Real Options

Finance 581
Advance Valuation Techniques

Managerial Decisions

- Investment decision
  - Invest now
  - Wait
  - Miss opportunity
- Operational decision
  - Expand
  - Status quo
  - Close
  - Abandon

Take into consideration time and price variabilities
Discounted Cash Flow Analysis

- DCF analysis approach
  - Unknown risky future cash flows are summarized by their expected (mean) values
  - Discounted to the present at a RADR
  - Compared to current costs to yield NPV
- Problem is sterilized of many problems
  - Managerial options are ignored.

What is an Option?

- An option gives the holder the right, but not the obligation to buy (call option) or sell (put option) a designated asset at a predetermined price (exercise price) on or before a fixed expiration date
- Options have value because their terms allow the holder to profit from price moves in one direction without bearing (or, limiting) risk in the other direction.
Two Sides of Uncertainty

Economic uncertainty
- Correlated with economy
- Exogenous, so learn by waiting
- Delays investment (NPV>0?)

Technical uncertainty
- Not correlated with economy
- Endogenous, so learn by doing
- Incentives for starting the investment (NPV<0?)

Investment: Governed quantitatively by the “bad news” principle (fear)
Abandon: Governed quantitatively by the “good news” principle (hope)

Expected value

Value of flexibility to alter decisions as info becomes available

Expected value with flexibility
Some Option Basics

<table>
<thead>
<tr>
<th>As _____ increase</th>
<th>Option Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call price</td>
<td>↑</td>
</tr>
<tr>
<td>Put price</td>
<td>↓</td>
</tr>
<tr>
<td>Asset price</td>
<td>↑</td>
</tr>
<tr>
<td>Exercise price</td>
<td>↓</td>
</tr>
<tr>
<td>Maturity</td>
<td>↑</td>
</tr>
<tr>
<td>Volatility</td>
<td>↑</td>
</tr>
<tr>
<td>Interest rate</td>
<td>↑</td>
</tr>
</tbody>
</table>

Some Terms
- In-the-money
- Out-of-the-money
- Intrinsic value
- Time value

What is a Real Option?

- An option on a non-traded asset, such as an investment project or a gold mine
- Options in capital budgeting
  - Delay a project (wait and learn)
  - Expand a project (“follow-on” investments)
  - Abandon a project
- Real options allow managers to add value to their firms by acting to amplify good fortune or to mitigate loss.
Investment Opportunities as Real Options

- Executives readily see why investing today in R&D, a new marketing program, or certain capital expenditures can generate the possibility of new products or new markets tomorrow.
- However, the journey from insight to action is often difficult.

Management’s Interest

- Experts explain what option pricing captures that DCF and NPV don’t.
  - Often buried in complex mathematics.
- Managers want to know how to use option pricing on their projects.
- Thus, need a framework to bridge the gap between real-world capital projects and higher math associated with option pricing theory.
  - Show spreadsheet models with “good enough” results.
Corporate Investments

- Corporate investment opportunity is like a call option
  - Corporation has the right but not the obligation to acquire something
- If we can find a call option sufficiently similar to the investment opportunity, the value of the option would tell us something about the value of the opportunity
  - However, most business opportunities are unique
  - Thus, need to construct a similar option.

Mapping a Project onto an Option

- Establish a correspondence between the project’s characteristics and 5 variables that determine value of a simple call option on a share of stock
  - Slide 7 shows the variables
- Use a European call
  - Exercised on only one date, its expiration date
  - Not a perfect substitute, but still informative.
Mapping

Investment opportunity
- PV of a project's operating assets to be acquired
- Expenditure required to acquire the project assets
- Length of time the decision may be deferred
- Time value of money
- Riskiness of the project assets

Call option
- Stock price
- Exercise price
- Time to expiration
- Risk-free rate of return
- Variance of returns on stock

NPV & Option Value Identical

Investment decision can no longer be deferred

\[ \text{NPV} = (\text{value of project assets}) - (\text{expenditure required}) \]

This is \( S \). This is \( X \).

So: \( \text{NPV} = S - X \)

We decide to “go” or “no go”. Here it’s “exercise” or “not”.

