Tutorial of the Black-Scholes Model

Assumptions

1. Risk-free rate = 8.5%
2. Strike (exercise) price = $50
3. Time to maturity = 1 year
4. Current stock price = $46.50
5. No dividends paid
6. Option issuers and holders are risk neutral—they would accept $1 for taking a 1 in 2 chance of losing $2 or a 1 in 3 chance of losing $3 and so forth.

The present value of the strike price = $50 / (1.085) = $46.00. According to the risk-neutral assumption, the present value of the contingent liability is $46.08 multiplied by the probability the liability will be paid.

Logic of the Black-Scholes Model

1. Compute the total return that would be realized if the stock grew from $46.50 to $50 in one year

Future value = Present value x e(rate of return x time)
Rate of return x time = ln(future value / present value)
Rate of return = ln($50 / $46.50) / 1 = ln(1.0753) = 7.26%

Question: What is the probability of the actual return on the stock for the next year will be at or above 7.26%?

2. Let’s compute the probability of a return greater than 7.26%.

Z = Volatility / 2
   - ln(present value of stock/present value of exercise price) / volatility

Volatility = standard deviation of the stock’s rate of return

Let’s assume volatility = 30%.

Z = .30 / 2 – ln($46.50 / $46.00) / .30 = .114

Look up the value 0.114 in a traditional normal distribution table. The interpretation is that 45.45% of the time, the stock’s rate of return will be higher than the 7.26% threshold rate or .114 standard deviations above
average. Thus, the present value of the hypothetical contingent liability is $46 \times 45.45\% = $20.91.

3. Compute the value of the hypothetical security greater than $50.

There are many possible future stock prices greater than $50, each with its own probability of occurrence. For a stock that pays no dividends, the current stock price can be thought of as the sum of the present values of all future possible stock prices zero or greater multiplied by their individual probabilities of occurrence.

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Z^* = -\frac{\text{Volatility}}{2} - \ln\left(\frac{\text{present value of stock}}{\text{present value of exercise price}}\right) / \text{volatility}
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Z^* = \frac{-0.30}{2} - \ln\left(\frac{46.50}{46.00}\right) / 0.30 = -0.186
\]

The probability is 57.37\% that the stock’s rate of return will be higher than the \(Z^*\) threshold of -0.186 or 0.186 standard deviations below average. This means the present value of the hypothetical security is \(46.50 \times 57.37\% = 26.68\).


**Complications**

1. Early exercise not allowed under the Black-Scholes model. FAS No. 123 addresses this problem by stipulating that the option life is the expected time until the option is exercised, instead of its contractual term.
2. If a stock pays dividends, it is necessary to reduce the current stock price by the value of dividends to be paid during the life of the option, since most options do not give holders the benefit of dividends paid before exercise.
3. Volatility is for the expected life of the option.