Explaining cross-racial differences in teenage labor force participation: Results from a two-sided matching model

Tom Ahn a, Peter Arcidiacono b,*, Alvin Murphy c, Omari Swinton d

a University of Kentucky, United States
b Duke University, United States
c Olin Business School, Washington University, United States
d Howard University, United States

A B S T R A C T

White teenagers are substantially more likely to search for employment than black teenagers. This differential occurs despite the fact that, conditional on race, individuals from disadvantaged backgrounds are more likely to search. While the racial wage gap is small, the unemployment rate for black teenagers is substantially higher than that of white teenagers. We develop a two-sided search model where firms are partially able to search on demographics. Model estimates reveal that firms are more able to target their search on race than on age. Employment and wage outcome differences explain half of the racial gap in labor force participation rates.

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1. Introduction

Differences in labor market outcomes between blacks and whites have long been a focus of the labor economics literature. 1 While much of the literature has focused on the black–white wage gap, increasing attention is being paid to differences in employment rates. In particular, the channel through which discrimination plays a role may have less to do with wages paid but more to do with whether or not the individual is hired in the first place. Indeed, results from Bertrand and Mullainathan (2004) suggest that even having an African–American name on your resume can result in a lower probability of being called in for an interview.

In this study we examine differences in labor market outcomes between black and white teenagers living in southern states. Data from the Current Population Survey (CPS) show that white teenagers in southern states earn wages that are 6.5% higher than their black counterparts. However, this gap is small relative to the gap in unemployment rates: black teenagers face unemployment rates that are 72% higher than their white counterparts.

Productivity differences would seem to be unable to account for this employment gap when we break down the data by age. Nineteen-year-old black teenagers receive wages identical to those of eighteen-year-old whites yet their unemployment rate is 62% higher. Further, nineteen-year-old blacks have unemployment rates virtually identical to those of sixteen-year-old whites, despite having wages that are 16% (over 90 cents an hour) higher. While the within-race pattern of higher wages being coupled with lower unemployment rates holds across ages, older blacks may have higher wages and higher unemployment rates than younger whites.

These cross-race differences in the probabilities of finding work have a compounding effect, as teenagers take into account labor market conditions when deciding to participate in the labor market. White teenagers have labor force participation rates that are over 40% higher than those of their black counterparts. This holds despite the fact that blacks come from more disadvantaged backgrounds—a feature that is generally associated with higher labor force participation rates for teenagers. 2

To explain these trends, we formulate a two-sided matching model that incorporates search discrimination. Individuals and firms match through a one-shot game where both the labor supply of workers and the search rates of firms are endogenous.

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1 See Altonji and Blank (1999) for a review.

2 See Ahn et al. (forthcoming).
Individuals who search emit signals regarding their characteristics and firms can target their search based upon these signals. The employment level associated with each signal is then determined by a matching function. Since the firms are identical, expected zero profit conditions must hold across signals. The degree to which signals are correlated across race and age will then dictate how well firms are able to discriminate at the search stage through targeting their search. Matched firms and workers then negotiate over the wage according to Rubinstein bargaining.

We estimate the model using a twelve-year band of the basic monthly outgoing rotation files of the CPS from 1989 to 2000. Model estimates show that productivity differs significantly by race and age. Firms are able to target their search much more easily on the basis of race than on the basis of age. These racial differences in employment and wage prospects are then shown to explain approximately half of the difference between black and white labor force participation rates.

We use the estimates of the model to simulate how removing targeted search affects the labor market outcomes for blacks and whites. Removing the targeting breaks a large portion of the tie between race and unemployment, with some gap remaining due to the average value of a match being lower for a black teenager than for a white teenager. Lower match values for blacks means that pooling blacks with whites leads to higher unemployment rates for whites. This leads to a feedback effect in that whites respond to the higher unemployment rates by becoming less likely to search.

The rest of the paper proceeds as follows. Section 2 describes the data and the patterns that the model must be able to explain. Section 3 proposes the model. Section 4 shows how the data can be used to structurally estimate the model. Results are presented in Section 5 with policy simulations conducted in Section 6. Section 7 concludes.

2. Data

We now describe the data that motivates the model. We use twelve years of the basic monthly outgoing rotation groups (ORG) survey files of the CPS from 1989 to 2000. The CPS ORG survey is ideal for estimating our model as the hourly wage variable is obtained directly from the survey without imputation. We use black and white male teenager workers aged 16 to 19 during non-summer months to look for evidence of discrimination at the employment stage. These teenagers are matched with their parents to obtain household characteristics. We focus our analysis on data from southern states as defined by the CPS due to the much larger percentage of blacks in this area. Although we pool both teenagers who are enrolled in school and those who are not, the same basic patterns hold within these two groups.

From the CPS, we collect the hourly wage, the individual’s employment status, whether the individual is looking for work, and demographic characteristics. We define individuals as being in the labor force when they are either employed or looking for work. Particularly relevant for the teenage labor market is the minimum wage. In all of the southern states, the binding minimum wage is the federal minimum wage.

Table 1 gives descriptive statistics by race and age. One surprising feature of the descriptive statistics is that blacks are forty percent less likely to be in the labor force than their white counterparts. Given that black teenagers are coming from worse family situations, we would expect that black teenagers would be more likely to search than whites. The fact that this is not so is suggestive that blacks are facing substantially different labor markets compared to whites.

Support for the black teenage labor market being different from the white teenage labor market can be found in examining the differences in unemployment rates across the races. Here we see that blacks have unemployment rates that are over seventy percent higher than those of whites. Wages for blacks are lower than those for their white counterparts, with whites earning about six and a half percent (forty cents an hour) more than blacks. However, this six and a half percent difference is small relative to the large differences in employment rates.

White teenagers are more likely to search as age is increased and this may be because of better labor market outcomes due to increases in their own skill level. Unemployment rates are eighty-seven percent higher for sixteen-year-old whites than for nineteen-year-old whites. Sixteen-year-old whites who do find jobs have wages that are on average one dollar and sixty cents lower than those of their nineteen-year-old counterparts. Similar age trends are seen for blacks. Nineteen-year-old blacks are almost three times more likely to be in the labor market than sixteen-year-old blacks. Conditional on finding a job, sixteen-year-old blacks earn on average more than a dollar less than nineteen-year-old blacks.

The most striking feature is how the sample statistics vary across races. Table 1 debunks the notion that the differences in employment rates are driven solely by differences in productivity by blacks and whites. Nineteen-year-old blacks earn almost a dollar more an hour than sixteen-year-old whites and yet face virtually the same unemployment rates. The data suggest that search may be an important explanation in the differences in black–white outcomes, with firms more easily able to target their search based upon race than age.