When \( t = 0 \), \( \sigma^2 \) and \( r_f \) do not affect call option value. Only \( S \) and \( X \) matter. At expiration, call option value is greater of \( S - X \) or 0.
Divergence

- When do NPV & option pricing diverge?
  - Investment decisions may be deferred
- Deferral gives rise to *two* sources of value
  - Better to pay later than sooner, all else equal
  - Value of assets to be acquired can change
    - If value increases, we haven’t missed out -- simply need to exercise the option
    - If value decreases, we might decide not to acquire them
- Traditional NPV misses the deferral opportunity
  - It assumes the decision can’t be put off.

1st Source: Capture Time Value

- Suppose you just put enough money in the bank now so that when it’s time to invest, that money plus interest it earned is sufficient to fund the required expenditure
- How much money is it?
  \[ PV(X) = \frac{X}{(1 + r_f)^t} \]
- Extra value = \( r_f \times PV(X) \) compounded \( t \) periods or \( X - PV(X) \)
- Conventional NPV misses the extra value.
"Modified" NPV

- NPV = S - X
- Rewrite using PV(X) instead of X
  
  \[
  \text{"Modified" NPV} = S - PV(X)
  \]
  
  \(S\) is value; \(PV(X)\) is cost adjusted for TVM
- "Modified" NPV \(\geq\) NPV
  - Implicitly includes interest to be earned while waiting
  - Modified NPV can be positive, negative, or zero
  - Express the relationship between cost and value so that the number > 0.

NPV as a Quotient

- Instead of expressing modified NPV as a difference, express it as a quotient
  - Converts negative value to decimals between 0 and 1
  
  \[
  NPV_q = S \div PV(X)
  \]
- \(NPV\) and \(NPV_q\) are not equivalent
  - \(S = 5, PV(X) = 7, NPV = -2\) but \(NPV_q = 0.714\)
  - When modified \(NPV > 0, NPV_q > 1\)
  - When \(NPV < 0, NPV_q < 1\)
  - When modified \(NPV = 0, NPV_q = 1\).
### Relationships: \( NPV \) & \( NPV_q \)

\[
\begin{align*}
\text{NPV} < 0 & \quad \text{NPV} = S - X & \text{NPV} > 0 \\
\text{NPV}_q < 1 & \quad \text{NPV}_q = S / PV(X) & \text{NPV}_q > 1
\end{align*}
\]

When time runs out, projects here are rejected (option isn’t exercised).

When time runs out, projects here are accepted (option is exercised).

### Interpretation of Real Options

- \( NPV_q > 1 \) \( \Rightarrow \) Positive NPV & call options "in the money"
  - \( NPV_q = \text{Asset value} / \text{PV(exercise price)} \)
- \( NPV_q < 1 \) \( \Rightarrow \) Negative NPV & call options "out of the money"
- Call option value increases as
  - \( NPV_q \) increases
  - Cumulative variance increases
- Traditional DCF treats management as passive
- Real options treat management as active.
Cumulative Volatility

- Asset value can change while you wait
  - Affect investment decision
  - Difficult to quantify since not sure asset values will change, or if they do, what the future value will be
- Don’t measure change in value directly
  - Measure uncertainty and let option-pricing model quantify the value
- Two steps
  - Identify a sensible way to measure uncertainty
  - Express the metric in a mathematical form.

Measure Uncertainty

- Most common probability-weighted measure of dispersion is variance
  - Summary measure of the likelihood of drawing a value far away from the average value
  - The higher the variance, the more likely it is that the values drawn will be either much higher or much lower than average
    - High-variance assets are riskier than low-variance assets
- Variance is incomplete because need to consider time.
Time Dimension

- How much things can change while we wait depends on how long we can afford to wait
  - For business projects, things can change a lot more if we wait 2 years than if we wait only 2 months
- Must think in terms of variance per period
  - Total uncertainty = \( \sigma^2 \times t \)
    - Called cumulative variance
      - Option expiring in 2 periods has twice the cumulative variance of an identical option expiring in one period, given the same variance per period.

