CSE 555 HOMEWORK TWO

DUE 19 FEBRUARY 2016, START OF CLASS

(1) (a) Let $\text{MINSTATE}_{\text{DFA}} = \{ \langle M \rangle : M \text{ is a DFA for which no DFA } M' \text{ with } L(M) = L(M') \text{ has fewer states than } M \}$. Prove or disprove: $\text{MINSTATE}_{\text{DFA}}$ is decidable.

(b) Let $\text{MINFIN}_{\text{DFA}} = \{ \langle M \rangle : M \text{ is a DFA for which no DFA } M' \text{ with } L(M) = L(M') \text{ has fewer final states than } M \}$. Prove or disprove: $\text{MINFIN}_{\text{DFA}}$ is decidable.

(c) Let $\text{MINSTATE}_{\text{NFA}} = \{ \langle M \rangle : M \text{ is an NFA for which no NFA } M' \text{ with } L(M) = L(M') \text{ has fewer states than } M \}$. Prove or disprove: $\text{MINSTATE}_{\text{NFA}}$ is decidable.

(2) (a) Let $\text{MINVAR}_{\text{CFG}} = \{ \langle G \rangle : G \text{ is a CFG in Chomsky normal form (CNF) for which no CFG } G' \text{ in CNF with } L(G) = L(G') \text{ has fewer variables than } G \}$. Prove or disprove: $\text{MINVAR}_{\text{CFG}}$ is decidable.

(b) Let $\text{MINRULE}_{\text{CFG}} = \{ \langle G \rangle : G \text{ is a CFG in Chomsky normal form (CNF) for which no CFG } G' \text{ in CNF with } L(G) = L(G') \text{ has fewer rules than } G \}$. Prove or disprove: $\text{MINRULE}_{\text{CFG}}$ is decidable.

(3) An instance of the Post Correspondence Problem (PCP) consists of a finite set of tiles $P = \left\{ \left[ \begin{array}{c} t_i \\ b_i \end{array} \right] : 1 \leq i \leq \gamma; t_i, b_i \in \Sigma^* \right\}$. An instance $P$

- has alphabet size $as(P) = |\Sigma|$,
- has length $\max \ell m(P) = \max \{ \max \{|t_i|, |b_i| : 1 \leq i \leq \gamma \} \}$,
- has tile count $tc(P) = \gamma$,
- has a tiling if there is a finite sequence $i_1, \ldots, i_k$ for which $t_{i_1} \cdots t_{i_k} = b_{i_1} \cdots b_{i_k}$.

(a) Prove or disprove: For each fixed integer $\tau$, $\{ \langle P \rangle : P \text{ has a tiling and } tc(P) \leq \tau \}$ is decidable.

(b) Prove or disprove: For each fixed integer $\sigma$, $\{ \langle P \rangle : P \text{ has a tiling and } as(P) \leq \sigma \}$ is decidable.

(4) This has the same setup as Problem 3.

(a) Prove or disprove: For each fixed integer $\ell$, $\{ \langle P \rangle : P \text{ has a tiling and } \ell m(P) \leq \ell \}$ is decidable.

(b) Prove or disprove: For each fixed integer $\ell$ and fixed integer $\tau$ and fixed integer $\sigma$, $\{ \langle P \rangle : P \text{ has a tiling and } \ell m(P) \leq \ell \text{ and } tc(P) \leq \tau \text{ and } as(P) \leq \sigma \}$ is decidable.

(5) We consider an infinite sequence of (deterministic) TMs $M_0, M_1, M_2, \ldots, M_\nu, \ldots$. For $i \geq 0$, each TM $M_i$, when run on empty input, outputs $\langle M_{i+1} \rangle$. If the output does not properly encode a TM, we interpret this as a TM that always rejects and writes no output.

For every fixed positive integer $\nu$, describe a way to produce such a sequence with $\langle M_i \rangle \neq \langle M_j \rangle$ for $0 \leq i < j < \nu$ and $\langle M_0 \rangle = \langle M_\nu \rangle$. 

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