Valuing an 8% 4-year Treasury bond

- The bond has a coupon rate of 8%, a face value of $1000 and a maturity of 4 years
- Each year you receive an interest payment of $80.
- In the year the bond matures you receive the final $80 interest payment and the $1000 face value.

\[
\begin{array}{cccc}
80 & 80 & 80 & 1080 \\
\hline
1 & 2 & 3 & 4
\end{array}
\]
Determine PV of bond’s cash flows

Suppose that similar risk investments offer a 9% return.

PV=

• Note that the bond can be valued in two pieces:
  – Annuity of $80 per year for four years.
  – Lump sum of $1000 at the end of 4 years

Alternative question: If the bond sells for $967.60, what return do investors expect? Yield to maturity and internal rate of return.

Valuing a semiannual coupon bond

In practice Treasuries make semi-annual payments.

\[
\begin{array}{cccccc}
0 & 1 & 2 & N-1 & N \\
\text{C/2} & \text{C/2} & \text{C/2} & \text{C/2} & \text{C/2} & \text{C/2} & \text{C/2} & \text{F} \\
\end{array}
\]

• Two Pieces:
  – Annuity of C/2 for 2N periods where C is total annual coupon payment.
  – Lump sum of F (face value) received at the end of 2N periods.

\[
P_0 = C/2 \left[ \frac{1}{r/2} - \frac{1}{(r/2)(1 + r/2)^{2N}} \right] + \frac{F}{(1 + r/2)^{2N}}
\]
Valuing a semiannual coupon bond: An example

• Dupont issued 30 year maturity bonds with a coupon rate of 7.95%. These bonds currently have 28 years remaining to maturity and are rated AA.
• Newly issued AA bonds with maturities greater than 10 years are currently yielding 7.73%. The bonds have a par value of $1000.
• What is the value of a Dupont bond today?

Dupont example (continued)

• Annual coupon payment=0.0795*$1000=$79.50
• Semiannual coupon payment=$39.75
• Semiannual discount rate=0.0773/2=0.03865
• Number of semiannual periods=28*2=56

\[
P_0 = 39.75 \left[ \frac{1}{0.03865} - \frac{1}{(0.03865)(1+0.03865)^{56}} \right] + \frac{1000}{(1+0.03865)^{56}} = $1025.06
\]
The effect of changes in interest rates on bond prices

• Consider two identical 8% coupon bonds except that one matures in 4 years, the other matures in 10 years.
• Calculate the change in the price of each bond if interest rates fall from 8% to 6%.
• Compare and discuss the relative price changes.

Stock valuation terminology

• $D_t$ or $\text{Div}_t =$ expected dividend at time $t$
• $P_0 =$ market price of stock today (time 0)
• $P_t =$ expected price of stock at time $t$
• $g =$ expected growth rate of dividends
• $r_s =$ required rate of return
• $D_1 / P_0 =$ expected one-year dividend yield
• $(P_1 - P_0) / P_0 =$ expected one-year capital gain
Valuing common stock

• As noted previously, the return on a share of stock is given by:

\[ r_s = \frac{\text{Div}_1 + P_1 - P_0}{P_0} \]

• Suppose that investors require a rate of return of \( r_s \) to hold the stock. The price an investor would be willing to pay is:

\[ P_0 = \frac{\text{Div}_1 + P_1}{1 + r_s} \]

where \( \text{Div}_1 \) is the expected dividend and \( P_1 \) is the expected price of the stock in period 1.

Common stock valuation (continued)

• What determines \( P_1 \)?

• An investor purchasing the stock at time 1 and holding it until time 2 would be willing to pay:

\[ P_1 = \frac{\text{Div}_2 + P_2}{1 + r_s} \]

• Substituting into the equation for \( P_0 \), the price at time zero is:

\[ P_0 = \frac{\text{Div}_1}{1 + r_s} + \frac{1}{1 + r_s} \left( \frac{\text{Div}_2 + P_2}{1 + r_s} \right) \]
Common stock valuation (continued)

• This process can be repeated into the future, for example, to period H, so that:

\[ P_0 = \frac{\text{Div}_1}{(1 + r_s)} + \frac{\text{Div}_2}{(1 + r_s)^2} + \frac{\text{Div}_3}{(1 + r_s)^3} + \cdots + \frac{\text{Div}_H + P_H}{(1 + r_s)^H} \]

\[ = \sum_{t=1}^{H} \frac{\text{Div}_t}{(1 + r_s)^t} + \frac{P_H}{(1 + r_s)^H} \]

• What happens to the last term as the time horizon gets long (H approaches infinity)?

