 5.76 (p. 206).

Also listed in the Table are three symmetrical isomers of V_{36} (see Fig. A4.3). The "tennis ball" is so named as the pentagons form a continuous edge-sharing strip that goes

round the polyhedron in the same fashion as the seam goes round a tennis ball (which has the same symmetry). This is the smallest 5-6 polyhedron in which there are no vertices 5^3 (i.e. vertices at which three pentagons meet).

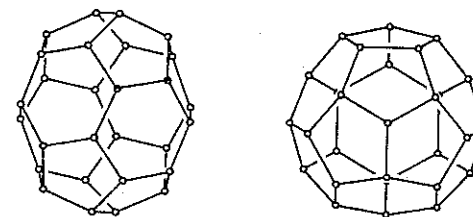


Fig. A4.2. The isomers of V_{28} (Table A4.1). Left: D_2 viewed down a 2-fold axis. Right: T_d viewed down a 3-fold axis.

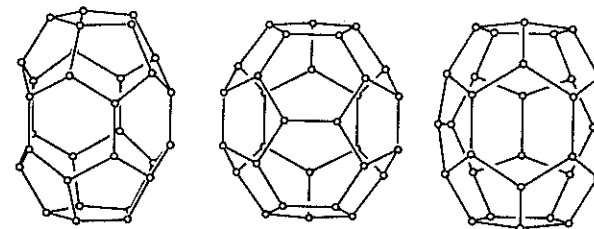


Fig. A4.3. The isomers of V_{30} (Table A4.1). Left: D_{5h} (pentagonal barrel). Middle and right: the two C_{2v} isomers viewed down the 2-fold axis.

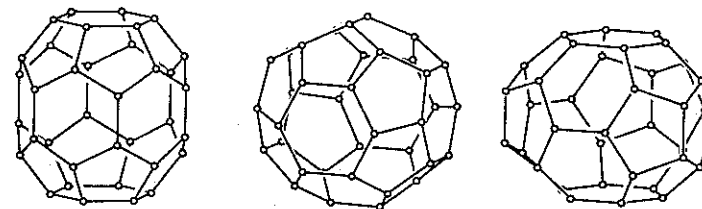


Fig. A4.4. Three isomers of V_{36} . Left: D_{6h} (hexagonal barrel). Middle: D_{2d} (tennis ball). Right: D_{3h} .

A4.3 Fullerene polyhedra

Large polyhedral carbon molecules C_n ($n \geq 60$) have come to be known collectively as *fullerenes*. These have structures based on 5-6 polyhedra in which pentagons are completely surrounded by hexagons. This restriction (known as the isolated pentagon rule

or IPR), which can be justified by simple chemical arguments, greatly reduces the number of possible structures. The simplest possibility, and the easiest to prepare pure, is C_{60} which has the structure of a truncated icosahedron (5.6^2). The next possibility is C_{70} which is also fairly easy to prepare. Thereafter all stoichiometries with an even number of vertices are realizable as IPR polyhedra. Fig. A4.5 illustrates the only possibilities for C_{70} , C_{72} and C_{74} and Fig. A4.6 illustrates the two possibilities for C_{76} ; interestingly, the D_2 isomer with 19 distinct vertices is favored over the T_d isomer with only 5 kinds of vertex.¹

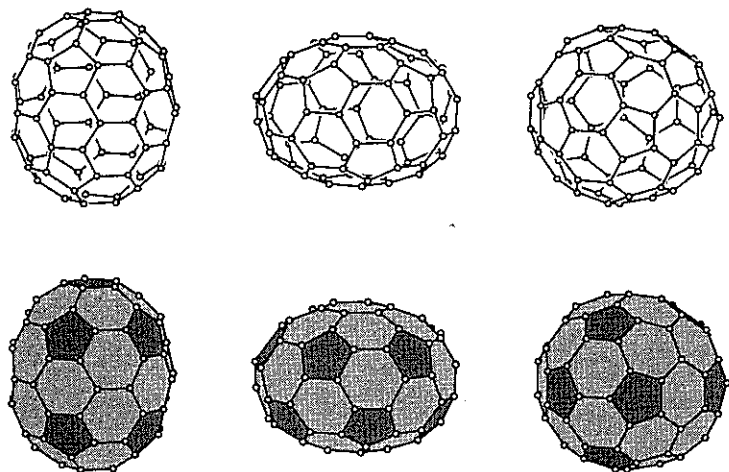


Fig. A4.5. From left to right: the structures of the fullerenes C_{70} , C_{72} and C_{74} . On the top as "ball and stick" models, and below as opaque polyhedra.

