CSE 355: Homework 2 Sample Solution

September 30, 2015

(1)
$$E(q_0) = \{ q_0, q_1, q_3, q_4, q_5 \}$$
$$E(q_1) = \{ q_0, q_1, q_3, q_4, q_5 \}$$
$$E(q_2) = \{ q_2 \}$$
$$E(q_3) = \{ q_6, q_7 \}$$
$$E(q_4) = \{ q_4 \}$$
$$E(q_5) = \{ q_4, q_5 \}$$
Let \( \text{superseq}(L) = \{ x : x \text{ has a subsequence } y, \text{ such that } y \in L \} \).

Claim: \( L \) is regular \( \iff \) \( \text{superseq}(L) \) is regular.

To prove the claim, let \( L \) be a regular language and let \( M = (Q, \Sigma, \delta, q_0, F) \) be a DFA that recognizes \( L \). We construct an NFA \( N \) that recognizes \( \text{superseq}(L) \). The construction is simple: For each symbol of \( \Sigma \) we add a self loop to each of the states in \( Q \). Formally, let \( N = (Q, \Sigma, \delta', q_0', F') \), where \( \delta'(q, a) = \{ \delta(q, a), q \} \) for all \( q \in Q \) and all \( a \in \Sigma \), \( q_0' = q_0 \), and \( F' = F \). Convince yourself that \( N \) recognizes \( \text{superseq}(L) \) (i.e. \( N \) accepts \( w \iff w \in \text{superseq}(L) \)).

Because \( \text{Extseq}(L', L) = L \cap \text{superseq}(L') \), using closure under intersection, and applying the claim, \( \text{Extseq}(L', L) \) is regular when \( L \) and \( L' \) are.

The language in this part is exactly \( L \cap \overline{\text{superseq}(L')} \). Again by the claim in the (a) part and closure of regular languages under intersection and complement this language is regular when \( L \) and \( L' \) are regular.

No, we cannot generate any language using exponentiation operator that we could not generate with the standard definition. Any time an extended expression uses \( R \uparrow k \), for some non-negative integer \( k \), we can replace it by the standard regular expression \( \underbrace{R.R.R \ldots .R}_{k \text{ times}} \). By using this replacement repeatedly (a finite number of times) we can transform any regular expression in the extended form to a finite length regular expression in the standard form.

Let \( \Sigma = \{0,1,\ldots,v-1\} \) be a \( v \)-ary alphabet given in the natural increasing order, and let \( \text{balanced}(\Sigma) = \{ w \in \Sigma^* : e(w) = f(w) \} \). When \( |\Sigma| = 1 \), \( \text{balanced}(\Sigma) = \Sigma^* = 0^* \). So in this case \( \text{balanced}(\Sigma) \) is a regular language.

When \( |\Sigma| = 2 \), \( \text{balanced}(\Sigma) \) is again regular. In fact, \( \text{balanced}(\Sigma) \) contains precisely the empty string and all strings in which the first character is the same as the last character. Convince yourself that this is true, and then it is easy to produce an NFA for this language.

When \( |\Sigma| = 3 \), suppose to the contrary that \( \text{balanced}(\Sigma) \) is regular. Then because \( (012)^*(210)^* \) is a regular expression and hence has a regular language, \( \text{balanced}(\Sigma) \cap (012)^*(210)^* \) is regular (closure under intersection). Let \( p \) be its pumping length. Choose \( w = (012)^p(210)^p \). Then \( |w| = 6p > p \), and \( w \in \text{balanced}(\Sigma) \cap (012)^*(210)^* \) because \( e(w) = f(w) = 3p - 1 \).

Now we try to pump \( w \). If \( w = xyz \) with \( |xy| \leq p \) and \( |y| \geq 1 \), \( y \) belongs to \( (\varepsilon \cup 2 \cup 12)(012)^*(\varepsilon \cup 0 \cup 01) \).

Case 1. \( y \) does not have the same number of 0s, 1s, and 2s

Then \( xy^2z \not\in (012)^*(210)^* \), which is a contradiction.

Case 2. \( y \) has a 0s, a 1s, and a 2s, with \( a \geq 1 \). Then \( e(xy^2z) = 3p - 1 + 2a \) and \( f(xy^2z) = 3p - 1 + a \), so \( xy^2z \not\in \text{balanced}(\Sigma) \), also a contradiction.

These are the only two cases for \( y \), so \( \text{balanced}(\Sigma) \cap (012)^*(210)^* \) is not regular. Then (as noted above) \( \text{balanced}(\Sigma) \) is not regular.

When \( |\Sigma| > 3 \), the same proof as for three symbols works. Hence \( \text{balanced}(\Sigma) \) is not a regular language for any \( \Sigma \) with \( |\Sigma| \geq 3 \).