MAT 270 **Test 1 Fall 1999** Name___________________

Each problem is worth 12 points. Be sure to explain, illustrate, and/or justify your method of solution.

1. Company A has fixed operating costs of \$600,000 per year and pays each employee an average of \$30,000 in salary and benefits while Company B has fixed operating costs of \$800,000 per year and pays each employee \$25,000 in salary and benefits. Write an equation which models each company's yearly operating costs and find how many employees each company would have if their yearly operating costs were equal.

$y_1 = 600 + 30x$ $\qquad$ $y_2 = 800 + 25x$

$600 + 30x = 800 + 25x$

$5x = 200$

$x = 40$ EMPLOYEES

2. The median price of a home in a certain area rose from \$50,000 in 1970 to \$100,000 in 1990. If it is assumed that homes prices are rising exponentially, find equations of the form $y = ab^t$ and $y = ae^{kt}$ which model home prices and use one of them to predict the year in which the median home price will be \$142,000.

$y = 50\, b^t$

$100 = 50\, b^{20}$

$2 = b^{20}$

$b = 2^{\frac{1}{20}}$

$b = 1.03526$

$y = 50\,(1.03526)^t$

or

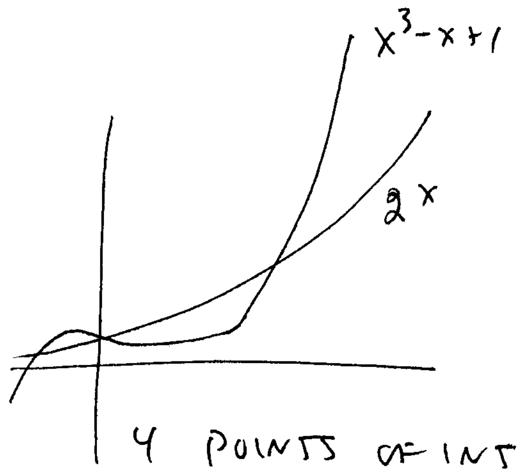
$y = 50\, e^{.03465t}$

$142 = 50\,(1.03526)^t$

$$t = \frac{\log\left(\frac{142}{50}\right)}{\log(1.03526)}$$

$= 30.1 \longrightarrow 2000$

3. Solve correct to 3 decimal places: $2^x = x^3 - x + 1$



$x = -1.212, 0, 1.488, 9.916$

4. Find the inverse of $f(x) = \dfrac{e^x}{1+e^x}$

$$x = \frac{e^y}{1+e^y}$$

$$y = \ln\left(\frac{x}{1-x}\right)$$

$$x + x e^y = e^y$$

$$x e^y - e^y = -x$$

$$e^y(x-1) = -x$$

$$e^y = \frac{-x}{x-1}$$

5. If $g(x) = 3 + x + e^x$, find $g^{-1}(2)$

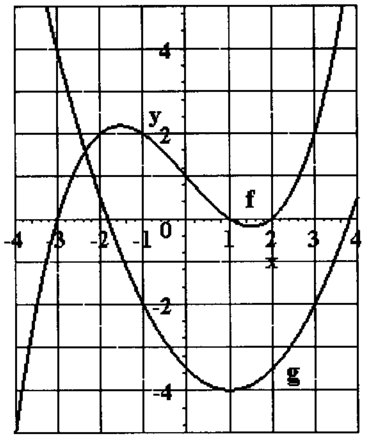
$$2 = 3 + x + e^x$$

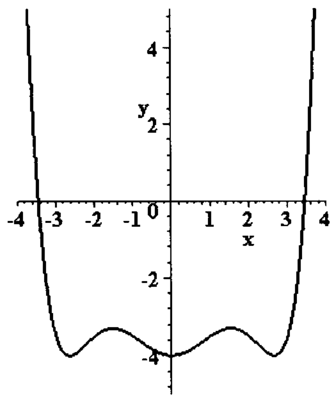
$$x = -1.278$$

SOLVE BY GRAPHING OR SOLVER

Name____________________

6. The graphs of f and g are shown below. Use them to sketch the graph of $g(f(x))$.





7. Find the limit if it exists. If it doesn't exist, show why.

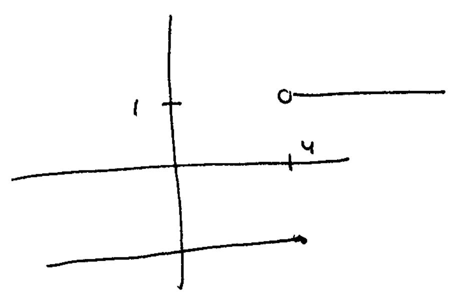
a) $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} \cdot \frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}$

$= \lim_{x\to 0} \frac{\cancel{x}\ 1}{\cancel{x}(\sqrt{2+x}+\sqrt{2})}$

$= \lim_{x\to 0} \frac{1}{\sqrt{2+x}+\sqrt{2}}$

$= \frac{1}{2\sqrt{2}}$

b) $\lim_{x\to 4} \frac{|x-4|}{x-4}$



LEFT + RIGHT HAND LIMITS $\neq$, SO LIMIT DNE

8. The function $f(t)=10t-4.9t^2$ gives the height of a ball in meters t seconds after it was thrown into the air with an initial velocity of 10 m/sec. Find the average velocities for the periods between $t=2$ and $t=2.5, 2.1,$ and 2.01. Then find an expression for the slope of the secant line between the points $(2, f(2))$ and $(2+h, f(2+h))$ and use it to find the instantaneous velocity of the ball at $t=2$.

$$\frac{f(2.5)-f(2)}{.5} = -12.05$$

$$\frac{f(2.1)-f(2)}{.1} = -10.09$$

$$\frac{f(2.01)-f(2)}{.01} = -9.649$$

$$\frac{f(2+h)-f(2)}{h} = \frac{10(2+h)-4.9(2+h)^2-(20-19.6)}{h}$$

$$= \frac{20+10h-4.9(4+4h+h^2)-20+19.6}{h}$$

$$= \frac{\cancel{20}+10h-\cancel{19.6}-19.6h-4.9h^2-\cancel{20}+\cancel{19.6}}{h}$$

$$= \frac{-9.6h-4.9h^2}{h}$$

$$= -9.6-4.9h$$

$$\lim_{h\to 0}(-9.6-4.9h) = -9.6$$