#From: A Course in Ordinary Differential Equations, 2nd Ed. by Stephen A. Wirkus & Randall J. Swift

#Chapter 1: Computer labs input and output

restart

#Maple Example 1

\[ f := x \rightarrow \cos(x) + e^{2x} + \ln(x) \]  
\[ \text{(1)} \]

\[ \text{diff} \left(f(x), x\right) \]  
\[ -\sin(x) + 2 e^{2x} + \frac{1}{x} \]  
\[ \text{(2)} \]

\[ \frac{d}{dx} f(x) \]  
\[ -\sin(x) + 2 e^{2x} + \frac{1}{x} \]  
\[ \text{(3)} \]

\[ \text{diff} \left(f(x), x \}$2) \]  
\[ -\cos(x) + 4 e^{2x} - \frac{1}{x^2} \]  
\[ \text{(4)} \]

\[ \frac{d^2}{dx^2} f(x) \]  
\[ \# \text{the second derivative symbol above was modified from the palette entry by inserting the power of 2} \]
\[ -\cos(x) + 4 e^{2x} - \frac{1}{x^2} \]  
\[ \text{(5)} \]

\[ \text{diff} \left(f(x), t\right) \]  
\[ 0 \]  
\[ \text{(6)} \]

\[ \frac{d}{dt} f(x) \]  
\[ 0 \]  
\[ \text{(7)} \]

\[ \text{int} \left(f(x), x\right) \]  
\[ \sin(x) + \frac{1}{2} e^{2x} + x \ln(x) \]  
\[ \text{(8)} \]

\[ \int f(x) \, dx \# \text{entered from Expression; if from Common Symbols, need to use d from there too} \]
\[ \sin(x) + \frac{1}{2} e^{2x} + x \ln(x) \]  
\[ \text{(9)} \]

\[ \text{int} \left(f(x), x = 0 .. \Pi\right) \]  
\[ -\frac{1}{2} + \ln(\Pi) \Pi + \frac{1}{2} e^{2 \Pi} - \Pi \]  
\[ \text{(10)} \]

\[ \int_{0}^{\Pi} f(x) \, dx \]  
\[ -\frac{1}{2} + \ln(\Pi) \Pi + \frac{1}{2} e^{2 \Pi} - \Pi \]  
\[ \text{(11)} \]
\[ \int f(x) \, dt \] (12)

\[ \left( \cos(x) + e^{2x} + \ln(x) \right) t \]

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\[ g := x \rightarrow \int_0^x f(t) \, dt \] (14)

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\[ g(\pi) \] (15)

\[ -\frac{1}{2} + \ln(\pi) \pi + \frac{1}{2} e^{2\pi} - \pi \]

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\[ \text{eval}(g(\pi)) \] (16)

\[ 267.7005103 \]

\# Maple Example 2

\texttt{restart}\# checking a solution, plotting implicit solutions

\texttt{eq1} := \frac{d^2}{dx^2} y(x) + 2 \cdot \frac{d}{dx} y(x) + y(x) = 0 \] (17)

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\[ \text{eval}\left(\text{subs}\left(y(x) = e^{-x}, \text{eq1}\right)\right) \text{# note that e is entered from the palette} \] (18)

\[ 0 = 0 \]

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\[ \text{eval}\left(\text{subs}\left(y(x) = x \cdot e^{-x}, \text{eq1}\right)\right) \text{# this is NOT a solution} \] (19)

\[ 0 = 0 \]

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\[ \text{eval}\left(\text{subs}\left(y(x) = e^{x}, \text{eq1}\right)\right) \] (20)

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\( x - 4\cdot y(x) + x^2 \cdot (y(x)^2 - 3) \cdot \frac{d}{dx} y(x) = 0 \] (21)

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\[ \text{sol2} := -\frac{1}{x} + \frac{2}{x^2} + \frac{1}{y} - \frac{1}{y^3} = C \] (22)