Adjustments to Cumulative Variance

- Don’t use variance of project values
  - Use variance of project returns
    - Instead of working with actual dollar values of the project, we’ll work with percentage gain or loss per year
    
    \[
    \text{Return} = \frac{\text{Future value} - \text{present value}}{\text{Present value}}
    \]
  - Express uncertainty in terms of standard deviation
    - Denominated in same units as the thing being measured
  - Convert to cumulative volatility = \( \sigma \sqrt{t} \)
Valuing the Option

- Call-option metrics \( NPV_q \) and \( \sigma \sqrt{t} \) contain all the info needed to value a project as a European call option
  - Capture the extra sources of value associated with opportunities
  - Composed of the 5 fundamental option-pricing variables onto which we map our business opportunity
    - \( NPV_q: S, X, r_f, \) and \( t \)
    - Cumulative volatility combines \( \sigma \) with \( t \).

Linking the Metrics to Black-Scholes

- Investment opportunity
  - PV of a project's operating assets to be acquired
  - Expenditure required to acquire the project assets
  - Length of time the decision may be deferred
  - Time value of money
  - Riskiness of the project assets

Combining values allows us to work in 2-space.
Locating the Option Value

Call option value increases in these directions:
- Lower values of NPV_q:
  - Lower X;
  - Higher S, r_f, or t.
- Higher σ and t increase the option value.

Higher NPV_q:
- Higher X;
- Lower S.

Locating various projects reveals their relative value to each other.

"Pricing the Space"

Black-Scholes value expressed as % of underlying asset NPV_q

<table>
<thead>
<tr>
<th>σ√t</th>
<th>.45</th>
<th>.96</th>
<th>.98</th>
<th>1.00</th>
<th>1.02</th>
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<tr>
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<td>16.2</td>
<td>17.0</td>
<td>17.8</td>
<td>18.6</td>
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<tr>
<td>.50</td>
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<td>.55</td>
<td>20.1</td>
<td>20.9</td>
<td>21.7</td>
<td>22.4</td>
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</table>

Suppose S = $100, X = $105, t = 1 year, r_f = 5%, σ = 50% per year.
Then NPV_q = 1.0 and σ√t = 0.50.
Table gives a value of 19.7%.
Viewed as a call option, the project has a value of:
- Call value = 0.197 * $100 = $19.70
- Conventional NPV = $100 - $105 = -$5.
Interpret the Option Value

- Why is the option value of $19.70 less than the asset value (S) of $100?
  - We've been analyzing sources of extra value associated with being able to defer an investment
- Don’t expect the option value > S = $100; rather expect it to be greater than {NPV} = S - PV(X)
  - For {NPVq = 1}, then S / PV(X) = 100 / ($105 / 1.05)
- Thus, conventional {NPV} = S - X = $100 - $105 = -$5.

Estimate Cumulative Variance

- Most difficult variable to estimate is σ
- For a real option, σ can’t be found in a newspaper and most people don’t have a highly developed intuition about uncertainty
- Approaches:
  - A(n educated) guess
  - Gather some data
  - Simulate σ.
A(n Educated) Guess

- $\sigma$ for returns on broad-based U.S. stock indexes = 20% per year for most of the past 15 years
  - Higher for individual stocks
  - GM’s $\sigma = 25\%$ per year
- $\sigma$ of individual projects within companies $>$ 20%
- Range within a company for manufacturing assets is probably 30% to 60% per year.

Gather Some Data

- Estimate volatility using historical data on investment returns in the same or related industries
- Computed implied volatility using current prices of stock options traded on organized exchanges
  - Use Black-Scholes model to figure out what $\sigma$ must be.
Simulate

- Spreadsheet-based projections of a project’s future cash flows, together with Monte Carlo simulation techniques, can be used to synthesize a probability distribution for project returns
  - Requires educated guesses about outcomes and distributions for input variables
  - Calculate $\sigma$ for the distribution.