The numbers of IPR isomers and symmetries of small fullerene polyhedra are given in Table A4.2.² The numbers grow rapidly; for 82 vertices there are 9 isomers, for 84 vertices there are 24 isomers and for 96 vertices there are 196 isomers.

¹There have been very many reviews, conference proceedings, etc. describing fullerenes. A collection of articles appears in *Accounts of Chemical Research* 25, No. 3 (1992). A good review with many illustrations and references is *Electronic Structure Calculations on Fullerenes and Their Derivatives* by J. Cioslowski [Oxford University Press (1995)]. Like other new materials that have brought so much excitement to solid state chemistry in recent years (quasicrystals, oxide superconductors) fullerenes are very easy and inexpensive to prepare. Indeed, it has been said, with some justification, that the greatest obstacle to fullerene synthesis was recognition of how remarkably simple it could be; and, as was the case for the other materials mentioned, it took non-chemists (W. Krätschmer and D. R. Huffman in this instance) to lead the way.

²We give the highest symmetry of polyhedra; molecules which are predicted to have degenerate ground states with that symmetry will probably distort to lower symmetry in accordance with the predictions of the Jahn-Teller theorem (this is the case for $T_d C_{76}$ and $I_h C_{80}$ for example).

Possible cubic symmetries for fullerene polyhedra are $T(23)$, $T_d(\bar{4}3m)$ and $T_h(m\bar{3})$. Compositions for T_d include C_{76} , and C_{84} . Fig. A4.7 illustrates C_{116} with T_h symmetry (the smallest IPR fullerene with this symmetry) and C_{120} with T_d symmetry. The smallest IPR polyhedron with T symmetry is C_{88} .¹ Icosahedral polyhedra are discussed in the next section.

Table A4.2 Symmetries of the isomers of the first few fullerenes.

V	isomers	symmetries
60	1	I_h
70	1	D_{5h}
72	1	D_{6d}
74	1	D_{3h}
76	2	T_d, D_2
78	5	$D_{3h}(2), D_3, C_{2v}(2)$
80	6	$I_h, D_{5d}, D_{5h}, D_2, C_{2v}(2)$

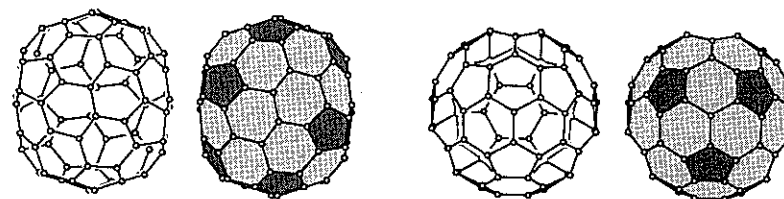


Fig. A4.6. The structures of the isomers of C_{76} . Left: D_2 projected down a 2-fold axis. Right: T_d projected down a 3-fold axis (note that some vertices and edges are superimposed in the projection).

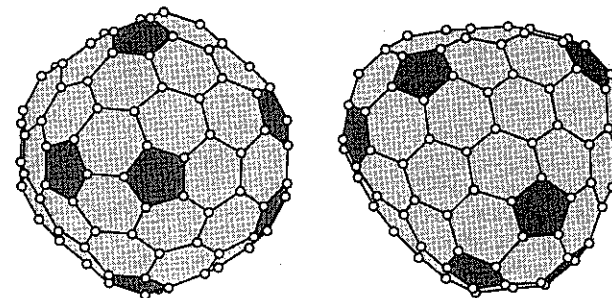


Fig. A4.7. The structures of the fullerene polyhedra $T_h C_{116}$ (left) and $T_d C_{120}$ (right).