3. Model

In this section we present a model designed to capture the key features of the data. Namely, our model needs to be able to explain why older blacks earn more than younger whites yet have higher unemployment rates. Further, the model needs to incorporate blacks responding to the poorer labor market conditions by being less likely to enter the labor market. Finally, we want the model to be simple enough such that we can take it to the data.

Capturing these features necessitates departing from the traditional search literature which has generally focused on the intersection of search and wages. For example, the approach taken in Wolpin (1992), Eckstein and Wolpin (1995, 1999), and Bowlus and Eckstein (2002) treats the arrival rates of job offers as reduced form parameters and does not model the labor force

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3 Household characteristics are only calculated for those who are living with a parent or a guardian.

4 The southern states are defined by the CPS are Alabama, Arkansas, Delaware, the District of Columbia, Florida, Georgia, Kentucky, Louisiana, Maryland, Mississippi, North Carolina, Oklahoma, South Carolina, Tennessee, Texas, Virginia, and West Virginia.

5 The demographic characteristics we use are parental education, whether the teenager comes from a single-parent home, and the employment status of the head of household. All income variables are adjusted to 2000 dollars using the CPI. To correct for misreporting of the hourly wages, if a teenager’s reported wage is below the minimum wage but within twenty-five cents, we attribute the minimum wage to them. Teenagers who report an hourly wage more than twenty-five cents below the minimum wage, and those who report being employed but do not report an hourly wage, are excluded from our sample.

6 In nominal terms, the minimum wage was $3.35 in 1989. The federal minimum wage increased to $3.80 on April 1st, 1990 and increased again on April 1st, 1991 to $4.25. The minimum wage was increased again on October 1st, 1996 to $4.75, with the final minimum wage change over the sample period occurring on September 1st, 1997 to $5.15. The minimum wage was also changed in the District of Columbia, Florida, Georgia, Kentucky, Louisiana, Maryland, Mississippi, North Carolina, Oklahoma, South Carolina, Tennessee, Texas, Virginia, and West Virginia.
participation decision.\(^7\) Bowlus (1997) uses a search model with non-participation to analyze differences in male and female labor market outcomes but offers arrival rates are exogenous. Here we want to develop and estimate a model where the key features of the data fall out of zero expected profit conditions, and hence endogenous offer arrival rates,\(^8\) while coupling this with labor force participation decisions that respond to market decisions. The one paper that has endogenous labor force participation and offer arrival rates that fall out of zero profit conditions, as well as incorporating dynamic search, is Flinn (2006). While Flinn has a fully dynamic model conditional on participating in the labor force, the labor force participation decision is a one-shot game.

We follow Ahn et al. (AAW) \textit{forthcoming} in developing a two-sided matching model with the primary extension being that firms can partially target their search. Individuals and firms match through a one-shot game where both the labor supply of workers and the search rates of firms are endogenous and the employment level is determined by a matching function. Matched firms and workers then negotiate over the wage using generalized Nash bargaining. By fully incorporating the search process in one step, it is possible to make other parts of the model much more complicated. For example, it is easy to incorporate both endogenous labor supply and endogenous firm vacancies. The latter results from zero expected profits from posting a vacancy while the former results from workers having heterogeneous values of leisure. The cost is that we do not model the dynamics of search. We believe that this is reasonable in this situation where minimum wages bind for some in every age group and in every race and the heterogeneity of wages is small relative to the heterogeneity in employment probabilities.\(^9\)

The model then has four components:

1. The decisions by individuals regarding whether to search given their expectations regarding labor market outcomes and their value of leisure.
2. The decisions by firms to search such that, in equilibrium, a zero expected profit decision is satisfied.
3. The process by which workers and firms are paired.
4. The process governing wages.

Each of these is described below.

3.1. Labor force participation

We assume that there are \(K\) types of worker, where \(k\) indexes the type. Each worker is a member of only one type. Let \(N_k\) index the number of type \(k\) individuals in the population. The number of workers of each type who search is endogenous. Let \(N^*_k\) indicate the number of searching workers of type \(k\). The different types of worker may differ in their average productivity and their attachment to the labor force. They also differ in the signal they emit to the market, with different signals associated with different probabilities of matching with an employer. The probability of emitting a particular signal depends only on the individual’s type. There are \(m \in M\) possible signals, with the probability of the \(k\)th type emitting signal \(m\) given by \(\lambda_{mk}\). The probability of matching with a firm conditional on emitting signal \(m\) is \(p_m\).

Individuals are differentiated in their reservation values, \(R\), for not working, where reservation values for the \(k\)th type are drawn from the cumulative distribution function \(F_k(R)\) with support \([0, \infty)\). This reservation value can be leisure or any outside option for workers. For instance, the reservation values could be the value of schooling for teenagers, with the treatment effect of education varying across the population and across type.

Denote \(C_{k1} \in [C_1, \infty)\) as the search cost, where \(C_1 > 0\) and is paid whether an individual matches with a firm or not. Individuals are risk neutral, with the value of searching (not searching) for individual \(i\) of type \(k\) denoted by \(V_{Sik}(V_{Nik})\). The payoff of matching with a firm is the wage, \(W_k\), if the wage is above the individual’s reservation value. If the wage is below the individual’s reservation value, the match will be rejected and the payoff is the reservation value. There is uncertainty with regard to the wage which will be explained later in the paper. \(V_{Sik}\) and \(V_{Nik}\) are then given by

\[
V_{Sik} = \sum_{m=1}^{M} \lambda_{mk} p_m (\max[W_k, R_k] + (1 - p_m) R_{ik} - C_{k1})
\]

\[
V_{Nik} = R_{ik}.
\]

Here, the probability of matching with a firm sums over the probability of emitting each of \(m\) signals times the probability of matching conditional on the \(m\)th signal. Individuals then weigh the probability of matching with a firm times the expected payoff of matching against the search cost. Differenting the value of searching for individual \(i\) of type \(k\) against the value of not searching yields the net expected value of searching, \(V_{ik}\), and is given by

\[
V_{ik} = \sum_{m=1}^{M} \lambda_{mk} p_m (\max[W_k - R_{ik}, 0] - C_{k1}),
\]

with individuals searching when \(V_{ik} > 0\).

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\(^7\) Eckstein and Wolpin (1995) derive a model where the arrival rates fall out of profit maximization but estimate a model where arrival rates are allowed to vary flexibly by race.

