

Sections Covered: 2.1–2.9, 3.1, 3.2, 3.4–3.11, 4.1–4.7, 4.9, 4.10, 5.1–5.5

Sample Problems (Not necessarily all inclusive. The final exam may include questions on any topic covered in the course)

Find  $\frac{dy}{dx}$

1.  $y = (2x + 1)^2$

2.  $y = \frac{1}{3x^2 + 1}$

3.  $y = \frac{x-1}{x+1}$

4.  $y = \frac{2x^2 - 1}{3x + 5}$

5.  $y = \sin^3 x$

6.  $y = \cos(3x^2 + 11x)$

7.  $y = (3x^3 - 11x)^7$

8.  $y^3 + 7y = x^3$

9.  $4x^2y - 3y = x^3 - 1$

10.  $y = x \sec x$

11.  $y = \sin^{-1} e^x$

12.  $y = \tan^{-1}\left(\frac{1-x}{1+x}\right)$

13.  $y = \ln x^3 + (\ln x)^3$

14.  $y = \frac{\ln x}{x^2}$

15.  $y = x^2 e^x$

16.  $\frac{1}{\sqrt{1+x^2}} e^{xy} + y = 2$

17.  $y = 6^{x^2}$

18.  $y = 2^{e^x}$

19. Find the equation of the tangent line to  $y = \frac{1}{x^2 + 1}$  at the point  $(1, \frac{1}{2})$  and use it to estimate  $f(1.06)$ .

20. A ball is thrown upward from the top of a building 50 meters high with an initial velocity of 20 m/sec. With what speed did it hit the ground? ( $a = -9.8 \text{ m/sec}^2$ )

21. Water is pouring into a conical cistern at the rate of  $3 \text{ m}^3/\text{min}$ . If the height of the cistern is 4 meters and the radius of the top is 1 meter, how fast is the water rising when the water is 2 meters deep? ( $V = (1/3)\pi r^2 h$ )

22. A plane flying north at 640 miles per hour passes over a certain town at noon and a second plane going east at 600 miles per hour passes over the same town 15 minutes later. How fast will they be separating at 1:15 p.m.?

23. Find the limits

a)  $\lim_{x \rightarrow \infty} \frac{2x^2 - 3x + 1}{x^2 - 2}$

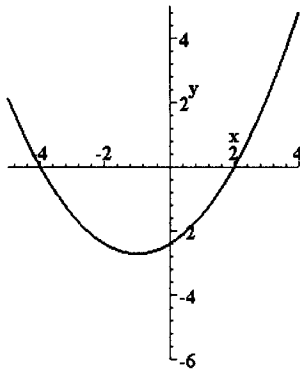
b)  $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x}$

c)  $\lim_{x \rightarrow 2^+} \frac{3}{x - 2}$

d)  $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

24. An open box with a capacity of 36,000 cubic inches is needed. If the box is to be twice as long as it is wide, what dimensions would require the least amount of material?

25. A page of a book is to contain 27 square inches of printed matter. If the margins at the top, bottom, and one side are 2 inches and the margin at the other side is one inch, what size page would use the least amount of paper?
26.  $f(x) = x^3 - 3x - 5$  has a root between 2 and 3. Find it correct to 5 decimal places using Newton's method.
27. Approximate  $\int_0^2 (4 - x^2) dx$  using the average of the left and right sums with 4 subintervals.
28. If the brakes of a car, when fully applied, produce a constant deceleration of 11 feet per second per second, what is the shortest distance in which the car can be braked to a stop from a speed of 60 miles per hour? (88 ft./sec.)
29. Given the graph of  $f'$ , sketch possible graphs of  $f$  and  $f''$ .



