Please note that there is more than one way to answer most of these questions. The following only represents a sample solution.

Problem 1: Linz 2.1.7(b)(c)(g), 2.2.7. and 2.2.11

2.1.7: Find dfa’s for the following languages on Σ = \{a, b\}

(b): $L = \{w : |w| \mod 5 \neq 0\}$

A dfa for $L$ is given by the following transition graph:

(c): $L = \{w : n_a(w) \mod 3 > 1\}$

A dfa for $L$ is given by the following transition graph:
(g): $L = \{w : |w| \mod 3 = 0, |w| \neq 6\}$

A dfa for $L$ is given by the following transition graph:

2.2.7: Design an nfa with no more than five states for the set $\{abab^n : n \geq 0\} \cup \{aba^n : n \geq 0\}$.

An nfa for the set is given by the following transition graph:

2.2.11: Find an nfa with four states for $L = \{a^n : n \geq 0\} \cup \{b^na : n \geq 1\}$.

An nfa for $L$ is given by the following transition graph:
Problem 2: Linz 2.39 and 2.3.12

2.39: Let $L$ be a regular language that does not contain $\lambda$. Show that there exists an nfa without $\lambda$-transitions and with a single final state that accept $L$.

Since $L$ is regular there exists a dfa, $D = (Q, \Sigma, \delta, q_0, F)$, with an associated transition graph, $G_D$, such that $L(D) = L$. We will construct an nfa $N = (Q \cup \{q_f\}, \Sigma, \delta', q_0, \{q_f\})$ where $q_f \notin Q$ by giving its transition graph $G_N$ as follows:

1. From $G_D$, remove the final label from every final state (making them nonfinal states).
2. Add a new state $q_f$ and label it as a final state.
3. For every state $q_i$, if there is a transition from $q_i$ to a state in $F$ on input $a \in \Sigma$, then add a transition from $q_i$ to $q_f$ on input $a$.

Clearly, $N$ has a single accept state, $q_f$, and no $\lambda$-transitions (since $D$ is a dfa and we did not add any $\lambda$-transitions in our construction of $N$). We will now show that $L(N) = L$. First note that since $\lambda \notin L$, every $w \in L$ can be written as $w = va$ for some $v \in \Sigma^*$ and an $a \in \Sigma$.

Now, $w = va \in L$ iff there is a walk on $G_D$ labeled with $w$ from $q_0$ to $q_i$ with $q_i \in F$
iff there is a walk on $G_D$ labeled with $v$ from $q_0$ to $q_j$ and a transition from $q_j$ to $q_i$ on input $a$
iff there is a walk on $G_N$ labeled with $v$ from $q_0$ to $q_j$ and a transition from $q_j$ to $q_f$ on input $a$
(since every transition in $G_D$ is a transition in $G_N$ and from step (3) in the construction of $G_N$)
iff there is a walk on $G_N$ labeled with $w$ from $q_0$ to $q_f$
iff $w \in L(N)$.

Thus, $w \in L$ iff $w \in L(N)$. Therefore we conclude that $L(N) = L$ and that for any regular language that does not contain $\lambda$, there exists an nfa without $\lambda$-transitions and with a single final state that accept $L$.

2.3.12: Show that if $L$ is regular, so is $L^R$.

Since $L$ is a regular language, we can construct a corresponding dfa, $N$, such that $L(N) = L$ (For every regular language, there is a corresponding dfa, by definition, and for every dfa, there is an equivalent nfa).