\(^8\) The theoretical search literature has developed models with endogenous arrival rates. See Black (1995), Sattinger (1998), Mailath et al. (2000), and Arcidiacono (2003).

\(^9\) As we show in the subsection on wages, under assumptions standard in the literature, a binding minimum wage in a Rubinstein bargaining game makes the continuation value of search irrelevant to the wage.
Higher search costs and higher reservation values both then make search less attractive though in different ways. Consider two individuals, one with a high reservation wage and a low search cost and another with a low reservation wage and a high search cost. It is possible to find a combination of wages and probabilities of matching such that the first individual searches and the other does not. But it is also possible, due to the interaction between the probability of matching interacting with the reservation wage, to find a wage/probability of matching combination such that the second individual searches and the first does not. This case will occur at a higher probability of matching and a lower reservation wage: individuals with relatively high search costs and low reservation wage are willing to take a lower wage for a higher probability of matching.

3.2. Firms

The number of firms in the signal \( m \) market, \( J_m \), is endogenous. All firms are identical. Each firm chooses whether or not to search in at most one market for one worker. The revenue for firm \( j \) for matching with worker \( i \) of type \( k \) is given by \( Y_{ijk} \). Let \( V_k \) denote the average revenue from matching with a worker of type \( k \). The stochastic portion of \( Y_{ijk} \) is given by \( \epsilon_{ijk} \), which is then drawn from the cumulative distribution function \( G_{\epsilon}(\epsilon_{ijk}) \) with support \([\epsilon_{min}, \epsilon_{max}]\). \( Y_{ijk} \) is then given by

\[
Y_{ijk} = \bar{V}_k + \epsilon_{ijk}.
\]

Like workers, firms pay a search cost, \( C_2 \), whether or not they find a match. Firms enter until all firms have zero expected profits in each of the \( M \) markets. Let the probability of finding a worker of type \( k \) in the signal \( m \) market be given by \( q_{mk} \). Expected profits for searching in the signal \( m \) market are then given by

\[
\sum_{k=1}^{K} q_{mk} E(\text{max}(Y_k - W_k, 0)) - C_2 = 0,
\]

as firms will reject matches where \( Y < W \). Here we are summing over the probabilities of matching with each type of worker times the expected share of the revenue that the firm gets from each type of worker. Suppose for a particular number of searching firms, positive profits exist in market \( m \). Firms will enter the signal \( m \) market leading to a fall in \( q_{mk} \) for all \( k \). Entry continues until the expected zero profit condition given in (4) holds.

In order for the zero profit condition to be satisfied, there exists a trade-off between the probability of matching with a worker and the expected profit the firm makes on the worker. Suppose the expected zero profit condition is satisfied for market \( m \) and \( m' \) but that higher profits conditional on matching are found in market \( m' \). It must then be the case that firms in market \( m' \) have a lower probability of matching than in market \( m \) for both expected zero profit conditions to hold.

As firms become less able to target their search, teenagers from groups with low match revenues benefit. Because they are pooled with groups with higher match revenues, demand is higher. In contrast, those groups with higher match revenues are hurt as targeting decreases because they are pooled with groups with low match revenues. This effect is magnified with endogenous labor supply. By partially removing targeting, members of the groups with high match revenues become less likely to participate while members of groups with low match revenues become more likely to participate. These effects further lower the equilibrium probabilities of both groups finding employment.

The size of the groups also matters. If the groups with low match revenue values are relatively small, they will have little impact on the labor market outcomes of those groups with high match revenue values. As long as targeting is not perfect, groups with low match revenue values prefer to be outnumbered by those from groups with high match revenue values.

3.3. Matching

With the search decisions for workers and firms defined above, we now describe how workers are allocated to firms. Workers and firms are matched using a Cobb–Douglas matching function for each search signal with the restriction that the number of matches can be no greater than either the number of searching workers or the number of searching firms. Let \( x_m \) index the number of matches in the signal \( m \) market and be given by

\[
x_m = \min\{\sum_{j=1}^{M} q_{jk} \bar{V}_k, J_m, N_m \}.
\]

While individuals choose to search, they do not control the signal they send to the labor market. Rather, \( \lambda_{mk} \), the probability of being assigned signal \( m \) conditional on being the \( k \)th type, is taken as exogenous. The number of workers assigned to signal \( m \) is then given by

\[
N_m = \sum_{k=1}^{K} \lambda_{mk} N_k
\]

where \( \sum_{m=1}^{M} \lambda_{mk} = 1 \) for all \( k \).

We assume that, conditional on the signal, all workers have the same probability of being matched, \( p_m = x_m/N_m \). With workers only knowing the probabilities of being assigned particular signals, the probability of a worker with type \( k \) matching with a firm is given by

\[
p_k = \sum_{m=1}^{M} \lambda_{mk} p_m.
\]

Firms who search on signal \( m \) also have identical probabilities of matching, \( q_m = x_m/J_m \). Furthermore, firms targeting the same signal also have identical probabilities of matching with a particular type of worker. The probability of matching with a worker of type \( k \) emitting signal \( m \) is then

\[
q_{mk} = q_m \Lambda_{mk}
\]

where \( \Lambda_{mk} \) is given by

\[
\Lambda_{mk} = \frac{\lambda_{mk} N_k}{\sum_{k'=1}^{K} \lambda_{mk'} N_{k'}}.
\]

3.4. Wages

We now specify a wage-generating process that is similar to that of AAW. Matched pairs split the revenue generated by the match, \( Y \), according to a Rubinstein bargaining game where the time between offers and the discount factors may vary for the firm and the worker. The bargaining game operates under the constraint that a successful match pays at least the minimum wage, \( W \). Building on the work by Binmore et al. (1989) and Binmore et al. (1986), we show in the appendix that, under certain assumptions, the unique subgame perfect equilibrium of the bargaining game for all matches where \( Y \geq \max(W, R) \) yields the following expression for wages:

\[
W_{ijk} = \max\{\beta Y_{ijk}, R_{ijk}, W\}.
\]

\( \beta \) can then be interpreted as the bargaining strength, and it represents a combination of the discount factors of the firm and worker as well as the differing time between offers given by the firm and worker. Matches where \( Y < \max(W, R) \) are unsuccessful.

10 Note that the match may be rejected by either the firm or the worker.
Note that the reservation value does not affect the match revenue division unless the reservation value is higher than both the minimum wage and \( \beta \) times the revenue of the match.

