CSE 355 HOMEWORK ONE

DUE 22 SEPTEMBER 2014, START OF CLASS

(1) Give a state diagram and the table specifying the transition function, for a DFA to recognize
   (a) \( L_1 = \{ w \in \{0,1\}^* : w \text{ either starts with } 00 \text{ or ends with } 11 \text{ (or both)} \} \)
   (b) \( L_2 = \{ w \in \{0,1\}^* : w \text{ contains } 00 \text{ as a substring at least twice} \} \)
   (c) \( L_3 = \{ w \in \{0,1\}^* : w \text{ is the binary representation of a number that is divisible by } 3 \} \)
   (d) (not to be graded) \( L_4 = \{ w \in \{0,1\}^* : w \text{ is the binary representation of a number that is divisible by } 11 \} \)
   (e) (not to be graded) \( L_5 = \{ w \in \{0,1\}^* : w \text{ is the binary representation of a number that is relatively prime to } 33 \} \)

(2) Give a state diagram for an NFA with as few states as you can to recognize
   (a) \( L_6 = \{ w \in \{0,1\}^* : w \in 0^+ 1^+ \} \)
   (b) \( L_7 = \{ w \in \{0,1\}^* : w \in (0^+ \cup 1^*)^* \cup (1^+ \cup 0^*)^* \} \)
   (c) \( L_8 = \overline{L_7} \).
   (d) (not to be graded) \( L_9 = \{ w \in \{0,1\}^* : w \text{ ends with } 010101 \} \)
   (e) (not to be graded) \( L_{10} = \{ w \in \{0,1\}^* : w \text{ has an odd number of inversions} \} \) (An inversion in \( w \) is a pair of consecutive symbols that are different.)

(3) Using the languages from Question 1 and the method of Theorem 1.25, give a state diagram and/or the table specifying the transition function, for a DFA to recognize
   (a) \( L_1 \cap L_2 \)
   (b) \( L_1 \cup L_3 \)
   (c) \( L_1 \cap \overline{L_3} \)
   (d) \( L_3 \cup \overline{L_3} \)
   Do not simplify the DFA produced.

(4) It seems hard for a DFA to determine whether an input string consists of two copies of a string from a regular language \( L \), i.e. a string of the form \( ww \) with \( w \in L \). But my friend tweets me the following: “Because \( L \) is regular, \( L \) has a DFA. Regular languages are closed under concatenation, so \( L L \) has a DFA \( M \), which recognizes \( \{ ww : w \in L \} \).” What precisely is my friend doing wrong?

(5) You are given a DFA \( M \) to recognize a regular language \( L \). You want to make an NFA that recognizes all substrings of strings in \( L \), that is \( \text{substr}(L) = \{ w \in \Sigma^* : xwy \in L \text{ for some } x, y \in \Sigma^* \} \). Devise (and explain) a general method for producing an NFA \( M' \) that recognizes \( \text{substr}(L) \), given only the description of \( M \). Justify why your method works.