Large fullerene (IPR) polyhedra can be generated from smaller 5-6 polyhedra by a

¹The interested reader will find that knowing the number of vertices and the symmetry makes it easy to construct models of the more-symmetrical polyhedra that are not illustrated (see Notes).

process known as "leap-frogging." In this method, edges joining two new vertices are placed as a perpendicular bisector of each of the edges of the parent polyhedron. The new vertices form (smaller) hexagons inside the original hexagons and pentagons inside the original pentagons; additionally each of the old vertices is in the center of a new hexagon as shown in Fig. A4.8. It should be clear that even if the original polyhedron had adjoining pentagonal faces, the new polyhedron will have isolated pentagonal faces. Indeed the truncated icosahedron 5.6^2 (V_{60}) is obtained by leap-frogging from the dodecahedron 5^3 (V_{20}). A polyhedron that is obtained by leap-frogging must have three times the number of vertices of the original polyhedron and hence a multiple of six vertices. Note that only one of the five isomers of C_{78} can be obtained by leap-frogging from V_{26} .

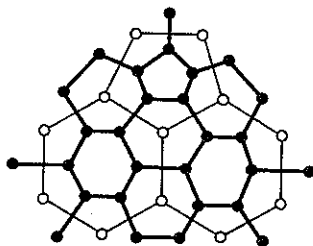


Fig. A4.8. Generating part of a larger polyhedron (heavy lines and filled circles) from a smaller polyhedron (lighter lines and open circles) by leap-frogging.

A4.4. Icosahedral polyhedra

We discuss here only 5-6 polyhedra and their duals.¹ The icosahedral 5-6 polyhedra are of interest as symmetrical isomers of larger fullerenes and their duals are of considerable interest in connection with virus structures and have been extensively investigated in that connection.² To have icosahedral symmetry each of the twelve pentagons centers have to be on 5-fold axes. The remaining surface of the polyhedron is made up of hexagons. The arcs of great circles joining each pair of 5-fold axes of icosahedral symmetry enclose an equilateral spherical triangle covering $1/20$ of a sphere as illustrated in Fig. A4.9.

To see how the polyhedron is developed, it is convenient to consider such a triangular patch which will be made up of a 6^3 net with the hexagons at the corners replaced by

¹These icosahedral polyhedra are sometimes called Goldberg polyhedra, as they appear to have first been described by M. Goldberg, *Tohoku Math. J.* 43, 104 (1937). We must admit to not having read this paper. After discovery of the fullerenes, many authors (including us) independently "rediscovered" the icosahedral polyhedra. A treatment similar to the present one is to be found in T. G. Schmalz *et al.*, *J. Amer. Chem. Soc.* 110, 1113 (1988) who give further references and describe some other polyhedra.

²A beautiful picture of the polyoma virus which is based on the dual of 2,1 (see the discussion below) appeared on the cover of the February 13, 1992 issue of *Nature*. See J. P. Griffith *et al.*, *Nature* 335, 652 (1992).

pentagons.

Fig. A4.10 shows a fragment of 6^3 with a triangular tile that is half a unit cell outlined. The tile contains one point. Suppose we take a larger hexagonal cell with new axes \mathbf{a}' and \mathbf{b}' related to the axes \mathbf{a} and \mathbf{b} of the elementary cell by:

$$\begin{pmatrix} \mathbf{a}' \\ \mathbf{b}' \end{pmatrix} = \begin{pmatrix} p+q & q \\ -q & p \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} \quad \text{A4.1}$$

The new cell is larger by a factor equal to the determinant of the matrix above, i.e. $p^2 + pq + q^2$ and a triangular tile (again one half of a unit cell) will contain $p^2 + pq + q^2$ points. If now the hexagons at the corners of the triangle are changed to pentagons we will have a patch corresponding to $1/20$ of a polyhedron. The example of $p, q = 3, 2$ is shown in Fig. A4.11 and basic units for some smaller polyhedra shown in Fig. A4.12.