Given the rules governing individual and firm decisions to search, the matching process, and the wage process, that an equilibrium exists follows directly from proposition 1 in AAW. The intuition for why equilibrium exists is straightforward: profits by firms in market \( m \) are continuously decreasing in entry by other firms in market \( m \). Hence, at any level of search by workers, including the equilibrium level, there exists a number of firms in each market such that the expected zero profit conditions hold.

3.5. Example: Targeting on race but not on age

In the data, we observe older black teens having higher realized wages but lower employment rates compared to younger white teens. We now show how the model can generate and explain this puzzling trend.

Assume there are two races, black and white, and two ages, young and old. Therefore, there are four types. We denote \( k \) black/old (\( B \)), black/young (\( b \)), white/old (\( W \)), and white/young (\( W' \)). Assume that the signals firms receive allow them to perfectly target their search based upon race but are unable to target their search at all on the basis of age. We then denote \( m \in \{ B, W \} \), where

\[
\begin{align*}
\lambda_{B, 1} &= 1, \\
\lambda_{B, 0} &= 1, \\
\lambda_{W, 1} &= 0, \\
\lambda_{W, 0} &= 0
\end{align*}
\]

and for white teens,

\[
\begin{align*}
\lambda_{B', 1} &= 0, \\
\lambda_{B', 0} &= 0, \\
\lambda_{W', 1} &= 1, \\
\lambda_{W', 0} &= 1
\end{align*}
\]

Expected revenues are such that older individuals have higher expected revenues than younger individuals but that within ages whites have higher expected revenues than blacks:

\[
\begin{align*}
\mathbf{Q}_{W'} > \mathbf{Q}_{W} > \mathbf{Q}_{B} > \mathbf{Q}_{B'}.
\end{align*}
\]

To keep the exposition simple, we focus on the case where search costs are low enough and match revenues are high enough such that all individuals find it optimal to search. Further, the lowest value of match revenue is above the highest reservation value, implying that all matches are successful. Finally, assume that the ratio of older individuals to younger individuals is the same across races.

Given the productivity orderings, the expected revenue from a match in the two markets is such that the expected revenue in the signal \( W \) market is higher than the expected revenue in the signal \( B \) market:

\[
E(Y_{W}) > E(Y_{B}).
\]

The expected zero profit condition then implies that the probability of a firm matching with a white worker is lower than the probability of a firm matching with a black worker,

\[
q_{W} < q_{B}.
\]

Note that \( P_{B} = \frac{x_{B}}{N_{B}} = P_{B} \) and \( P_{W} = \frac{x_{W}}{N_{W}} = P_{W} \), because workers cannot signal their age. Writing \( P_{B} (P_{W}) \) as a function of \( q_{B} (q_{W}) \), we get

\[
\begin{align*}
P_{B} &= A^{\frac{1}{\lambda_{B}}} \cdot q_{B}^{\frac{1}{\lambda_{B}}} = P_{B}^{0} \\
P_{W} &= A^{\frac{1}{\lambda_{W}}} \cdot q_{W}^{\frac{1}{\lambda_{W}}} = P_{W}^{0}.
\end{align*}
\]

Since \( q_{W} < q_{B} \), it trivially follows that \( P_{B} < P_{W} \), and by extension, \( P_{B} < P_{W} \). Therefore, the probability that an older black teen matches with a firm is lower than the probability that a low ability white teen matches with a firm.

However, upon matching, the firm and worker split the revenue that is generated. Since, by assumption, \( \mathbf{Q}_{B'} > \mathbf{Q}_{W} \), then \( W_{\mathbf{B'}} > W_{\mathbf{W}} \). Therefore, older black teenagers, conditional on matching, receive higher expected wages than younger white teenagers.

4. Empirical specification

We now describe our estimation procedure for the structural model. The estimation has three components. First, for those individuals who successfully match, we observe wages. Second, we need to estimate the parameters of the zero profit condition. Although we do not observe the probability of a firm finding a match, we are able to rewrite the zero profit condition as a function of the individual’s probability of finding a match. Finally, we observe decisions by individuals as to whether to search. We use these decisions to estimate the supply side parameters.

4.1. Parameterizing wages

Recall that \( Y_{ijkl} \) is the revenue generated by a match between individual \( i \) of type \( k \) with firm \( j \) in location \( l \). In order to conserve on notation, we allow \( f \) to denote a state–time combination (e.g., Alabama, 4th quarter, 1990). We assume that \( \ln(Y_{ijkl}) \) is given by

\[
\ln(Y_{ijkl}) = X_{ijkl} \theta + \epsilon_{ijkl} \tag{8}
\]

where \( X_{ijkl} \) includes the individual’s age, race, state, quarter, and year and the \( \epsilon \)’s are then the match-specific components of the revenue.

The wage-generating process under Rubinstein bargaining yields \( W_{ijkl} = \max[\beta Y_{ijkl}, W_{0}, R_{ijkl}] \). Under certain assumptions on the primitives, any worker who chooses to participate in the labor market will have a reservation value that is lower than the minimum wage, in which case \( W_{ijkl} = \max[\beta Y_{ijkl}, W_{0}] \). Namely, suppose the following condition, \( NR \), holds:

\[
NR \quad \text{For all } R > W, \quad Pr(Y \geq R) (E[\max(\beta Y, R)|Y \geq R] - R) - c_{1} < 0.
\]

The expression on the left-hand side of the inequality is the value of searching given the lowest possible search cost without the probability of matching. Given that the probability of matching would range between zero and one, only workers who have reservation values below the minimum wage will search when \( NR \) holds.\(^{11}\) This is effectively an assumption on the distribution of match revenues, \( Y \), relative to the lowest search cost, \( c_{1} \). When the spread of possible revenues is small relative to the search costs, those with reservation values above the minimum wage do not find it worth the risk to search on the off-chance that, should they match, the draw on the match revenue will be at least as high as their reservation wage. To keep the model tractable, we make this assumption throughout the rest of the paper.\(^{12}\)

When the minimum wage does not bind, log wages are then given by

\[
\ln(W_{ijkl}) = X_{ijkl} \theta + \ln(\beta) + \epsilon_{ijkl} \tag{9}
\]

where \( \beta \) is the bargaining power of the worker. In the presence of a minimum wage, the wage distribution is then distributed truncated log-normal with censoring at the minimum wage. The truncation occurs when the match value is so low that the firm rejects the match. This occurs whenever \( W_{ijkl} > Y_{ijkl} \). There are then three relevant regions for the quality of the match\(^{13}\):

\(^{11}\) When \( R > W \), individuals search when \( pPr(Y \geq R) (E(W|Y \geq R) - R) - c_{1} > 0 \).