Capital Budgeting Example

<table>
<thead>
<tr>
<th>Year</th>
<th>Phase 1 FCF</th>
<th>Investment</th>
<th>Terminal value</th>
<th>Net FCF</th>
<th>Present value</th>
<th>Net present value</th>
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<tr>
<td></td>
<td>0.0</td>
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<td>190.0</td>
<td>-125.0</td>
<td>-125.0</td>
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<tr>
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<tr>
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<td>12.1</td>
<td>12.1</td>
<td>12.1</td>
<td>12.1</td>
<td>12.1</td>
<td>12.1</td>
</tr>
</tbody>
</table>

Hurdle rate: 12%
Growth: 5%
Discount factor: 0.892857, 0.797194, 0.71178, 0.635518, 0.567427, 0.506631

Terminal value changes as hurdle rate changes.
Capital Budgeting Example

<table>
<thead>
<tr>
<th>Phase 1 FCF</th>
<th>0.0</th>
<th>9.0</th>
<th>10.0</th>
<th>11.0</th>
<th>11.6</th>
<th>12.1</th>
<th>12.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment</td>
<td>-125.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Terminal value</td>
<td>190.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Present value</td>
<td>-125.0</td>
<td>8.0</td>
<td>8.0</td>
<td>7.8</td>
<td>7.4</td>
<td>6.9</td>
<td>6.4</td>
</tr>
<tr>
<td>Net present value</td>
<td>18.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase 2 FCF</td>
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<td>25.4</td>
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<tr>
<td>Investment</td>
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<td>26.4</td>
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<td></td>
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<tr>
<td>Net present value</td>
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<td>14.7</td>
<td>14.4</td>
<td>227.0</td>
<td></td>
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<td></td>
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<td>Combined FCF</td>
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<td>10.0</td>
<td>11.0</td>
<td>34.7</td>
<td>37.5</td>
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<tr>
<td>Investment</td>
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<td>0.0</td>
<td>-382.0</td>
<td>0.0</td>
<td>0.0</td>
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<td>Terminal value</td>
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<td></td>
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<tr>
<td>Net present value</td>
<td>-125.0</td>
<td>32.3</td>
<td>26.4</td>
<td>227.0</td>
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<td>Net present value</td>
<td>-55.2</td>
<td>14.7</td>
<td>14.4</td>
<td>227.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Valuing the Option

- Combine the option-pricing variables into our two option-value metrics:
  \[ NPV_q = S + PV(X) = \frac{255.7}{382 \times (1.055)^3} = 0.786 \]
- Look up \( \sigma \sqrt{t} = 0.4 \times \sqrt{3} = 0.693 \)

About 19% of underlying asset \( S \) or $48.6 million.
Value of Project

Project value = NPV(phase 1) + call value (phase 2)
Project value = $16.3 + $48.6 = $64.9
- Original estimate = $0.2
- A marginal DCF analysis project is in fact very attractive
- What to do next?
  - Check and update assumptions
  - Check for disadvantages to deferring investment
  - Simulate, ...

Black-Scholes Model

Call = S N(d_1) - E \( e^{-rt} \) N(d_2)
\[
d_1 = \left[ \ln(S/E) + (r + \sigma^2/2)t \right] / \sigma \sqrt{t}
\]
\[
d_2 = d_1 - \sigma \sqrt{t}
\]
Put = E \( e^{-rt} \) + C - S

- Known as put-call parity
- No early exercise or payment of dividends
- Inputs are consistent on time measurement
  - All weekly, quarterly, etc...

S = stock price
N(d) = cumulative normal distribution
E = exercise price
r = continuous risk-free rate
t = time to maturity
\( \sigma \) = std deviation in returns
Interpretation of N(d)

- Think of N(d) as risk-adjusted probabilities that the option will expire in-the-money
- Example:
  - \( S/E >> 1.0 \Rightarrow \) Stock price is high relative to exercise price, suggesting a virtual certainty that the call option will expire in-the-money
    - Thus, N(d) terms will be close to 1.0 and call option formula will collapse to \( S - E e^{-rt} \Rightarrow \) Intrinsic value of option
  - \( S/E << 1.0 \Rightarrow \) Both N(d) terms close to zero and option value close to zero as it is deep out-of-the-money.