There are distinct icosahedral polyhedra corresponding to each distinct pair p, q with $p \geq q \geq 0$. The number of vertices is $V = 20(p^2 + pq + q^2)$. The number of edges is $3V/2$ and the number of faces (of which 12 are pentagons) is $V/2 + 2$. For the symmetry to be I_h , the edges of the triangles must lie on mirror planes; it should be evident that this is only the case if $p = q$ or $q = 0$, otherwise the symmetry is I . Note that there can be distinct icosahedral polyhedra with the same number of vertices; the first case is for $p, q = 7, 0$ and $5, 3$ each of which have 980 vertices.

The first few simple icosahedral polyhedra with hexagon and pentagon faces are listed in Table A4.3 below. For their duals interchange V and F . As the number of vertices increases the shape of the polyhedron tends to that of an icosahedron with pentagons at the twelve vertices. Fig. A4.13 illustrates V_{240} .



Fig. A4.9. Some of the icosahedral symmetry elements. On the left $2, \bar{5}$ and $\bar{3}$ axes of I_h are shown with heavy lines representing the traces of mirror planes. On the right are shown the corresponding 2, 3 and 5 axes of I .

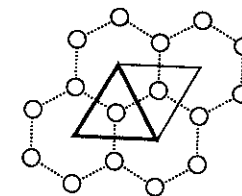


Fig. A4.10. A fragment of 6^3 showing a unit cell and a triangular tile.

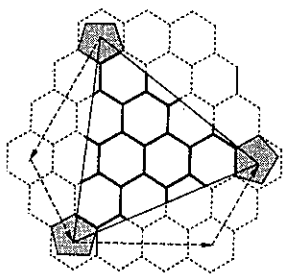


Fig. A4.11. A tile of the icosahedral polyhedron 3,2 (triangular outline). The arrows correspond to vectors of length $3a$ and $2a$ (a is the distance between centers of hexagons). It might be verified that there are $3^2 + 3 \times 2 + 2^2 = 19$ vertices in the tile.

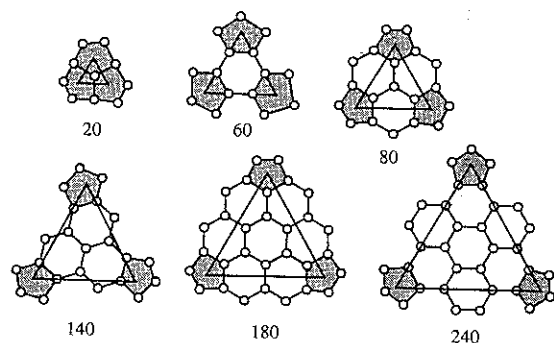


Fig. A4.12. Triangular patches of icosahedral polyhedra. The number under each diagram is the number of vertices in the full polyhedron.

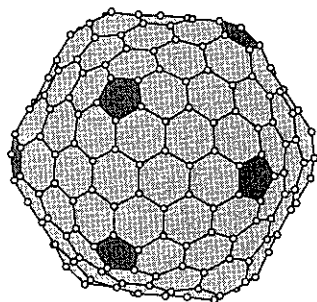


Fig. A4.13. The icosahedral polyhedron V_{240} .

Table A4.3 Properties of the first few icosahedral 5-6 polyhedra. "pg" is the point group and k_V , k_E and k_F are respectively the numbers of topologically-distinct vertices, edges and faces.

p,q	V	E	F	pg	k_V	k_E	k_F
1,0	20	30	12	I_h	1	1	1
1,1	60	90	32	I_h	1	2	2
2,0	80	120	42	I_h	2	2	2
2,1	140	210	72	I	3	4	2
3,0	180	270	92	I_h	3	4	3
2,2	240	360	122	I_h	3	3	4
3,1	260	390	132	I	5	7	3
4,0	320	480	162	I_h	5	6	4
3,2	380	570	192	I	7	10	4
4,1	420	630	212	I	7	11	5
5,0	500	750	252	I_h	7	9	5
3,3	540	810	272	I_h	6	9	5

