Table of \( z \) values and probabilities for the standard normal distribution. \( z \) is the first column plus the top row. Each cell shows \( P(X \leq z) \). For example \( P(X \leq 1.04) = .8508 \). For \( z < 0 \) subtract the value from 1, e.g., \( P(X \leq -1.04) = 1 - .8508 = .1492 \).

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Table of $t$ values and right tail probabilities. Degrees of freedom are in the first column (df). **Right tail probabilities** are in the first row. For example for $d.f. = 7$ and $\alpha = .05$ the critical $t$ value for a two-tail test is 2.365 and for $d.f. = 10$ and $\alpha = .1$ the critical $t$ value for a one-tail test is 1.372.

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Chapter 1: no key formulas.
Chapter 2: Relative Frequency = freq. of the class/n.
Approx. Class Width:
= (largest value - smallest value) / number of classes.
Chapter 3: sample and population means
\[ \bar{x} = \frac{\sum x_i}{n} \text{ and } \mu = \frac{\sum x_i}{N} \]
Weighted mean and geometric mean
\[ \bar{x} = \frac{\sum w_i x_i}{w_i} \text{ and } \bar{x}_g = \left( [x_1(x_2) \ldots (x_n)]^{1/n} \right. \]
Interquartile Range: IQR = Q_3 - Q_1.
Population and sample variance
\[ \sigma^2 = \frac{\sum (x_i - \mu)^2}{N} \text{ and } s^2 = \frac{\sum (x_i - \bar{x})^2}{n - 1} \]
Population and sample standard deviation
\[ \sigma = \sqrt{\sigma^2} \text{ and } s = \sqrt{s^2} \]
Coefficient of Variation
\[ \left( \frac{\text{Standard deviation}}{\text{Mean}} \times 100 \right) \%
\]
Z-Score: \[ z_i = \frac{x_i - \bar{x}}{s} \]
Population and Sample Covariance
\[ \sigma_{xy} = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{N} \text{ and } s_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n - 1} \]
Population and Sample Pearson Correlation
\[ \rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \text{ and } r_{xy} = \frac{s_{xy}}{s_x s_y} \]
Chapter 4: Counting Rule for Combinations
\[ C^n_N = \binom{N}{n} = \frac{N!}{n!(N-n)!} \]
Counting Rule for Permutations
\[ P^n_N = n! \binom{N}{n} = \frac{N!}{(N-n)!} \]
Probability Rules: \( P(A) = 1 - P(A^C) \)
Chapter 4 continued:
\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
\[ P(A|B) = \frac{P(A \cap B)}{P(B)} \]
\[ P(A \cap B) = P(B)P(A|B) = P(A)P(B|A) \]
Multiplication Law for Independent Events
\[ P(A \cap B) = P(B)P(A) \]
Bayes’ Theorem
\[ P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \cdots + P(A_n)P(B|A_n)} \]
Chapter 5: Discrete Uniform Probability Mass Function: \( f(x) = 1/n \).
Expected Value of a Discrete R.V.: \( E(x) = \mu = \sum xf(x) \).
Variance of a Discrete R.V.:
\[ Var(x) = \sigma^2 = \sum (x - \mu)^2 f(x) \]
Number of Experimental Outcomes Providing Exactly \( x \) Successes in \( n \) Trials
\[ \binom{n}{x} = \frac{n!}{x!(n-x)!} \]
Binomial Probability Mass Function
\[ P(X = x) = f(x) = \binom{n}{x} p^x (1-p)^{n-x} \]
Expected Value for Binomial Distribution: \( E(x) = \mu = np \).
Variance for Binomial Dist.: \( Var(x) = \sigma^2 = np(1-p) \).
Poisson Probability Mass Function:
\[ P(X = x | \mu) = f(x) = \frac{\mu^x e^{-\mu}}{x!} \]
Chapter 6: Uniform PDF
\[ f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases} \]
Normal PDF The density function is
\[ f(x) = \frac{1}{\sqrt{2\pi \sigma^2}} \exp \left( -\frac{(x - \mu)^2}{2\sigma^2} \right) \]
Converting to the Standard Normal rv:
\[ z = \frac{x - \mu}{\sigma} \]
Exponential PDF and CDF for \( x \geq 0 \)
\[ f(x) = \mu^{-1}e^{-x/\mu} \text{ and } P(x \leq x_0) = 1 - e^{-x_0/\mu} \]
Chapter 7: expected value of \( \bar{x} \)
\[ E(\bar{x}) = \mu \]
Standard Deviation of \( \bar{x} \) (Standard Error)
\[ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \]
Expected Value and Std Dev (Standard Error) of \( \bar{p} \)
\[ E(\bar{p}) = p \text{ and } \sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} \]
Finite Pop. Correction Factor: \( \sqrt{(N-n)/(N-1)} \).
Chapter 8: Interval Estimate of Population Mean, \( \sigma \) known and unknown
\[ \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \text{ and } \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \]
Necessary Sample Size for Interval Estimate of \( \mu \)
\[ n = \frac{(z_{\alpha/2})^2 \sigma^2}{E^2} \]
Chapter 8 continued: Interval Estimate of $p$