N(d): Risk-Adjusted Probabilities

- \( \ln(S/E) = \% \) amount the option is in or out of the money
  - \( S = 105 \) and \( E = 100 \), the option is 5\% in the money
    - \( \ln(S/E) = 4.9\% \)
  - \( S = 95 \) and \( E = 100 \), the option is 5\% out of the money
    - \( \ln(S/E) = -5.1\% \)
- \( \sigma \sqrt{t} \) adjusts the amount by which the option is in or out of the money for the volatility of the stock price over the remaining life of the option.
Another Example Using NPVq: “Follow-on” Investment Option

<table>
<thead>
<tr>
<th>Year</th>
<th>1997</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
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<tbody>
<tr>
<td>Op. CF</td>
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<td>159</td>
<td>295</td>
<td>185</td>
<td>0</td>
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<td>Cap. Invest.</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Inc. WC</td>
<td>0</td>
<td>50</td>
<td>100</td>
<td>100</td>
<td>-125</td>
<td>-125</td>
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<tr>
<td>Net CF</td>
<td>-450</td>
<td>60</td>
<td>59</td>
<td>195</td>
<td>310</td>
<td>125</td>
</tr>
</tbody>
</table>

NPV at 20% = -$46.45 million. Project fails to meet hurdle rate. If the company doesn’t make the investment now, it will probably be too cost prohibitive later. By investing now, the opportunity exists for later “follow-on” investments. The project gives its own cash flows & the call option to go to the next step.

Valuing the “Follow-on” Option...

- “Follow-on” investment must be made in 3 years
- New investment = 2 * initial investment ($900 M)
- Forecast cash inflows = 2 * initial inflows
  - PV = $800 M in 3-years; $463 M today @ 20%
- Future cash flows highly uncertain
  - Standard deviation = 35% per year
- Annual risk-free rate = 10%
- Interpretation:
  - The opportunity to invest is a 3-year call option on an asset worth $463 M with a $900 M exercise price.
Valuing the “Follow-on” Option

\[ NPV_q = \frac{\text{Underlying asset value}}{\text{PV (exercise price)}} \]
\[ = \frac{\$463}{\left[ \frac{\$900}{(1.1)^3} \right]} = 0.68 \]

Cumulative variance = \( \sigma \sqrt{\text{time}} = 0.35 \sqrt{3} = 0.61 \)

Call value = Asset value * BS value as % of asset
\[ = \$463 \times 11.9\% = \$55 \text{ M} \]

- Value of project = -$46 M + $55 M = $9 M
- Interpretation:
  - “Follow-on” has a NPV = -$100, 3 years from now. The project may be very profitable because of its high variance.
  - The call option allows you to cash in on the opportunity.

A Binomial Approach to Pricing Options

- Sometimes it is inappropriate to use Black-Scholes
- Examples
  - Abandonment value
  - Timing option.
Option to Abandon an Asset

PV = $553,000

E(CF) = Pr(High demand) * $738 + Pr(Low demand) * $415
= .6 * $738 + .4 * $415 = $609

PV = $609 / 1.1 = $553,000

If the project is not successful in the 1st year, it is better to abandon for $500,000 rather than retain an asset worth only $415,000.

Value of Option to Bail Out

- PV of asset without option = $553,000
- Exercise price = $500,000
- Maturity = 1 year
- Risk-free rate = 5%
- FV of asset with high demand = $738,000
- FV of asset with low demand = $415,000
- Pretend that individuals are risk-neutral and then value the expected payoffs from the option.
The Binomial Option Methodology

- You are indifferent to risk and content to earn the risk-free rate of 5%
- The asset will either increase 33% \((\frac{738}{553})\) or decline 25% \((\frac{415}{553})\) from its PV
  - True probability of rise in value = 60%
  - Don’t need to know this to value the option
- Solve this equation for \(Pr(\text{rise})\):
  \[
  Pr(\text{Rise}) \times 33\% + [1 - Pr(\text{Rise})] \times (-25\%) = 5\%
  \]
  \[
  \therefore Pr(\text{Rise}) = 52\%.
  \]