A4.5 Space filling packings of 5-6 polyhedra

The structures of two clathrate hydrates (Types I and II) were mentioned in § 7.6. These are based on space-filling packings of dodecahedra (V_{20}) and, respectively, 14-hedra (V_{24}) and 16-hedra (V_{28}). It is also possible to fill space with combinations of V_{20} , V_{24} and V_{26} . These structures are difficult to illustrate clearly, but it is easy to make models of them (see § A4.7). The simplest is a combination of V_{20} , V_{24} and V_{26} in the ratio 3:2:2, for want of a better name we call this structure III.¹ Crystallographic data for unit edge length are:

Packing III	$P6/mmm$, $a = 4.401$, $c = 4.399$
	5.5.5.5.5.5 in 4 h , $x = 1/3$, $y = 2/3$, $z = 0.1137$
	5.5.5.5.5.6 in 6 l , $x = 0.1312$, $y = 2x$, $z = 0.0$
	5.5.5.5.6.6 in 6 k , $x = 0.2272$, $y = 0$, $z = 1/2$
	5.5.5.5.5.5 in 12 o , $x = 0.2085$, $y = 2x$, $z = 0.1386$
	5.5.5.5.5.6 in 12 n , $x = 0.3579$, $y = 0$, $z = 0.3140$

Another combination is known in the hydrate of tetra n -butyl ammonium benzoate which we call structure IV.² The relative proportions of polyhedra and the average ring size, $\langle n \rangle$, are listed in the table below.

The small range of average ring size is striking. Obviously $5 \leq \langle n \rangle < 6$ for any packing of 5-6 polyhedra, but it would be nice to have tighter bounds and to know the largest

¹This is the framework of the hydrate of tetra *iso*-amyl ammonium fluoride. The real crystal structure is orthorhombic [D. Feil & G. A. Jeffrey, *J. Chem. Phys.* **35**, 1863 (1961)].

²This is a rather complex structure: there are 172 vertices of 17 crystallographic kinds in the tetragonal cell. See M. Bonamico, G. A. Jeffrey & R. K. McMillan, *J. Chem. Phys.* **37**, 2219 (1962). It should be remarked that much of our knowledge of hydrate structures is due to the work of Jeffrey and collaborators published in the early 1960s.

polyhedron which can participate in such packings. We are not aware of a *proof* that a space filling by polyhedra topologically equivalent to pentagonal dodecahedra is impossible. Notice that these packings only involve polyhedra with isolated hexagons.

structure	V_{20}	V_{24}	V_{26}	V_{28}	$\langle n \rangle$
I	1	3			5.111
II	2			1	5.100
III	3	2	2		5.106
IV	5	8	2		5.109

A4.6 Large coordination polyhedra

In § 5.1.7 (p. 143) we described the Frank-Kasper polyhedra which are simplicial polyhedra with 3^5 and 3^6 vertices and commonly found as coordination polyhedra in intermetallic compounds. The 16-vertex tetra-capped truncated tetrahedron (Friauf polyhedron) is the largest such polyhedron without adjacent 3^6 vertices (i.e. 3^6 vertices sharing an edge). To have larger coordination numbers without adjacent 3^6 vertices, the coordination polyhedra must have n -gon faces with $n > 3$. An example is provided by the snub cube ($3^4.4$) which has triangular and quadrangular faces and occurs as the $\{\text{Na}\}\text{Zn}_{24}$ polyhedron in NaZn_{13} (p. 273); it is shown again in Fig. A4.14.

There are polyhedra with $3^4.4$, 3^5 , and 3^6 vertices in which the 3^6 vertices are not adjacent. The largest is perhaps the polyhedron obtained by capping the eight hexagonal faces of the truncated octahedron (4.6^2). The resulting polyhedron (Fig. 14.14) has 32 vertices (8×3^6 and $24 \times 3^4.5$).