$\hat{p} \pm z_{\alpha/2} \frac{p(1-p)}{\sqrt{n}}$

Necessary Sample Size for Interval Estimate of $p$

$n = \left(\frac{z_{\alpha/2}}{p(1-p)}\right)^2$ \(\frac{E^2}{p(1-p)}\)

Chapter 9: Test Statistic for Hypothesis Tests About $\mu$, $\sigma$ known and unknown

$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$ and $t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$

Test Stat for Hypothesis About $p$

$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0) / n}}$

Chapter 10 continued: Test Statistic (Matched Samples)

$t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}}$

ANOVA Related:

$\bar{x}_j = \frac{\sum_{i=1}^{n_j} x_{ij}}{n_j}$ \hspace{1cm} $\bar{y}_j^* = \frac{\sum_{i=1}^{n_j} y_{ij}}{n_j}$ \hspace{1cm} $\bar{x} = \frac{\sum_{j=1}^{k} \sum_{i=1}^{n_j} x_{ij}}{n}$

$MSTR = \frac{SSTR}{k-1}$ \hspace{1cm} $SSTR = \sum_{j=1}^{k} n_j (\bar{x}_j - \bar{x})^2$ \hspace{1cm} $MSE = \frac{SSE}{n-k}$

$SSE = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (y_{ij} - \bar{y}_j)^2$ \hspace{1cm} $F = MSTR / MSE$

$SST = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (y_{ij} - \bar{y})^2$ \hspace{1cm} $SSR = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (\bar{y}_j - \bar{y})^2$

$R^2 = 1 - \frac{SSE}{SST}$ \hspace{1cm} $R^2 = \frac{MSR}{MSE}$

$MSR = \frac{SSR}{p}$ \hspace{1cm} $MSE = \frac{SSE}{n-p-1}$ \hspace{1cm} $F = \frac{MSR}{MSE}$

Chapter 11: not covered in this course

Chapter 12: y = $\beta_0 + \beta_1 x + \epsilon$

$E(y) = \beta_0 + \beta_1 x_0$ \hspace{1cm} $\hat{y} = b_0 + b_1 x$ \hspace{1cm} $b_0 = \bar{y} - b_1 \bar{x}$

$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$ \hspace{1cm} $SSE = \sum (y_i - \hat{y}_i)^2$

$SST = \sum (y_i - \bar{y})^2$ \hspace{1cm} $SSR = \sum (\bar{y}_j - \bar{y})^2$ \hspace{1cm} $\hat{y} = b_0 + b_1 x$

$r^2 = \frac{SSR}{SST}$ \hspace{1cm} $r_{xy} = (\text{sign of } b_1) \sqrt{r^2}$ \hspace{1cm} $s^2 = MSE = \frac{SSE}{n-2}$

Standard Error of the Estimate, $s = \sqrt{MSE}$.

$s_{b_1} = \frac{s}{\sqrt{\sum (x_i - \bar{x})^2}}$ \hspace{1cm} $b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$

For simple regression, $MSR = SSR$ because there is only one independent variable.

$F = \frac{MSR}{MSE}$ \hspace{1cm} $s_{\hat{y}} = \frac{s}{\sqrt{\sum (x_i - \bar{x})^2}}$ \hspace{1cm} $t = \frac{b_1}{s_{b_1}}$

Confidence Interval for $E(y^*)$: $\hat{y}^* \pm t_{\alpha/2}s_{\hat{y}}$

$s_p\overline{md} = \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$

Other Math Rule Reminders:

$c^2 = \exp(x)$

$\ln 1 = 0$ \hspace{1cm} $\ln e = 1$

$x! = (x)(x-1)(x-2) \cdots (2)(1)$

$0! = 1$ \hspace{1cm} $\chi^2 = 1$
VERSION A

Choose the best answer. Do not write letters in the margin or communicate with other students in any way. If you have a question note it on your exam and ask for clarification when your exam is returned. In the meantime choose the best answer. Neither the proctors nor Dr. Cox will answer questions during the exam.