\(^{12}\) The value of \( \theta \) that leads to the highest value of the expression on the left-hand side of the inequality is one. Setting \( p = 1 \) yields the left-hand side expression in \( NR \).

\(^{13}\) We estimated reduced form wage equations to see if the factors that influenced the reservations values (for example, parental education) also influenced the wage. We found no evidence that higher (lower) reservation values were associated with higher (lower) wages.

\(^{14}\) The three regions provide a structural interpretation to Meyer and Wise (1983a,b) who estimate a model where those who would normally make less than the minimum wage either make exactly the minimum wage or are unemployed.
\[ \beta Y_{ijkl} \geq W_j \Rightarrow \{W_{ijkl} = \beta Y_{ijkl}\} \]

\[ Y_{ijkl} \geq W_j \geq \beta Y_{ijkl} \Rightarrow \{W_{ijkl} = W_i\} \]

\[ W_i > Y_{ijkl} \Rightarrow \{\text{Nomatch}\} \]

We then observe successful matches for those who are employed either at or above the minimum wage.

The shape of the distribution of the \( e \)’s plays an important role in distinguishing the probability of receiving a match value so low that the firm rejects the match from the probability of receiving a match that leads to a minimum wage job. We allow the \( e \)’s to be drawn from one of two normal distributions where both the means and the variances are allowed to vary across distributions. The probability of the draw coming from the \( r \)th distribution is then given by \( \pi_r \). Identification of the parameters of the mixture distribution then comes from the shape of the observed wages, variation across locations in productivity and therefore the fraction of observations that are censored, and variation in minimum wages over time.

Let \( N_{1kl} \) and \( N_{2kl} \) indicate the number of individuals of type \( k \) in location \( l \) who have wage observations above and at the minimum wage, respectively. Defining \( \Phi \) and \( \phi \) as the CDFs and PDFs of the standard normal distribution, the likelihood for these observations is that given in Box I. This likelihood is conditional on the firm not rejecting the match. The denominator in both expressions is one minus the probability that the revenue of the match is so low that the firm would rather not match than pay the minimum wage. The first expression then gives the conditional probability of wages above the minimum wage while the second expression is the conditional likelihood of receiving exactly the minimum wage.

### 4.2. Parameterizing firms

Although we have no information on firms, we can infer the parameters of the profit function by rewriting the zero profit condition as a function of an individual’s probability of finding a match. We first show that we can rewrite the zero profit condition as a function of \( p_{ml} \), the probability of a worker finding a match after being assigned signal \( m \) in location \( l \), rather than as a function of \( q_{ml} \), the probability of a firm in location \( l \) finding a match in the signal \( m \) market. These probabilities are given by

\[ q_{ml} = A \left( \frac{N_{ml}}{J_{ml}} \right)^{1-a}, \quad p_{ml} = A \left( \frac{J_{ml}}{N_{ml}} \right)^{a}, \]

implying that we can write \( q_{ml} \) as

\[ q_{ml} = A^\frac{a}{2} \frac{a-1}{p_{ml}^2}. \]

The expected zero profit condition for firms in location \( l \) who choose the signal \( m \) market can be written as

\[ q_{ml} E(\max(Y_{ml} - W_{ml}, 0)) - C_2 = 0. \]

Substituting for \( q_{ml} \) as a function of \( p_{ml} \) yields

\[ A^\frac{a}{2} \frac{a-1}{p_{ml}^2} E(\max(Y_{ml} - W_{ml}, 0)) - C_2 = 0. \]

Solving for \( p_{ml} \) yields

\[ p_{ml} = \delta E(\max(Y_{ml} - W_{ml}, 0))^a. \]

where

\[ \delta = C_2^{1-a} A^{\frac{a}{2}}. \]

Note that the expectations above are taken with respect to both the match-specific component of revenue and the probabilities of meeting the different types of worker. We can now substitute in for \( E(\max(Y_{ml} - W_{ml}, 0)) \) with the corresponding probabilities and expected values conditional on type:

\[ p_{ml} = \delta \left( \sum_{k=1}^K \Lambda_{mlk} E(\max(Y_{kl} - W_{kl}, 0)) \right)^{\frac{a-1}{a}}. \]

Given the assumed distribution of \( Y \) and the parameters of the wage-generating process, we can calculate \( E(\max(Y_{kl} - W_{kl}, 0)) \), the expected revenue from matching with a \( k \)-type worker in location \( l \). This revenue can be broken down into three parts for each type of worker: (1) when the match value is high enough such that the minimum wage does not bind, \( \bar{Y}_{1kl} \), (2) when the match value is such that the minimum wage binds, \( \bar{Y}_{2kl} \), and (3) when the match value is so low that the firm rejects the match. The last of these parts yields an expected revenue of zero. The first and second parts are then given by

\[ \bar{Y}_{1kl} = \sum_{r=1}^2 \pi_r \left[ \exp(X_{kl} \theta + \ln(1 - \beta) + \mu_r + \sigma_r^2/2) \Phi \left( \frac{\sigma_r^2 - \ln(W_i) + X_{kl} \theta + \ln(\beta) + \mu_r}{\sigma_r} \right) \right] \]

\[ \bar{Y}_{2kl} = \sum_{r=1}^2 \pi_r \left[ \exp(X_{kl} \theta + \mu_r + \sigma_r^2/2) B_{rkl} - (\Phi \left( \frac{\ln(W_i) - X_{kl} \theta - \mu_r - \ln(\beta)}{\sigma_r} \right) - (\Phi \left( \frac{\ln(W_i) - X_{kl} \theta + \mu_r}{\sigma_r} \right)) W_i \right] \]

where

\[ B_{rkl} = \Phi \left( \frac{\sigma_r^2 - \ln(W_i) + X_{kl} \theta + \mu_r}{\sigma_r} \right) - (\Phi \left( \frac{\sigma_r^2 - \ln(W_i) + X_{kl} \theta + \mu_r}{\sigma_r} \right) + (\Phi \left( \frac{\sigma_r^2 - \ln(W_i) + X_{kl} \theta - \mu_r}{\sigma_r} \right)). \]

We then define \( \bar{Y}_k \) such that

\[ \bar{Y}_k = E(\max(Y_{kl} - W_{kl}, 0)) = \bar{Y}_{1kl} + \bar{Y}_{2kl}. \]

\[ p_{ml} \] can then be rewritten as

\[ p_{ml} = \delta \left( \sum_{k=1}^K \Lambda_{mlk} \bar{Y}_{kl} \right)^{\frac{a-1}{a}}. \]

