Final Exam Review

Sections Covered: 2.1–2.9, 3.1, 3.2, 3.4–3.11, 4.1–4.7, 4.9, 4.10, 5.1–5.5

Sample Problems (Not necessarily all inclusive. The final exam may include questions on any topic covered in the course)

Find \( \frac{dy}{dx} \)

1. \( y = (2x + 1)^2 \)
2. \( y = \frac{1}{3x^2 + 1} \)
3. \( y = \frac{x-1}{x+1} \)
4. \( y = \frac{2x^2 - 1}{3x+5} \)
5. \( y = \sin^3 x \)
6. \( y = \cos(3x^2 + 11x) \)
7. \( y = (3x^2 - 11x)^7 \)
8. \( y^3 + 7y = x^3 \)
9. \( 4x^2 y - 3y = x^3 - 1 \)
10. \( y = x \sec x \)
11. \( y = \sin^{-1} e^x \)
12. \( y = \tan^{-1} \left( \frac{1-x}{1+x} \right) \)
13. \( y = \ln x^3 + (\ln x)^3 \)
14. \( y = \frac{\ln x}{x^2} \)
15. \( y = x^2 e^x \)
16. \( \frac{1}{\sqrt{1+x^2}} e^y + y = 2 \)
17. \( y = 6^x \)
18. \( y = 2^{e^x} \)
19. Find the equation of the tangent line to \( y = \frac{1}{x^2 + 1} \) at the point \( \left( 1, \frac{1}{2} \right) \) and use it to estimate \( f(1.06) \).

20. A ball is thrown upward from the top of a building 50 meters high with an initial velocity of 20 m/sec. With what speed did it hit the ground? \( \left( a = -9.8 \text{ m/sec}^2 \right) \)

21. Water is pouring into a conical cistern at the rate of 3 \( \text{m}^3/\text{min} \). If the height of the cistern is 4 meters and the radius of the top is 1 meter, how fast is the water rising when the water is 2 meters deep? \( \left( V = \frac{1}{3} \pi r^2 h \right) \)

22. A plane flying north at 640 miles per hour passes over a certain town at noon and a second plane going east at 600 miles per hour passes over the same town 15 minutes later. How fast will they be separating at 1:15 p.m.?

23. Find the limits
   a) \( \lim_{x \to \infty} \frac{2x^2 - 3x + 1}{x^2 - 2} \)
   b) \( \lim_{x \to 0} \frac{\cos x - 1}{x} \)
   c) \( \lim_{x \to 2} \frac{3}{x - 2} \)
   d) \( \lim_{x \to 1} \frac{x^2 - 1}{x - 1} \)

24. An open box with a capacity of 36,000 cubic inches is needed. If the box is to be twice as long as it is wide, what dimensions would require the least amount of material?
25. A page of a book is to contain 27 square inches of printed matter. If the margins at the top, bottom, and one side are 2 inches and the margin at the other side is one inch, what size page would use the least amount of paper?

26. $f(x) = x^3 - 3x - 5$ has a root between 2 and 3. Find it correct to 5 decimal places using Newton's method.

27. Approximate $\int_0^2 (4 - x^2) \, dx$ using the average of the left and right sums with 4 subintervals.

28. If the brakes of a car, when fully applied, produce a constant deceleration of 11 feet per second per second, what is the shortest distance in which the car can be braked to a stop from a speed of 60 miles per hour? (88 ft./sec.)

29. Given the graph of $f'$, sketch possible graphs of $f$ and $f''$.

30. The acceleration function (in m/sec$^2$) and the initial velocity (in m/sec) are given for a particle moving along a line. Find the distance traveled during the given time interval.
\[ a(t) = t + 4, \quad v(0) = 5, \quad 0 \leq t \leq 3 \]

Integrate

31. $\int_0^1 |x - 1| \, dx$

32. $\int x \sqrt{x^2 - 4} \, dx$

33. $\int \frac{dx}{(x + 2)^2}$

34. $\int (5x^4 - 3x^2 + 5) \, dx$

35. $\int \frac{x^2 + 1}{\sqrt{x^3 + 3x}} \, dx$

36. $\int \frac{1 - x^4}{2x^2} \, dx$

37. $\int \sin^2 3x \cos 3x \, dx$

38. $\int \frac{\sin x + \cos x}{\cos x} \, dx$

39. $\int \frac{e^x \cos x}{1 + e^{2x}} \, dx$

40. $\int \frac{dx}{\sqrt{1 - 3x^2}}$

41. $\int \frac{x \, dx}{\sqrt{9x^2 - 9}}$

42. $\int \frac{x \, dx}{9x^2 - 9}$
43. For the following function state where \( f \) is discontinuous and where \( f \) is not differentiable.

\[
f(x) = \begin{cases} 
0 & \text{if } x \leq 0 \\
5 - x & \text{if } 0 < x < 4 \\
\frac{1}{5 - x} & \text{if } x \geq 4 
\end{cases}
\]

44. Use the limit definition of the derivative to find \( f'(x) \) if \( f(x) = 4 - x^2 \).

Sample Answers

1. 8x + 4
2. \( \frac{-6x}{(3x^2 + 1)^2} \)
3. \( \frac{2}{(x + 1)^2} \)
4. \( \frac{6x^2 + 20x + 3}{(3x + 5)^2} \)
5. \( 3\sin^2 x \cos x \)

6. \(-(6x + 11)\sin(3x^2 + 11x) \)
7. \( 7(9x^2 - 11)(3x^3 - 11x)^6 \)
8. \( \frac{3x^2}{3y^2 + 7} \)
9. \( \frac{3x^2 - 8xy}{4x^3 - 3} \)

10. \( \sec x + x \sec x \tan x \)
11. \( \frac{e^x}{\sqrt{1 - e^{2x}}} \)
12. \( -\frac{1}{1 + x^2} \)
13. \( \frac{3 + 3(\ln x)^2}{x} \)
14. \( \frac{1 - 2\ln x}{x^3} \)

15. \( x^2e^x + 2xe^x \)
16. \( \frac{y(y - 2)}{2x - xy + 1} \)
17. \( 2x(\ln 6)6^x \)
18. \( e^x(\ln 2)2^x \)
19. \( y = -\frac{1}{2}x + 1, \ 0.47 \)
20. 37.13 m/sec.
21. 3.82 m/min.
22. 872 mph
23. a) 2   b) 0   c) \( \infty \)   d) 2
24. 30 x 60 x 20
25. 7.5 x 10
26. 2.27902
27. L = 6.25, R = 4.25, Av. = 5.25
28. 352 feet

29.

30. 37.5 m1
31. 5
32. \( \frac{1}{3}(x^2 - 4)^\frac{3}{2} + C \)
33. \( \frac{-1}{x + 2} + C \)

34. \( x^5 - x^3 + 5x + C \)
35. \( \frac{2}{3}\sqrt{x^3 + 3x} + C \)
36. \( \frac{-1}{2x} - \frac{x^3}{6} + C \)
37. \( \frac{1}{9}\sin^3 3x + C \)

38. \( -\ln|\cos x| + x + C \)
39. \( \tan^{-1} e^x + C \)
40. \( \sin^{-1}(\sqrt{3}x) + C \)
41. \( \frac{1}{3}\sqrt{x^2 - 1} + C \)
42. \( \frac{1}{18}\ln|9x^2 - 9| + C \)

43. Discontinuous at \( x = 0, \ x = 5 \), not differentiable at \( x = 0, \ x = 4, \ x = 5 \).

44. We all know the answer is \(-2x\), but did you find it using the correct method?