(1) Give a state diagram and the table specifying the transition function, for a DFA to recognize
(a) \( w \in \{0, 1\}^* : \) w has at least three 0s and at most two 1s
(b) \( w \in \{0, 1\}^* : \) w contains the substring 101 but does not contain the substring 1101
(c) \( w \in \{0, 1\}^* : \) w does not have 00 or 111 as a substring
(d) (not to be graded) \( w \in \{0, 1\}^* : \) w \( \not\in \{1, 01\}^* \)
(e) (not to be graded) \( w \in \{0, 1\}^* : \) w has even length and an even number of 1s

(2) Give a state diagram for an NFA with the specified number of states to recognize
(a) \( w \in \{0, 1\}^* : \) w ends with at least one 0, with two states
(b) \( w \in \{0, 1\}^* : \) w \( \not\in \{1, 01\}^* \), with three states
(c) \( w \in \{0, 1\}^* : \) w has an even number of 1s or w has exactly two 0s, with six states
(d) (not to be graded) \( w \in \{0, 1\}^* : \) w has length 0, 1, or 2, with three states
(e) (not to be graded) \( w \in \{0, 1\}^* : \) w = \( \varepsilon \), with one state

(3) Let \( M_1 = (Q_1, \Sigma, \delta_1, q_0^1, F_1) \) and \( M_2 = (Q_2, \Sigma, \delta_2, q_0^2, F_2) \) be NFAs with \( Q_1 \cap Q_2 = \emptyset \). Let \( L_1 = L(M_1) \) and \( L_2 = L(M_2) \). Suppose that \( F_1 = \{f\} \) and \( f \neq q_0^1 \), i.e. \( M_1 \) has a single accepting state \( f \) which is not its start state. Form a new NFA \( M = (Q_1 \cup Q_2 \setminus \{f\}, \Sigma, \delta, q_0^1, F_2) \), where for each \( a \in \Sigma \),

\[
\delta(q, a) = \begin{cases} 
\delta_2(q, a) & \text{if } q \in Q_2 \setminus \{q_0^2\} \\
\delta_2(q_0^2, a) \cup \delta_1(f, a) & \text{if } q \in q_0^2 \\
\delta_1(q, a) \text{ with } f \text{ replaced by } q_0^2 & \text{if it appears, when } q \in Q_1 \setminus \{f\}
\end{cases}
\]

In other words, we join \( M_1 \) and \( M_2 \) together by making the final state of \( M_1 \) the same as the start state of \( M_2 \).

How does \( L(M) \) relate to \( L_1 L_2 \) in general? In particular, is \( L(M) \subseteq L_1 L_2 ? \) \( L(M) = L_1 L_2 ? \) \( L(M) \supseteq L_1 L_2 ? \) Show that your answer is correct.

(4) Let \( \Sigma = \{a, b\} \). Determine all possible DFAs having exactly two states, and briefly describe (in English) what language each one recognizes. (In choosing final states, note that if neither is final the language is empty, and if both are final the language is \( \Sigma^* \), so you need only consider the cases when there is exactly one final state. There are still 16 DFAs, and two choices of final state in each, so think carefully about how to present your answer!)

(5) (not to be graded) Let \( \Sigma = \{a, b\} \). How many different languages are recognized by NFAs with no \( \varepsilon \) transitions having exactly two states? List them. In this case, if neither state is final the language is still empty, but if both states are final the language need not be \( \Sigma^* \).

For questions 4 and 5, you might find it helpful to write a program to generate all of the DFAs and NFAs with two states.