We do not observe \( p_{ml} \). Rather, we observe whether a black or a white worker obtains employment conditional on searching. Recall that the probability of an individual of the \( k \)th type matching in location \( l \) is

\[ p_k = \sum_{m=1}^M \lambda_{mk} p_{ml}. \]

What we observe in the data is \( p_k \psi_{kl} \), where \( \psi_{kl} \) gives the probability of a successful match conditional on matching:

\[ \psi_{kl} = \sum_{r=1}^2 \pi_r \left( 1 - (\Phi \left( \frac{\ln(W_i) - X_{kl} \theta - \mu_r}{\sigma_r} \right)) \right). \]

Substituting in for the \( p_{ml} \) in the expression for \( p_{kl} \) and multiplying by \( \psi_{kl} \) yields

\[ p_{kl} \psi_{kl} = \psi_{kl} \sum_{m=1}^M \lambda_{mk} \delta \left( \sum_{k=1}^K \Lambda_{mlk} \bar{Y}_{kl} \right)^{\frac{a-1}{a}}. \]
Their parents. Given our actual data, we estimated a probit on the combination. We used census data combined with reduced form estimates of the probability of searching to form the number of searching workers in each group. The census data provided forecasts of the size of each of our eight groups. We then forecasted the share of each group that would be in school and living with their parents. Given our actual data, we estimated a probit on the probability of being in the labor force using year, year squared, state dummies, quarter dummies, and age dummies as regressors.

We estimated the probit separately for blacks and whites and then used the fitted values in forming the expected number of each group in the labor force for each state-quarter combination.

4.3. Parameterizing the individual

We now turn to the decision by individuals as to whether or not to search which follows directly from AAW. Recall that individual i in location l searches if

\[ \sum_{m=1}^{M} \lambda_{mk} p_m (E(W_{il}) - R_{il}) - C_{ikl} > 0. \]

With the estimates from the previous two stages it is possible to calculate expected wages and the probability of employment for each individual. We now need to parameterize the reservation values. In particular, we parameterize \( R_{il} \) such that all workers have positive reservation values:

\[ R_{il} = \exp(Z_{ikl} \gamma_1) + \eta_i \]

\( Z_{ikl} \) is then a vector of demographic characteristics which affect the individual’s outside option, the \( \gamma_1 \)’s are the coefficients to be estimated, and \( \eta_i \) is the unobserved portion of the reservation value. We also allow the search costs to vary, where

\[ C_{ikl} = \exp(Z_{2ikl} \gamma_2) \]

and \( Z_{2ikl} \) contains those variables that affect the cost of searching.

As in AAW, individuals who come from highly educated families may have high reservation values, making search less likely. However, these same individuals may also have lower search costs. What separately identifies search costs from reservation values is how individuals react to the probability of finding a job. In particular, those with low search costs but high reservation values will be more willing to trade off higher expected wages conditional on matching for lower probabilities of employment. In contrast, those with high search costs but low reservation values prefer lower wages coupled with higher match probabilities.15

Substituting in and solving for \( \eta_i \) shows that an individual will search when

\[ \eta_i < \ln \left( \frac{E(W_{il}) - \exp(Z_{2ikl} \gamma_2)}{p} \right) - Z_{ikl} \gamma_1. \]

We assume that the \( \eta_i \)’s follow a logistic distribution. Note that because of the log, any coefficient on \( E(W) \) will be factored into the intercept term of the reservation values. Since we do not observe the \( \eta_i \)’s, the likelihood function is then given by

\[ L_3 = \prod_{i=1}^{K} \prod_{k=1}^{N_{1il}} \left( \frac{1}{1 + \exp \left( \frac{-E(W_{il}) - \exp(Z_{2ikl} \gamma_2)}{p} \right) - Z_{ikl} \gamma_1} \right)^{\gamma_1 \text{ikl} - 1} \times \left( 1 - \frac{1}{1 + \exp \left( \frac{-E(W_{il}) - \exp(Z_{2ikl} \gamma_2)}{p} \right) - Z_{ikl} \gamma_1} \right)^{\gamma_1 \text{ikl} = 0} \]

15 While in theory it is possible to let all variables affect both the reservation wages and the search costs, in practice we place restrictions on which variables are allowed to affect the search costs. Namely, while reservation wages are allowed to vary with time and by state, the search costs are not.
where $N_{ik}$ is the number of potential searchers of type $k$ in location $l$, $S_{ikl}$ is an indicator for whether the $i$th individual chose to search, and $F$ is the standard logit CDF. In a standard logit, all coefficients are relative to the variance scale parameter, $\sigma$. Here we can actually estimate $\sigma$ as there is no other natural interpretation for the coefficient on the expression inside the log. The $\gamma$’s are then the $y$’s divided by the standard deviation of the $\eta$’s, $\sigma$.

While it is possible to estimate all three stages simultaneously, the additive separability of the log-likelihood function makes it possible to estimate the parameters in stages. In practice, we estimate the parameters of the wage generating process and of the zero profit condition jointly. Taking these parameters as given, we then estimate the parameters of individual’s decision to search.\(^{16}\)

### 5. Results

Estimates of the wage generating process are given in Table 2. Teenage wages are negatively impacted by the prime age male unemployment rate which operates as our exclusion restriction as a variable which affects wages but not the value of leisure. Nineteen-year-olds generate revenue values that are almost twenty-three per cent higher than those of sixteen-year-olds while blacks have revenue values that are over nine per cent lower than those of whites.\(^{17}\)

The parameters of the zero profit condition are also given in Table 2. These parameters are estimated jointly with the parameters of the wage distribution. The bargaining parameter is estimated at 0.78, suggesting that the market for teenage workers is fairly competitive. $\alpha$, which measures how sensitive the matching function is to the number of searching firm versus the number of searching workers, was set at 0.5. This is the standard estimate from the macroeconomics literature.\(^{18}\)

Of more interest are the targeting parameters which are presented in Table 3. Recall that these are logit parameters embedded in the zero profit condition. What these parameters imply is that firms are able to almost fully target their search by race with much less targeting by age. With the exception of white sixteen-year-olds, whites were generally assigned the signal one. Sixteen-year-old whites were assigned to all signals with roughly equal probabilities. Sixteen-to-eighteen-year-old blacks were generally assigned to signal four. Nineteen-year-old blacks were more spread out, though they too were heavily concentrated in signals three and four. The way in which in individuals were assigned to signals leads to different probabilities of matching across signals because of differences in productivities and because of the expected zero profit condition for each signal. Signal one had particularly high match probabilities relative to the other signals. With the parameter estimates, we can calculate the average probability of being matched conditional on age and race. These forecasts are given in Table 4. White teenagers above the age of sixteen face match probabilities between 78% and 82%. The corresponding figure for sixteen-year-old whites is 71.4%. In contrast, the range for black teenagers is between 65% and 71% with higher probabilities associated with older workers.