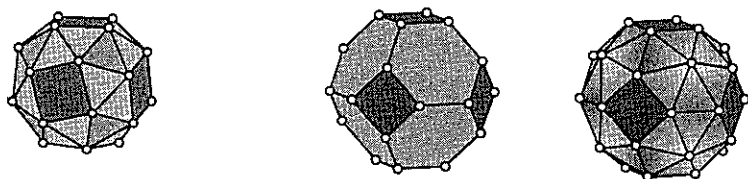


Fig. A4.14. Left: the snub cube ($3^4.4$). Right: the truncated octahedron (4.6^2) and the octacapped truncated octahedron with 32 vertices (in the conformation shown they are all equidistant from the center).

In the structures of BaHg_{11} and ThMn_{12} (these are the prototypes of fairly large families) there is a tetragonal ($4/mmm$) 20-vertex coordination polyhedron ($\{\text{Ba}\}\text{Hg}_{20}$ or $\{\text{Th}\}\text{Mn}_{20}$). This polyhedron can be considered to be derived by capping the hexagonal faces of a polyhedron [$3^8.4^2.6^4$] known as a "tetragonal hexagon prism"¹ to produce a polyhedron with four 3^6 vertices, eight 3^5 vertices and eight $3^5.4$ vertices as shown in Fig.

¹The term "tetragonal hexagon prism" is used by E. Hellner & W. B. Pearson [*Physik Daten / Physics Data* 16, Nr. 5 (1986)] who discuss its occurrence in intermetallic compounds. The polyhedron is not a prism in the usual sense.

A4.15.

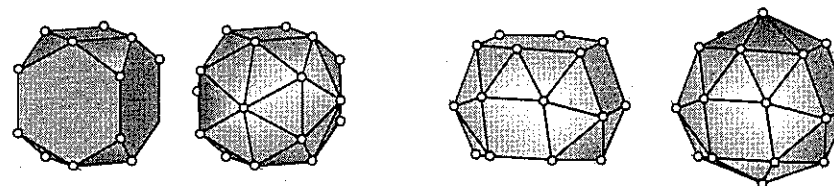


Fig. A4.15. Left: The "tetragonal hexagon prism" and the 20-vertex polyhedron obtained by capping its hexagonal faces. Right: a polyhedron with two hexagonal faces (top and bottom) and the polyhedron with 22 vertices obtained by capping the hexagonal faces.

Another family of structures is named for BaCd_{11} . In this structure there are $\{\text{Ba}\}\text{Cd}_{22}$ polyhedra. These are again tetragonal (now the symmetry is $\bar{4}m2$) and contain two 3^6 vertices, four 3^5 vertices and sixteen $3^4.5$ vertices. This is shown in Fig. A4.15 together with the polyhedron obtained by removing the two 3^6 vertices. The following table, listing the number of vertices and faces in the four polyhedra with triangular and quadrangular faces, suggests that there are more polyhedra between V_{24} and V_{32} .

polyhedron	3^5	3^6	$3^4.4$	triangles	quadrangles
V_{20}	8	4	8	32	2
V_{22}	4	2	16	32	4
V_{24}	0	0	24	32	6
V_{32}	0	8	24	48	6

A4.7 Models of large polyhedra

Most of the polyhedra describe here have irregular faces, so constructing models with (e.g.) cardboard faces is rather difficult. However satisfying "ball and stick" models can be made using tetrahedral or triangular stars. For smaller polyhedra, the tetrahedral star is more suitable (compare the angles of 108° in V_{20} with the tetrahedral angle of 109.5°).

The net III of § A4.5 may be made the following way: (a) construct a column of V_{24} polyhedra sharing hexagon faces, (b) surround the neck between alternate pairs of these polyhedra with a ring of six V_{20} sharing the exposed pentagon faces (it will be found that there is only one way to do this). It will now be found that a ring of V_{26} 's fits snugly (again sharing pentagon faces) in the depressions of the new structure. Remember to keep the hexagon faces of V_{26} parallel to the axis of the original column (which is the c axis).

To make large fullerene polyhedra 3-pointed "stars" are best. A good strategy is to construct the 12 isolated pentagons first, and then to explore ways in which they can be linked using the appropriate number of connectors of a different color (these are 6^3).