Dividend valuation model

• As H approaches infinity the last term goes to zero…

\[ P_0 = \frac{\text{Div}_1}{(1 + r_s)} + \frac{\text{Div}_2}{(1 + r_s)^2} + \frac{\text{Div}_3}{(1 + r_s)^3} + \cdots + \frac{\text{Div}_H + X}{(1 + r_s)^H} \]

\[ = \sum_{t=1}^{H} \frac{D_i v_t}{(1 + r_s)^t} \]

• The resulting **Dividend Valuation Model** posits that the price of a stock is equal to the present value of the stream of expected future dividends.
Constant dividend growth

• If the dividend payments on a stock are expected to grow at a constant rate, g, and the discount rate is $r_s$, the value of the stock at time 0 is:

\[ P_0 = \frac{D_0}{r_s - g} \]

• g must be less than $r_s$ to use this formula
• If g = 0, the formula reduces to the perpetuity formula

Dividend valuation example

• Geneva steel just paid a dividend of $2.10. Geneva’s dividend payments are expected to grow at a constant rate of 6%. The appropriate discount rate is 12%. What is the price of Geneva Stock?
• Div₀ = $2.10  \quad Div₁ = $2.10(1.06) = $2.226

\[ P₀ = \frac{-2.226}{0.12 - 0.06} = $37.10 \]
Estimating the capitalization rate

The growing perpetuity formula that explains price...

\[ P_0 = \frac{Div_1}{r_s - g} \]

… can be rearranged to get an estimate of \( r_s \)

\[ r_s = \frac{Div_1}{P_0} + g \]

The market capitalization rate equals the dividend yield \((Div_1/P_0)\) plus the expected rate of growth on dividends \((g)\).

Estimating the capitalization rate of Sears

- In early 1986 Sears stock was selling for $45 per share. Dividend payments for 1986 were expected to be $1.76. This implies a dividend yield of 0.039 or 3.9%.
- Estimating \( g \) is trickier. One approach is to start with the payout ratio, the ratio of dividends paid to earnings per share. The payout ratio for Sears has been around 45% of earnings per share (EPS).
- This means that each year the company plows back 55% of EPS into the business
- Plowback ratio = 1 - payout ratio
Capitalization rate of Sears (cont)

- In addition Sears’ return on equity (ROE) has been stable at 13%. ROE is the ratio of earnings per share to book equity per share.
- If the company earns 13% of book equity and reinvests 55% of that, then book equity will increase by \(.55 \times .13 = .072\) or 7.2%.
- Earnings and dividends per share will also increase by 7.2%.
- Dividend growth rate \(= g = \text{plowback ratio} \times \text{ROE}\)
- Assuming these relationships hold in the future, the equity capitalization rate for Sears is:

\[
  r_s = \frac{\text{Div}_1}{P_0} + g = \frac{1.76}{45} + 0.072 = 0.111 = 11.1\%
\]

Caveats on estimating rates of return

- It is difficult to estimate \(r_s\) using only a single stock. Use a large sample of equivalent risk securities.
- Do not apply the technique to firms with high current rates of growth. It is unlikely that supernormal growth can be sustained.
  - Why might this be the case?
Valuation of stocks with variable dividend growth

• Firms go through lifecycles
  – Fast growth
  – Growth that matches the economy
  – Decline

• A supernormal growth stock is one that is going through a period of rapid growth in dividends. This supernormal growth is generally only temporary.

Valuation of stocks with variable dividend growth

• Find the PV of dividends during the period of nonconstant growth.