30. The acceleration function (in  $m/sec^2$ ) and the initial velocity (in  $m/sec$ ) are given for a particle moving along a line. Find the distance traveled during the given time interval.  
 $a(t) = t + 4$ ,  $v(0) = 5$ ,  $0 \leq t \leq 3$

Integrate

- |   |  |
|---|--|
| 31. $\int_0^4  x-1  dx$                   | 32. $\int x\sqrt{x^2-4} dx$                  |
| 33. $\int \frac{dx}{(x+2)^2}$             | 34. $\int (5x^4 - 3x^2 + 5) dx$              |
| 35. $\int \frac{x^2+1}{\sqrt{x^3+3x}} dx$ | 36. $\int \frac{1-x^4}{2x^2} dx$             |
| 37. $\int \sin^2 3x \cos 3x dx$           | 38. $\int \frac{\sin x + \cos x}{\cos x} dx$ |
| 39. $\int \frac{e^x dx}{1+e^{2x}}$        | 40. $\int \frac{dx}{\sqrt{1-3x^2}}$          |
| 41. $\int \frac{x dx}{\sqrt{9x^2-9}}$     | 42. $\int \frac{x dx}{9x^2-9}$               |

43. For the following function state where  $f$  is discontinuous and where  $f$  is not differentiable.

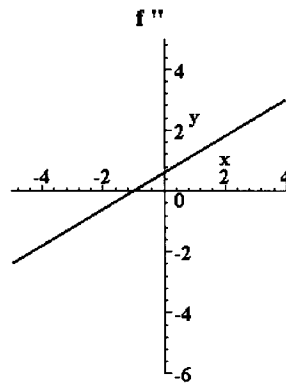
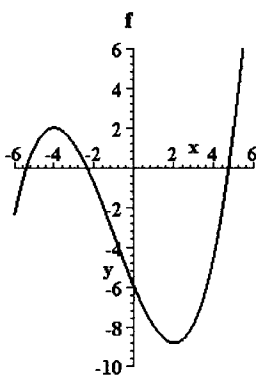
$$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 5 - x & \text{if } 0 < x < 4 \\ \frac{1}{5-x} & \text{if } x \geq 4 \end{cases}$$

44. Use the limit definition of the derivative to find  $f'(x)$  if  $f(x) = 4 - x^2$ .

Sample Answers

1.  $8x + 4$       2.  $\frac{-6x}{(3x^2 + 1)^2}$       3.  $\frac{2}{(x+1)^2}$       4.  $\frac{6x^2 + 20x + 3}{(3x+5)^2}$       5.  $3\sin^2 x \cos x$
6.  $-(6x+11)\sin(3x^2+11x)$       7.  $7(9x^2-11)(3x^3-11x)^6$       8.  $\frac{3x^2}{3y^2+7}$       9.  $\frac{3x^2-8xy}{4x^2-3}$
10.  $\sec x + x \sec x \tan x$       11.  $\frac{e^x}{\sqrt{1-e^{2x}}}$       12.  $-\frac{1}{1+x^2}$       13.  $\frac{3+3(\ln x)^2}{x}$       14.  $\frac{1-2\ln x}{x^3}$
15.  $x^2 e^x + 2x e^x$       16.  $\frac{y(y-2)}{2x-xy+1}$       17.  $2x(\ln 6)6^{x^2}$       18.  $e^x(\ln 2)2^{e^x}$       19.  $y = -\frac{1}{2}x + 1, 0.47$
20. 37.13 m/sec.      21. 3.82 m/min.      22. 872 mph      23. a) 2    b) 0    c)  $\infty$     d) 2
24. 30 x 60 x 20      25. 7.5 x 10      26. 2.27902      27. L=6.25, R=4.25, Av.=5.25
28. 352 feet

29.



30. 37.5 ml      31. 5      32.  $\frac{1}{3}(x^2 - 4)^{\frac{3}{2}} + C$       33.  $\frac{-1}{x+2} + C$
34.  $x^5 - x^3 + 5x + C$       35.  $\frac{2}{3}\sqrt{x^3 + 3x} + C$       36.  $\frac{-1}{2x} - \frac{x^3}{6} + C$       37.  $\frac{1}{9}\sin^3 3x + C$
38.  $-\ln|\cos x| + x + C$       39.  $\tan^{-1} e^x + C$       40.  $\sin^{-1}(\sqrt{3}x) + C$
41.  $\frac{1}{3}\sqrt{x^2 - 1} + C$       42.  $\frac{1}{18}\ln|9x^2 - 9| + C$

43. Discontinuous at  $x = 0$ ,  $x = 5$ , not differentiable at  $x = 0$ ,  $x = 4$ ,  $x = 5$ .

44. We all know the answer is  $-2x$ , but did you find it using the correct method?