Value of Abandonment Option

- If asset is successful, abandonment option = $0
- If asset is unsuccessful, can sell the asset and save $85,000 (= $500,000 - $415,000)
- \(E(\text{option}) = Pr(\text{rise}) \times 0 + [1 - Pr(\text{rise})] \times 85\)
  \[
  = 52\% \times 0 + 48\% \times 85 = 41,000
  \]
- PV option to abandon = $41,000 / 1.05 = $39,000
- Value of asset = $553,000 + $39,000 = $592,000.
Practical Considerations

- Abandon value may change over the life of the project making it difficult to use BS
- Abandoning may bring in costs and not a liquidation amount
  - Makes no sense to abandon
- Implications
  - Use annual contracts and not long-term contracts
  - Hire on a temporary basis rather than permanently
  - Lease physical assets on a short-term basis
  - Make financial commitments in stages.

The “Timing” Option

- Opportunity to invest in NPV > 0 is equivalent to an “in-the-money” call option
- Optimal investment timing means exercising the call option at the best time
- Example:
  - If the project is a winner, waiting to invest could mean the loss of early high cash flows
  - If the project is a loser, waiting could prevent a bad mistake.
"Timing" Option’s Profile...

Value of option to invest

Investment can be postponed

Investment now or never

You can delay construction for 1 year. Even though the project may have NPV ≤ 0 today, the call option has value because the year’s delay allows room for the market to improve.
"Timing" Option's Profile

Value of option to invest

Investment can be postponed

You can delay construction for 1 year. Even though the project may have NPV \( \leq 0 \) today, the call option has value because the year’s delay allows room for the market to improve.

Exercise now

Investment now or never

Delay exercising

0

Project NPV

Possible Cash Flows

Current value = $200. If demand is low in year 1, cash flow = $16 and value of project falls to $160. If demand is high in year 1, cash flow = $25 and value of project rises to $250. Although the project lasts indefinitely, assume that you can delay the decision only 1 year. By investing this year, you earn either $16 or $25 in cash flows. By delaying, you forego these cash flows but gain information.
"Pretend" Probability of High Demand

\[ PV = $200 \]

Cash flow = $16

Cash flow = $25

\[ E(r) = Pr(\text{High demand}) \times 37.5\% + [1 - Pr(\text{High demand})] \times (-12\%) = 5\% \]

Solve for \( Pr(\text{High demand}) = 34.3\% \)

Value a call option with an exercise price = $180

If low demand results, the option on the left leg = $0

If high demand result, the option on the right leg = $250 - $180 = $70.

Value of Timing Option Today

\[ E(r) = Pr(\text{High demand}) \times 37.5\% + [1 - Pr(\text{High demand})] \times (-12\%) = 5\% \]

Solve for \( Pr(\text{High demand}) = 34.3\% \)

Value a call option with an exercise price = $180

If low demand results, the option on the left leg = $0

If high demand result, the option on the right leg = $250 - $180 = $70.
Value of Option Today

Assume the risk-free rate is 5%.

Value a call option with an exercise price = $180
If low demand results, the option on the left leg = $0
If high demand results, the option on the right leg = $250 - $180 = $70

The current value of the option = (34.3% * $70 + 65.7% * $0) / 1.05 = $22.9 million

Value of Timing Option Today

The option is worth $22.9 million if you keep it open.
It is worth $20 million if exercised now:
\[ PV \text{ of } $200 - \text{exercise price of } $180 \]
Even though the project has NPV > 0, you should not invest now. A better strategy is to wait and see.
NPV Rules vs. Real Options

**NPV**
- Invest in all projects with NPV > 0
- Reject all projects with NPV < 0
- Among mutually exclusive projects, choose the higher NPV

**Real Options**
- Invest when the project is “deep in the money”
- Can recommend to start “strategic projects”
- Frequently chooses smaller projects sufficiently deep in the money

Practical Considerations

- Difficult to estimate project’s value and variance
- Behavior of prices over time may not conform to the price path assumed by option pricing models
- How long can the investment be deferred?
- Need to know the probability distribution for X and joint probability distribution of S and X
- Does uncertainty change over time?
- Is the option an American type as opposed to European?
- Do the Black-Scholes assumptions hold?
The End