• Find the price of the stock at the end of the nonconstant growth period. Using, for example, the constant growth model. Discount this price back to the present.

• Add these two present values to find the intrinsic value (price) of the stock.
Valuation of stocks with variable dividend growth: An example

- Batesco Inc. just paid a dividend of $1. The dividends of Batesco are expected to grow at 50% the next year (year 1) and 25% in the year after that (year 2). Batesco’s dividends are expected to grow at 6% per year in perpetuity beginning in year 3.
- The proper discount rate for Batesco is 13%.
- What price would you pay for a share of Batesco stock?

Example (continued)

- First, determine the dividends using g
  - \( D_0 = $1 \)  \( g_1 = 50\% \)
  - \( D_1 = $1(1.50) = $1.50 \)  \( g_2 = 25\% \)
  - \( D_2 = $1.50(1.25) = $1.875 \)  \( g_3 = 6\% \)
  - \( D_3 = $1.875(1.06) = $1.9875 \)

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<tr>
<th>Year</th>
<th>Dividend</th>
<th>Growth Rate</th>
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<tr>
<td>0</td>
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<td>50%</td>
</tr>
<tr>
<td>1</td>
<td>1.50</td>
<td>25%</td>
</tr>
<tr>
<td>2</td>
<td>1.875</td>
<td>6%</td>
</tr>
<tr>
<td>3</td>
<td>1.9875</td>
<td>6%</td>
</tr>
<tr>
<td>4</td>
<td>2.107</td>
<td></td>
</tr>
</tbody>
</table>
Example (continued)

- Supernormal growth period:
  \[ P_s = \frac{D_1}{(1+r_s)^1} + \frac{D_2}{(1+r_s)^2} = \frac{1.50}{(1.13)^1} + \frac{1.875}{(1.13)^2} = $2.796 \]

- Constant growth period. Value at time 2:
  \[ P_c = \frac{D_3}{r_s - g} = \frac{1.9875}{0.13 - 0.06} = $28.393 \]

- discount to time 0 and add to \( P_s \):
  \[ P_0 = P_s + \frac{P_c}{(1+r_s)^2} = 2.796 + \frac{28.393}{(1.13)^2} = $25.03 \]

Link between stock prices and earnings

- Consider a firm with a 100% payout ratio, where DIV = EPS in each period
- A new valuation model:
  \[ P_0 = \sum_{t=1}^{H} \frac{E P S_t}{(1 + r_s)^t} \]

- Example: \( r = .10 \), EPS = $10 in perpetuity
  \[ P = 10/.10 = $100 \]
- What is the P-E ratio?
- What is the E-P ratio?
- Rationale for use as multiplier, capitalization rate.
Suppose at time 0, the firm discovers a future investment opportunity...

... which it plans to finance with earnings.

The opportunity:
Invest: $10.00 per share at t = 1.
Returns: $1.50 per share per year in perpetuity starting in year 2.

What is the effect of the discovery of this new project on TODAY’s stock price?

Present value of all future growth opportunities (PVGO)

\[
PVGO = \sum_{t=1}^{H} \frac{NPV_t}{(1 + r)^i}
\]

Thus, another valuation model:

\[
P_0 = \frac{EPS}{r} + PVGO
\]

(capitalized value with no growth opportunities)

<table>
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<tr>
<th>Value</th>
<th>$100</th>
<th>$4.55</th>
</tr>
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</table>

What affect does the new investment opportunity have on the E/P ratio?

E/P ratio goes from .10 (=10/100)…
… to .0956 (= 10/104.55).

Is the E/P ratio the capitalization rate?

The E/P ratio underestimates the capitalization rate when the stock price (P) reflects the present value of future growth opportunities.

What affect does the new investment opportunity have on the P/E ratio?

P/E ratio goes from 10 (=100/10)…
… to 10.455 (= 104.55/10).

So what do high P/E ratios signify?
(1) Growth opportunities
(2) Safe earnings
or (3) Some combination of the two!

WARNINGS:
• Price is forward looking…EPS is historical.
• BE CAREFUL WITH ACCOUNTING NUMBERS