Table 4 also shows the average probability that a match is successful, where success is defined as having a revenue value at least at the level of the minimum wage. Here we see large differences across age and to some extent across race as well. Nineteen-year-old blacks are then primarily unemployed due to not matching with a firm. Compared with nineteen-year-old blacks, seventeen-year-old whites are more likely to be unemployed due to lower revenue values once a match has been obtained.

With the parameters of the wage process and zero profit condition in hand, we now turn toward the decision by teenagers regarding whether or not to search. The parameter estimates of the wage generating process and the zero profit condition are then used in the individual search decisions to form the probability of obtaining a successful match as well as the expected wage conditional on a successful match. These estimates are reported in Table 5.

The estimates for blacks suggest that they have higher reservation values and lower search costs. This seems unlikely and we may need a finer geographic definition of the labor markets. Those coming from single-parent families have lower reservation wages and higher search costs than their two-parent counterparts. In contrast, those who have parents with higher parental education have higher reservation values and lower search costs. Hence, individuals who come from poorer, less-educated families are more likely than their richer counterparts to be willing to trade off lower wages for a higher probability of finding a job.

Since the parameters of a logit are difficult to interpret, in Table 6 we forecast how the probability of searching changes as we vary either labor market conditions or the demographics of searchers. These simulations take all other characteristics as given and forecast the change in search behavior from changing the

---

\(^{16}\) Estimating the model in two stages reduces the computational burden but with a reduction in efficiency.

\(^{17}\) Surplus values refer to the total revenue which would include any compensation the firm would take for not wanting to hire blacks.

\(^{18}\) See (Petrongolo and Pissarides, 2001). We attempted to estimate $\sigma$, but separately identifying both $\sigma$ and the targeting parameters proved difficult.
Table 5
Parameter estimates of the utility function*. 

<table>
<thead>
<tr>
<th></th>
<th>In school</th>
<th>Search costs</th>
<th>Out of school</th>
<th>Search costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservation values</td>
<td>2.906  (0.132)</td>
<td>-0.191  (0.111)</td>
<td>4.371 (0.542)</td>
<td>-0.491  (0.110)</td>
</tr>
<tr>
<td>Black</td>
<td>0.426     (0.109)</td>
<td>0.090     (0.139)</td>
<td>0.564  (0.113)</td>
<td>-0.159  (0.119)</td>
</tr>
<tr>
<td>Household head unemployed</td>
<td>-0.098 (0.102)</td>
<td>-0.191  (0.111)</td>
<td>0.564  (0.113)</td>
<td>-0.159  (0.119)</td>
</tr>
<tr>
<td>Household head other b</td>
<td>0.559  (0.069)</td>
<td>-0.159  (0.119)</td>
<td>0.564  (0.113)</td>
<td>-0.159  (0.119)</td>
</tr>
<tr>
<td>Household head some college</td>
<td>0.131  (0.071)</td>
<td>-0.466  (0.216)</td>
<td>0.564  (0.113)</td>
<td>-0.159  (0.119)</td>
</tr>
<tr>
<td>Household head college</td>
<td>0.540  (0.096)</td>
<td>-0.972  (0.574)</td>
<td>0.564  (0.113)</td>
<td>-0.159  (0.119)</td>
</tr>
<tr>
<td>Household head post-college</td>
<td>0.637  (0.094)</td>
<td>-0.930  (0.614)</td>
<td>0.564  (0.113)</td>
<td>-0.159  (0.119)</td>
</tr>
<tr>
<td>Single parent</td>
<td>-0.092    (0.054)</td>
<td>0.114     (0.091)</td>
<td>0.564  (0.113)</td>
<td>-0.159  (0.119)</td>
</tr>
</tbody>
</table>

Notes: a Reservation values include age, state, year, and quarter fixed effects. Search costs include age fixed effects. N = 30,600.

b Other is defined as having a household head who is neither working nor unemployed.

Table 6
Changes in the probability of searching.

<table>
<thead>
<tr>
<th>Percentage change in probability of searching</th>
<th>In school (%)</th>
<th>Not in school (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% Change in expected wage</td>
<td>20.81</td>
<td>19.39</td>
</tr>
<tr>
<td>Black preferences to white preferences</td>
<td>24.14</td>
<td>20.34</td>
</tr>
<tr>
<td>Black labor market to white labor market</td>
<td>18.24</td>
<td>22.03</td>
</tr>
</tbody>
</table>

6. Policy simulations

We focus our policy simulations on changing the degree to which firms can target on the basis of race and age. We consider two policy changes, no targeting on race and no targeting on either race or age. We assume that the parameters of the zero profit condition are the same as with targeting except that now firms can no longer target according to the policy change.

Table 7 shows the changes in the probabilities of searching and matching for blacks and whites with the removal of targeting. All blacks search more and have a greater probability of matching successfully. This effect is largest for older blacks who were mainly pooled with sixteen-year-old blacks but are now pooled with older (and white) workers. The drops in search probabilities are larger for younger whites. These individuals were more likely to be at the margin of searching in the first place leading to larger labor supply responses to the lower probability of matching with a firm.

The removal of targeting based on both race and age reduces both the probability of search and the employment rate for older whites but helps sixteen-year-old whites. Recall that firms were able to partially screen out white sixteen-year-olds. Hence, white sixteen-year olds actually see increases in the probability of searching and matching. This is not the case for white teenagers who are older than sixteen. These teenagers face small drops in the probability of matching, which in turn leads to small drops in the probability of searching. Note that these drops have a reinforcing dimension. As nineteen-year-old whites drop out of the labor market, the expected match revenue falls from the firm’s perspective. With falling expected revenues, more teenagers will choose not to participate in the labor force. The losses faced by white teenagers over sixteen are counterbalanced by the gains for black teenagers. Black teenagers see their probability of matching increased by 10% to almost 19%, with higher increases associated with younger workers.

7. Conclusion

Differences in labor market outcomes between blacks and whites are stark. Wage differences are small relative to differences in unemployment. In fact, nineteen-year-old blacks earn more than seventeen-year-old whites despite having higher unemployment rates. The effect of these unemployment rates is magnified by the resulting lower search rates for black teenagers.

We propose and structurally estimate a search model with endogenous labor demand and labor supply. Unemployment has two sources in the model. First, unemployment comes from workers not matching with firms. Second, those who do match may draw match values so low that firms are unwilling to pay these workers the minimum wage. Firms are able to partially target their search and we estimate that firms find it easier to target their search on the basis of race than on the basis of age. The primary reason for unemployment among nineteen-year-old blacks then comes from the low probability of matching with a firm. In contrast, the main reason for unemployment among seventeen-year-old whites are match values below the minimum wage.

Removing firm targeting decreases the black–white unemployment gap. In response to the higher employment rates, more blacks search. However, pooling black and white workers leads to higher unemployment for whites as they are penalized for being pooled with their black counterparts. This has a reinforcing effect as whites respond to the higher unemployment rates by exiting the labor force.

Our model could easily be extended to examine targeting based on many other observable characteristics, such as gender,
education, and age. Possible avenues for future work include relaxing the assumption that workers can not influence the signal they emit or moving to a more dynamic model where the value of future search would influence the wage-setting stage. Future extensions should include older individuals as well, where the unemployment differences between blacks and whites are even more stark, as well as looking at the interplay between labor market conditions and education.

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Appendix. Derivation of wages from a Rubinstein bargaining game

Following the outlines of the proof in Binmore et al. (1989) (from hereon referred to as BSS) and Binmore et al. (1986), we define \( m_f \) and \( M_f \) as the infimum and supremum payoffs for the firm, respectively, and \( m_w \) and \( M_w \) as the infimum and supremum payoffs for the worker, respectively. Match revenue is \( Y \) and outside options are 0 and \( R \) for firms and workers, respectively.

In a Rubinstein bargaining game in which the firm moves first (in the absence of a minimum wage), the following inequalities hold:

\[
\begin{align*}
    m_f & \geq Y - \max \{\tau_w M_w, R\} \\
    Y - M_f & \geq \max \{\tau_w m_w, R\} \\
    m_w & \geq Y - \tau_f M_f \\
    Y - M_w & \geq \tau_f m_f.
\end{align*}
\]

Here, \( \tau_w \) represents the worker’s discount factor, and \( \tau_f \) represents the firm’s. We define \( \tau_i = 1 - e^{-\tau_i} \), where \( \tau \) is the common interest rate, and \( \Delta_i \) represents the length of the interval that elapses between i’s reaction to the other party’s offer. Therefore, if \( \Delta_w > \Delta_f \), it means the worker takes longer to respond to a firm’s offer compared to a firm’s response to a worker’s offer.

Inclusion of minimum wage means that the any bargaining offer (whether supremum or infimum) must be capped from below at the minimum wage: therefore, the inequalities change to

\[
\begin{align*}
    m_f & \geq Y - \max \{\tau_w M_w, W, R\} \\
    Y - M_f & \geq \max \{\tau_w m_w, W, R\} \\
    m_w & \geq Y - \tau_f M_f \\
    Y - M_w & \geq \tau_f m_f.
\end{align*}
\]

We will examine the case where \( W \geq R \) and \( W < R \) separately. First, when \( W \geq R \), we examine three regions, defined similarly to BSS:

\[
W \leq \tau_w m_w \ (\text{region } 1), \quad \tau_w m_w < W < \tau_w M_w \ (\text{region } 2), \quad \text{and} \quad W \geq \tau_w M_w \ (\text{region } 3).
\]

Focusing on region 1, the inequalities change to

\[
\begin{align*}
    m_f & \geq \frac{Y - \tau_w M_w}{1 - \tau_w} \\
    Y - M_f & \geq \frac{\tau_w m_w - \tau_f M_f}{1 - \tau_w} \\
    m_w & \geq \frac{Y - \tau_f M_f}{1 - \tau_w} \\
    Y - M_w & \geq \tau_f m_f.
\end{align*}
\]

It is easy to show that

\[
\frac{(1 - \tau_f)Y}{1 - \tau_f \tau_w} \leq m_w \leq \frac{(1 - \tau_f)Y}{1 - \tau_f \tau_w},
\]

Therefore, \( M_w = m_w = \frac{(1 - \tau_f)Y}{1 - \tau_f \tau_w} \). We now let \( \Delta_w \) and \( \Delta_f \) approach zero while keeping their ratios constant, such that we define \( \beta = \frac{\Delta_f}{\Delta_f + \Delta_w} \). Then, \( M_w = m_w = \beta Y \), implying that \( M_f = m_f = (1 - \beta)Y \).

We next show that region 2 yields a logical contradiction:

\[
\begin{align*}
    m_f & \geq \frac{Y - \tau_w M_w}{1 - \tau_w} \\
    Y - M_f & \geq \frac{W - \tau_w m_w}{1 - \tau_w} \\
    m_w & \geq \frac{Y - \tau_f M_f}{1 - \tau_w} \\
    Y - M_w & \geq \tau_f m_f.
\end{align*}
\]

which yields \( \frac{(1 - \tau_f)Y}{1 - \tau_f \tau_w} < m_w \leq \frac{(1 - \tau_f)Y}{1 - \tau_f \tau_w} \).

For region 3, the inequalities are

\[
\begin{align*}
    m_f & \geq \frac{Y - \tau_w M_w}{1 - \tau_w} \\
    Y - M_f & \geq \frac{W - \tau_f M_f}{1 - \tau_f} \\
    m_w & \geq \frac{Y - \tau_f M_f}{1 - \tau_w} \\
    Y - M_w & \geq \tau_f m_f.
\end{align*}
\]

This yields \( m_w = M_w = \frac{(1 - \tau_f)Y + \tau_f W}{1 - \tau_f} \) and \( m_f = M_f = Y - W \). Letting \( \Delta_f \) approach zero, we have \( m_w = M_w = W \) and \( m_f = M_f = Y - W \). When \( W \geq R \) and a worker successfully matches, his wage outcome is \( \max(\beta Y, W) \).

Now, repeating the exercise with \( W < R \), we see that, for regions 1 and 2, the results are identical (since we just replace \( W \) with \( R \)) and region 3 changes to \( m_w = M_w = R \) and \( m_f = M_f = Y - R \). Therefore, when \( W < R \) and a worker successfully matches, his wage outcome is \( \max(\beta Y, R) \).

Combining these two results, when a worker successfully matches \( (Y > W) \), the unique subgame perfect equilibrium outcome of the bargaining game is a wage offer of \( \max(\beta Y, W, R) \), which is accepted.

References