Please check each question and possible answers thoroughly as questions at the bottom of a page sometimes run onto the next page.

This exam has 25 questions.
Relax - do a great job!
1. I have checked that my ID is bubbled in correctly. If it is bubbled in incorrectly I will get this question wrong. I also understand that questions and their possible answers may run onto the next page and so I should always check the top of the next page for possible answers. I understand that if I have a question I should simply make a note on my exam and ask Dr. Cox afterwards. I should always choose the best answer.

(a) False.
(b) I didn’t read the directions.
(c) True.

2. If there are multiple low outliers (values more than 1.5 IQR below the first quartile) and there are more observations between Q1 and Q2 than there are between Q2 and Q3 then the median will be below the mean.

(a) True
(b) False
(c) It depends on whether the median is the middle value.

3. A data set cannot contain both time series data and cross sectional data.

(a) True
(b) False

4. The graph below is a

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The decimal point is 1 digit(s) to the right of the |
1 | 899
2 | 0112344
2 | 555566788
3 | 023444
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(a) histogram
(b) box and whiskers plot
(c) scatter plot
(d) stem and leaf display

5. Using the Poisson PMF, when the mean is 18, what is $P(X = 16)$?

(a) 0.282596198358993
6. Suppose that the number of times a college student changes their major follows a Poisson distribution with a mean of 1. What is the probability that a student will change their major exactly 1 time(s)?

(a) 0.36789441171442
(b) 0.651819161757163
(c) 0.190727664702865
(d) 0.632120558828558

7. Suppose at ASU 50% of students take out some form of student loan to pay for school. In a class of 390 what is the expected number of students with a loan?

(a) 205
(b) 165
(c) 195
(d) 169.65

8. Which of the following characterizes the Poisson distribution?

(a) the mean is twice the standard deviation.
(b) it applies to random variables that take on any values from the non-negative reals (real numbers that are positive or 0).
(c) it applies to random variables that take on any values from the non-negative rationals (rational numbers that are positive or 0).
(d) it applies to random variables that take on any values from the non-negative integers (integers that are positive or 0).
(e) the PMF lacks a closed form solution.

9. Suppose you have a random variable that is uniformly distributed with a maximum of 259 and a minimum of 88. What is the expected value of this random variable?

(a) 193.5
(b) 158.5
(c) 225.55
10. Suppose that starting salaries for new graduates are roughly exponentially distributed. That is, salaries tend to group around a low end and then are skewed right with a few “rock star” students earning high salaries. If the average salary is $52228 then what is the probability of getting a salary of at least $65000?

(a) 0.711927518007993  
(b) 0.288072481992007  
(c) 0.144036240996003  
(d) 0.576144963984013

11. Suppose that the known standard deviation for the numbers of hours that students work in a week is 14.9. If I draw a sample of 29 what is the standard error?

(a) 2.76686053883807  
(b) 2.21348843107046  
(c) 0.513793103448276  
(d) 3.5969187004895

12. A confidence interval is constructed as $\bar{x} \pm \text{margin of error}$. 

(a) true  
(b) false  
(c) true if $\alpha < .5$ and false if $\alpha \geq .5$

13. Suppose that in a recent sample of 33 recent graduates with business degrees the mean starting salary is 54789 and the standard deviation is 1770. Construct a 99% confidence interval for the mean starting salary for business majors.

(a) [54345, 55232]  
(b) [53945, 55632]  
(c) [53345, 56232]  
(d) [48550, 50068]

14. You are given the following, $s = 18.6$, $\bar{x} = 66$ and $n = 35$. Construct a 90% confidence interval. The resulting interval is

(a) [60.6837783972553, 71.3162216027447]
15. Suppose that you have a sample with 35 observations. You are going to use this sample to construct a confidence interval for the population mean. How many degrees of freedom are there?

(a) 5.91607978309962
(b) 35
(c) 18
(d) 34

16. Suppose that Steph Curry has a field goal percentage 51%. Suppose also that he attempts an average of 20.1 shots per game. What is the expected number of shots that he will make in an upcoming game?