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\[ \frac{1}{x^2} + \frac{2}{x^2} + \frac{1}{y} - \frac{1}{y^3} = C \]

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\[ dsol2 := \text{implicitdiff}\left(\text{sol2}, y, x\right) \] (23)

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\[
\text{collect}\left(\text{expand}\left(y(x)^4 \cdot x^3 \cdot \text{dsol2}ight), \frac{d}{dx} y(x)\right)
\]
\[
\left(-y(x)^2 \cdot x^3 + 3 \cdot x^3\right) \left(\frac{d}{dx} y(x)\right) + y(x)^4 \cdot x^4 \equiv 0
\]

\text{(24)}

\[
\text{slove}\left(\text{eq2}, \frac{d}{dx} y(x)\right)
\]
\[
\frac{y(x)^4 (x - 4)}{x^3 (y(x)^2 - 3)}
\]

\text{(25)}

\[
\text{slove}\left(\text{dsol2}, \frac{d}{dx} y(x)\right)
\]
\[
\frac{y(x)^4 (x - 4)}{x^3 (y(x)^2 - 3)}
\]

\text{(26)}

\text{with}\left(\text{plots}\right): \#\text{loads package for implicit plotting}

\text{implicitplot}\left(\text{subs}\left(C = -5, \text{sol2}\right), x = -2 \ldots 2, y = -2 \ldots 2, \text{gridrefine} = 4, \text{title} = "Implicit Plot with C=-5"\right)
From "SteelBlue", "LightBlue"], title = "Implicit Plot with C=-10,-5,-1,0,1,2,6,50", legend = ["C=-10", "C=-5", "C=-1", "C=0", "C=1", "C=2", "C=6", "C=50"] # see plot, colornames

Implicit Plot with C=-10,-5,-1,0,1,2,6,50

# Maple Example 3
restart
with(DEtools):
with(plots):

eq3 := \((x^2 + 1) \cdot \frac{d}{dx} y(x) + 4 \cdot x \cdot y(x) = x\)

DEplot(eq3, y(x), x = 0..4, [y(0) = 10], linecolor = blue, arrows = none, title = "Numerical Approximation of Solution of \((x^2 + 1)y'(x) + 4xy(x) = x, y(0)=10\)"")
Closed Form Solution of $(x^2+1)y''(x)+4xy(x)=x$, $y(0)=10$

\[
y(x) = \frac{\frac{1}{4}x^4 + \frac{1}{2}x^2 + 10}{(x^2 + 1)^2}
\]
Maple Example 4

restart
with(DEtools):
with(plots):

\[
\begin{align*}
\text{eq3} & := (x^2 + 1) \cdot \frac{d}{dx} y(x) + 4 \cdot x \cdot y(x) = x \\
& = (x^2 + 1) \left( \frac{d}{dx} y(x) \right) + 4 \cdot x \cdot y(x) = x
\end{align*}
\] (29)

\text{DEplot(eq3, y(x), x = 0 .. 4, \{y(0) = 10\}, linecolor = blue, arrows = none, title = "Numerical Approximation of Solution of (x^2+1)y'(x)+4xy(x)=x,y(0)=10")}

Numerical Approximation of Solution of (x^2+1)y'(x)+4xy(x)=x,y(0)=10

\[
\begin{align*}
\text{sol3} & := \text{dsolve} \{ \text{eq3}, y(0) = 10 \} \\
y(x) & = \frac{\frac{1}{4} x^4 + \frac{1}{2} x^2 + 10}{(x^2 + 1)^2}
\end{align*}
\] (30)

\text{plot(rhs(sol3), x = 0 .. 5, color = "Gray", title = "Closed Form Solution of (x^2+1)y'(x)+4xy(x)=x,y(0)=10")}
Closed Form Solution of \((x^2 + 1)y'(x) + 4xy(x) = x, y(0) = 10\)