Due on 17 March 2009, start of class

Note change in due date, see the news page. Please submit a paper copy in class.

CSE 355 Homework Three

3. Sipser 2.15 and 2.16, page 129.
4. Sipser 2.31 and 2.32, page 131.

EXERCISES

2.1 Recall the CFG $G_4$ that we gave in Example 2.4. For convenience, let's rename its variables with single letters as follows:

$$E \rightarrow E + T | T$$
$$T \rightarrow T \times F | F$$
$$F \rightarrow (E) | a$$

Give parse trees and derivations for each string.

a. a
b. a+a

c. a+a+a
d. ((a))

2.2 a. Use the languages $A = \{a^m b^n c^m | m, n \geq 0\}$ and $B = \{a^m b^n c^m | m, n \geq 0\}$ together with Example 2.36 to show that the class of context-free languages is not closed under intersection.

b. Use part (a) and DeMorgan's law (Theorem 2.20) to show that the class of context-free languages is not closed under complementation.

2.15 Give a counterexample to show that the following construction fails to prove that the class of context-free languages is closed under star. Let $A$ be a CFL that is generated by the CFG $G = (V, \Sigma, R, S)$. Add the new rule $S \rightarrow S'$ and call the resulting grammar $G'$. This grammar is supposed to generate $A^*$.

2.16 Show that the class of context-free languages is closed under the regular operations, union, concatenation, and star.

2.31 Let $B$ be the language of all palindromes over \{0,1\} containing an equal number of 0s and 1s. Show that $B$ is not context free.

2.32 Let $\Sigma = \{1,2,3,4\}$ and $C = \{tw \in \Sigma^* | \text{in } w, \text{ the number of 1s equals the number of 2s, and the number of 3s equals the number of 4s} \}$. Show that $C$ is not context free.

2.44 If $A$ and $B$ are languages, define $A \circ B = \{xy | x \in A \text{ and } y \in B \text{ and } |x| = |y|\}$. Show that if $A$ and $B$ are regular languages, then $A \circ B$ is a CFL.