CSE 555 MIDTERM EXAMINATION

TAKE-HOME: OUT 02/21 NOON, DUE 02/25 NOON

The completed test is to be submitted electronically to colbourn@asu.edu before the deadline, or time-stamped by the main office in CIDSE and placed in Prof. Colbourn’s physical mailbox.

Answer every question, whether or not you think your answer is complete or correct. You can use any materials desired, but this is an individual effort, so limit your discussions with fellow students, and under no circumstances should written work be shared.

Part of the exercise of doing the questions is understanding the questions. Read each carefully so as not to waste time.

All questions are equally weighted (but not necessarily equally difficult.)
1. Question 1.

Consider the statement $\forall w \forall x \exists y \forall z[(w + x = y) \land ((y = z) \lor ((w + x \neq z))].$ We operate in the universe of natural numbers with two relations, ‘equal’ and ‘plus’, having their natural interpretations.

(a) When $w, x, y, z$ are provided as input in binary, encoded in a 16-letter alphabet whose letters are the $2^4$ binary vectors of length four, devise a DFA that accepts precisely when $(w + x = y) \land ((y = z) \lor ((w + x \neq z))].$

(b) Transform the quantifications into nondeterminism and show the NFA produced for $\forall w \forall x \exists y \forall z[(w + x = y) \land ((y = z) \lor ((w + x \neq z))].$ Then state whether the statement is true or false, and why.
2. Question 2.

\[ \text{FINITE}_{LG} = \{(G) : \text{the language generated by linear grammar } G \text{ is finite}\} \]. Show that \text{FINITE}_{LG} \text{ is decidable.}
3. Question 3.

\( \text{REGULAR}_{TM} = \{ \langle M \rangle : \text{the language recognized by Turing machine } M \text{ is regular} \} \).

(a) Show that \( \text{REGULAR}_{TM} \) is not decidable.

(b) Determine whether \( \text{REGULAR}_{TM} \) is co-Turing recognizable, Turing-recognizable, or neither. Explain your answer carefully.

Show how to use the recursion theorem to produce two different Turing machines $M$ and $N$ so that when $M$ is run on any input, it prints $\langle N \rangle$, and when $N$ is run on any input, it prints $\langle M \rangle$. 
5. Question 5.

We have focussed on deterministic Turing machines.
(a) Justify this by showing that there is a computable function $t : \Sigma^* \mapsto \Sigma^*$ so that whenever $M$ is a nondeterministic Turing machine, $t(\langle M \rangle) = \langle M' \rangle$ where $M'$ is a deterministic Turing machine with $L(M) = L(M')$ (and if $x$ does not properly encode a nondeterministic Turing machine, then $t(x)$ does not properly encode a deterministic Turing machine).

(b) Can you show that the language of encodings of nondeterministic Turing machines with minimal length descriptions is not recognizable? Explain.