(a) 10.251
(b) 11.055
(c) 11.859
(d) 9.246

17. What is the probability of observing a z value less than -0.405?

(a) 0.6573
(b) 0.3427
(c) 0.4627
(d) 0.2727

18. In the assigned reading from the paper by Cellini and Goldin that was published in the American Economic Journal which of the following points could you learn?

(a) In several states for which they analyzed data the average tuition at T4 schools is approximately 3 times what it is at NT4 schools.
(b) In several states for which they analyzed data the average tuition at T4 schools is approximately 5 times what it is at NT4 schools.
(c) In several states for which they analyzed data the average tuition at T4 schools is approximately 6 times what it is at NT4 schools.
(d) In several states for which they analyzed data the average tuition at T4 schools is approximately 7 times what it is at NT4 schools.

(e) In several states for which they analyzed data the average tuition at T4 schools is approximately 2 times what it is at NT4 schools.

19. Here are the top 2016 US presidential candidates by contributions received. The data are from the Federal Election Commission available at http://www.fec.gov/disclosurep/pnational.do and I retrieved the data on 3/7/2016. What is the standard deviation of operating expenditures?

<table>
<thead>
<tr>
<th>Name</th>
<th>party</th>
<th>contributions received</th>
<th>operating expenditures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clinton</td>
<td>D</td>
<td>126.4</td>
<td>95.9</td>
</tr>
<tr>
<td>Sanders</td>
<td>D</td>
<td>95.4</td>
<td>80.7</td>
</tr>
<tr>
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<td>R</td>
<td>57.5</td>
<td>53.4</td>
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<td>R</td>
<td>32.6</td>
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<tr>
<td>Trump</td>
<td>R</td>
<td>25.2</td>
<td>23.7</td>
</tr>
</tbody>
</table>

(a) 10.77  
(b) 37.35  
(c) 28.02  
(d) 26.37  
(e) 27.54

20. Suppose that the data below are the entire population. Do not recalculate the standard deviation; suppose that the value you found in the previous problem is the population standard deviation. Suppose we draw a sample of Clinton and Sanders. What is the \( z \)-value for this sample? Note: the variable of interest is the operating expenditure.
2016 election cycle, values in millions of dollars

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(a) -.1606  
(b) 1.699  
(c) 1.919  
(d) .3944

21. Again you will use the data on presidential candidates. This time look at the variable “contributions received.” Which candidate has the largest z score in absolute terms (could be positive or negative)?

2016 election cycle, values in millions of dollars

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(a) Clinton  
(b) Sanders  
(c) Cruz  
(d) Rubio  
(e) Trump

22. \( \alpha \) is known as the level of significance or significance level.

(a) True
23. For a confidence coefficient of .96 we will have $\alpha$ of.

(a) .04
(b) .96
(c) .02
(d) 96%

24. What is the critical $t$ value when there are 25 degrees of freedom and the confidence coefficient is .8?

(a) 1.316
(b) 1.708
(c) 2.086
(d) .2

25. The central limit theorem tells us that as the sample size gets large (i.e., it approaches infinity) then the data will be normally distributed.

(a) true
(b) false
(c) true only for continuous data
Key

1. c

2. b, false for example consider the following data set: -1000, 2, 2, 2, 3, 5, 6, 7, 8. The median is 3 but the mean is negative.

3. b

4. d

5. c, plug in to the Poisson PMF which is on the formula sheet

6. a

7. c, use the formula for the expected value of a variable that follows the binomial distribution

8. d, if a variable follows the Poisson distribution it must be a count variable, non negative integer

9. d

10. b, the formula for the exponential distribution is on the formula sheet

11. a, the standard error is $\sigma/\sqrt{n}$

12. a, construct the interval as $\bar{x} \pm \text{margin of error}$.

13. b, when using the sample standard deviation find a t value instead of a z value.

14. a

15. d

16. a

17. b

18. b

19. e

20. c, you found a standard deviation of 27.54 and an overall mean of 50.929 and a sample average of 88.3 for Clinton and Sanders. Then you had something like $\frac{88.3-50.929}{27.54/\sqrt{2}} = 37.371/19.4737207538775=1.9190477501614$.

21. a, find the average which is 60.69 and see which is the furthest value from that.
22. a

23. a

24. a

25. b, the central limit theorem tells us how \( \bar{x} \) will be distributed and not how “the data” will be distributed.