By definition, $L^R$ consists of all strings in language $L$ in reverse order. We will construct a nfa, $N_R$, representing $L^R$ such that $L(N_R) = L^R$. $N_R$ will contain an additional start state with $\lambda$-transitions to the final states of $N$. The direction of every transition in $N$ is reversed. Also, the start state of $N$ will be the final state of $N_R$. The construction of nfa $N_R$ is as follows:

Let $N = (Q, \Sigma, \delta, q_n, F)$

$N_R = (Q \cup \{q_0\}, \Sigma, \delta_r, q_r, \{q_n\})$

Set of states of $N_R$ = set of states of $N$ along with $q_0 = Q \cup \{q_r\}$

$\Sigma = $ alphabet of $N_R = $ same as $N$

$q_r = $ start state of $N_R$

$\{q_n\} = $ set of final states of $N_R = $ start state of $N$

Transition function:

$\delta_r(q_r, a) = \{q_1 : \delta(q_1, a) = q\}$

$\delta_r(q_r, \lambda) = F$
\[ \delta_r(q_r, a) = \emptyset, \text{ if } a \neq \lambda \]

Now we will show that \( L^R = L(N_R) \). \( w \in L^R \) iff \( w^R \in L \) iff there is a walk on the transition graph of \( N \) with label \( w^R \) from \( q_n \) to some \( q_i \in F \) iff there is a walk on the transition graph of \( N_R \) from \( q_r \) to \( q_i \) with label \( \lambda \) and a walk from \( q_i \) to \( q_n \) with label \( w \) (Following the reverse of every transition in the original graph) iff \( w \in L(N_R) \).

Since \( L_R \) can be represented by a nfa, it is regular (by equivalence of nfa to dfa, and dfa to regular language).

**Problem 3: Linz 2.1.8**

2.1.8: A run in a string is a substring of length at least two, as long as possible and consisting entirely of the same symbol. For instance, the string \( abbaabaab \) contains a run of \( b \)'s of length three and a run of \( a \)'s of length two. Find dfa’s for the following languages on \( \{a, b\} \).

(a): \( L = \{ w : w \text{ contains no runs of length less than four} \} \).
(b): \( L = \{w : \text{every run of } a\text{'s has length either two or three}\} \).

(c): \( L = \{w : \text{there are at most two runs of } a\text{'s of length three}\} \).
(d): \( L = \{ w : \text{there are exactly two runs of } a \text{'s of length 3} \}. \)

Problem 4: Linz 2.2.22

2.2.22: Let \( L \) be a regular language on some alphabet \( \Sigma \), and let \( \Sigma_1 \subset \Sigma \) be a smaller alphabet. Consider \( L_1 \), the subset of \( L \) whose elements are made up only of symbols from \( \Sigma_1 \), that is,
\[ L_1 = L \cap \Sigma_1^* . \]
Show that \( L_1 \) is also regular.

Since \( L \) is a regular language, there should be a dfa, \( N \), representing \( L \) such that \( L(N) = L \), where \( N = (Q, \Sigma, \delta, q_0, F) \).

Since \( L_1 \) is made up of strings with alphabets from \( \Sigma_1 \), \( \Sigma_1 \subset \Sigma \), and \( L_1 \) is a subset of \( L \), \( L_1 \) contains only strings that are accepted by \( L \) as well. We can construct a dfa, \( M \), for \( L_1 \) as follows:

1. From the transition graph of \( N \), remove every transition that is labeled with some \( a \notin \Sigma_1 \).

Now we will show that \( L(M) = L_1 \). \( w = a_1a_2 \ldots a_n \in L_1 \) iff there is a walk on the transition graph of \( N \) with label \( w \) from \( q_0 \) to some \( q_i \in F \) and every \( a_i \in \Sigma_1 \) iff there is a walk on the transition graph of \( M \) from \( q_0 \) to \( q_i \) with label \( w \) (it will be the exact same path as it was in \( N \) iff \( w \in L(M) \).

Since \( L_1 \) can be represented by a dfa, it is regular.
Problem 5: Linz 2.3.3 and 2.3.8

2.3.3: Convert the following NFA into an equivalent DFA (see textbook for the diagram).

2.3.8: Find an NFA without λ-transitions and with a single final state that accepts $L = \{a\} \cup \{b^n : n \geq 1\}$.

Noting that $\lambda \notin L$, we can use the technique given in 2.3.9 (Problem 2) and we get the NFA given by the following transition graph: