Lecture 9: Leftovers, or random issues with OLS

- Functional form misspecification
- Proxy variables
- Measurement error
- Missing data
- Nonrandom samples
- Influential data
- Least absolute deviation (LAD)
- Scaling offending
Functional Form Misspecification

- Functional form misspecification is a special type of missing variable problem because the missing variable is a function of nonmissing variables.
  - Examples: missing a squared term, an interaction, or log(x).
- As such, it is possible to fix functional form misspecification if that’s your only problem.
- RESET test can identify general functional form problems but it can’t tell you how to fix them.
Functional Form Misspecification, RESET test

- The test is easy to implement. After your regression, generate fitted values, and powers of fitted values, usually just squared and cubed values.
- Estimate the original equation, adding the squared and cubed fitted values to the Xs:

\[ y = \beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k + \delta_1 \hat{y}^2 + \delta_2 \hat{y}^3 + e \]

- Test the joint hypothesis that \( \delta_1 \) and \( \delta_2 \) are equal to zero using either an LM or F test.
- If you reject the null, you have functional form misspecification.
In practice, in criminology, the only functional form misspecification that you might be asked about in a journal article review is age. If the ages of your sample span the curvy part of the age crime curve, you ought to have a squared age term in there.

If functional form is misspecified, ALL of your parameter estimates are biased.

Example from midterm conservative model . . .
In the social science in particular, we often cannot directly measure constructs that we are interested in. So we often have to use proxy variables as a stand-in for what we really want.

A good proxy:
- Is strongly correlated with what we really want to measure.
- Renders the correlation between included variables and the unobserved construct zero.
Proxy variables in criminology

- For self-control:
  - 11-item scale representing inclination to act impulsively (Mazzerolle 1998)
  - Enjoy making risky financial investments / taking chances with money (Holtfreter, Reisig & Pratt 2008)
  - Gambling, smoking & drinking (Sourdin 2008)
- For social support:
  - Marriage (Cullen 1998)
  - Ratio of tax deductible contributions to total number of returns (Chamlin et al. 1999)
- For social altruism:
  - United Way contributions (Chamlin & Cochran 1997)
- For violent crime:
  - Homicide
Lagged dependent variables as proxies

- Because of continuity in individual offending and macro-level crime rates, lagged dependent variables are very powerful predictors of crime.
- However, if your focal concern is the impact of some other variable, say gang membership for example, including a lagged dependent variable changes the nature of your parameter estimate for gang membership.
- It is now a question of whether gang membership leads to a change in offending.
- Furthermore, a lagged dependent variable can introduce measurement error in an independent variable that is correlated with measurement error in the dependent variable.
Not all error is created equally. The consequences of random and nonrandom measurement error are very different.

**Random measurement error**: there is no correlation between the true score and the error with which it is measured

- independent variables: unbiased estimates, but inefficient (standard errors go up, r-squared goes down)
- dependent variables: estimates biased downward for bivariate case, unknown bias for multivariate case
Measurement error

- **Non-random measurement error**: the degree to which a particular $x_j$ is measured with error is related to values of $x_k$, where $k$ may be equal to $j$ or not, and $x_k$ may or may not be observed.

- Effects of non-random measurement error depend on the specific nature of the error. But typically results in biased estimates.

- Systematic over- or under-estimation of an independent variable $X$ or the dependent variable $Y$, will bias the intercept only, and is therefore less concerning.
Nonrandom samples / missing data

- Ideally, you possess a random sample of data from the population you are interested in studying, with no missing data. Usually, however, this is not the case.
- If the nonrandomness is known, as is the case with stratified sampling, you can usually modify your regressions with sampling weights to obtain unbiased estimates.
- Exogenous sample selection: known nonrandomness based on an independent variable. This is not a problem either, but it changes the meaning of your parameters. You can no longer make inferences to the population of interest, but to the population that corresponds to your nonrandom sample.
  - Example: many variables in the NLSY97 are only asked of certain age cohorts. Using these requires dropping a large percentage of the data, but doesn’t bias the estimates for the represented age cohort.
Nonrandom samples / missing data

- Endogenous sample selection: based on the dependent variable
  - This biases your estimates.
- Missing data can lead to nonrandom samples as well.
- Most regression packages perform listwise deletion of all variables included in OLS. That means that if any one of the variables is missing, then that observation is dropped from the analysis.
- If variables are missing at random, this is not a problem, but it can result in much smaller samples.
- 20 variables missing 2% of observations at random results in a sample size that is 67% of the original (.98^20)
Nonrandom samples / missing data

- Usually data is not missing at random.
  - Ex: missing self-reported drug use, property offending, sexual behavior, etc.

- When data is not missing at random, and you run your models with listwise deletion, the resulting parameter estimates are biased for the population of interest.
Dealing with missing data

- It is advisable to compare data for the observations dropped from your sample and those retained.
- Create a dummy variable for being in your final sample. (1=in sample, 0=not in final sample)
- Demographic variables will typically be nonmissing for all cases, so you can compare those using independent samples t-tests.
- If you can find no significant differences between the included and excluded samples, you can make the case that data is missing at random, and proceed as usual.
- If you find many significant differences, you have a few options.
Dealing with missing data, cont.

- Describe the type of observations that make it into your regression analysis to indicate what population your parameters refer to. (weak)
- Correct for sample selection bias using the Heckman Two-Step Correction (see Bushway, Johnson & Slocum 2007) – we’ll cover this next time
- Perform multiple imputation (mi command in Stata)
  - Impute many datasets (~30)
  - Obtain estimates from each dataset
  - Recombine estimates
Influential Data

- Is there an observation in your sample so influential that removing it would substantially change your regression estimates?
- If so, what does this mean and what should be done?
  - Incorrect data? Fix it.
  - Observation drawn from different population. Drop it.
Residuals by themselves are not informative. Both of the circled points below are outliers, in different ways.

- The first has a large residual, but has little leverage (influence) over the regression line.
- The second has a small residual, but a lot of leverage.
Identifying Influential Data, graphs

- One way to check for influential data is to run scatter plots.
- In stata, you can call up a matrix of scatter plots all at once:
  \[
  . \text{graph matrix homrate poverty IQ het gradrate fem\_hh, half}
  \]
- You can also identify particular observations with labels:
  \[
  . \text{scatter homrate poverty, mlabel(state)}
  \]
Another way to identify influential data after running a regression model is to look at “hat values.”

Hat values are a measure of influence of each data point. They range from 1/n to 1, their mean is $k/n$ where $k$ is the number of regressors in the model, including the intercept.

The more unusual the Xs for any observation, the greater its influence on the regression model.

In stata use the “, hat” option for predict.
Identifying Influential Data, studentized residuals

- The difference between the $i$th observation, and what the regression line would be with that observation deleted is: $\varepsilon_i^* = y_i - \hat{\beta}(i)x_i$

- We would like a standardized version of this residual, which is given by:

$$
\varepsilon_i^* = \frac{y_i - \hat{\beta}(i)x_i}{s_i(i)} \sim N(0,1)
$$

- So look for absolute values $>1.96$

- Use the \texttt{, rstud} option for \texttt{"predict"} in Stata
Identifying Influential Data, dfbetas

- Studentized residuals give us a general sense of which observations are most influential.
- Dfbetas tell us which observations most impact specific parameters.

\[
DFBETAS_{ji} = \frac{\hat{\beta}_j - \hat{\beta}_j(i)}{S_{\hat{\beta}_j(i)}}
\]
Identifying Influential Data, dfbetas

- If the absolute value of dfbeta is greater than $2/\sqrt{N}$, then it’s considered problematic.
- In Stata you can create all the dfbeta estimates at once:

  . dfbeta
Identifying Influential Data, diagnostic graphs

- There are two kinds of graphs that are useful after a regression.
- Added-variable, or partial regression plots show the relationship between one independent variable and the dependent variable after adjusting for all other independent variables ("avplot x" or "avplots")
- Residual vs. leverage plots can show which observations have high residuals and high leverage, which can be the most problematic (lvr2plot)
Dealing With Influential Data

- So you have some influential data. What do you do about it?
- Look at it. Are there any data entry errors? If so, fix them. If you can’t, throw it out.
- Is this observation drawn from a different distribution (e.g. DC vs states)? If so, consider throwing it out.
- Otherwise, keep it. But what if your key independent variable is statistically significant if and only if you keep a single observation?
Least Absolute Deviation, or quantile regression

- In some applications, it is preferable to model the expected median given $x_1$ through $x_k$, or the expected 25th or 75th percentile.
- This is extremely uncommon in criminology.
- I could find one application of this method in a top criminology journal:
  "Modeling the distribution of sentence length decisions under a guidelines system: An application of quantile regression methods" by Chet Britt in JQC 2009
- If interested, start there and work backwards.
- "qreg" in Stata
The problem of scaling offending

• Should we combine measures of different types of offending into a single scale?
• Behaviors considered “criminal” are so disparate that illegality itself may seem like their only shared characteristic.
• How can they be combined?

*This topic was originally presented at ASC 2009.*
“Is one homicide to be equated with 10 petty offenses? 100? 1000? We may sense that these are incommensurables and so feel that the question of comparing their magnitude is a nonsense question.”

- Robert Merton (1961, emphasis in original)
Options for scaling offending

- Prevalence (0/1)
- Frequency
  - weighted by seriousness
- Variety
- Summed ordinal scale
  - Often transformed in some way: logged, z-score, factor weighted, etc.
- Latent trait estimated from Rasch models (item response theory)
- Other ad hoc method

- Limiting to one crime type or official data? See above.
History of Scaling Offending

• Guttman scaling
  • an ordered variety scale, popular in the 1960s, used as late as 2003 (Tittle et al.) to describe levels of self-control

• Sellin-Wolfgang seriousness scale
  • 21 different offenses (141 originally) scaled according to seriousness ratings in survey research
  • Ex: Homicide=26, serious assault=7, petty theft=1
History of Scaling Offending, cont.

- “Measuring Delinquency” (Hindelang, Hirschi & Weis 1981)
  - Modest support for unidimensional scale of offending
  - Recommended “ever variety” scale: number of types of offenses committed
    - Higher reliability than frequency scores
    - Higher correlation with official reports than frequency scores
Item Response Theory

- latent trait, theta (θ), accounts for the observed response patterns
- α reflects strength of relationship between a single item’s responses and latent trait
- $b$ reflects threshold for which question response category (or more serious category) is 50% likely
- After item-specific parameters estimated, theta is estimated for each person
Scaling Offending Today

• Of 130 individual-level quantitative articles in 5 criminology journals in 2007-8
  • 76 (58%) prevalence (0/1)
  • 53 (41%) frequency
  • 15 (12%) summed or transformed category
  • 12 (9%) variety
  • 5 (4%) weighted frequency
  • 5 (4%) IRT
Scaling example, data

- National Longitudinal Survey of Youth 1997
  - 8,984 youths 12-16 years old as of 12/31/1996
  - Wave 3 used, when youth were, on average 17.4 years old (s.d.=1.4), N=8209
  - 6 offending items: intentional destruction of property, petty theft (<$50), serious theft (>=$50), attacking with intent to hurt, selling drugs
## Item-specific descriptives

<table>
<thead>
<tr>
<th>Item</th>
<th>Prevalence</th>
<th>Frequency (s.d.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Destruction of property</td>
<td>.0798</td>
<td>.37 (3.32)</td>
</tr>
<tr>
<td>Theft &lt;$50</td>
<td>.0867</td>
<td>.55 (4.08)</td>
</tr>
<tr>
<td>Theft &gt;$50</td>
<td>.0277</td>
<td>.17 (2.37)</td>
</tr>
<tr>
<td>Other property crime</td>
<td>.0273</td>
<td>.19 (2.47)</td>
</tr>
<tr>
<td>Attacking to hurt</td>
<td>.0959</td>
<td>.37 (3.16)</td>
</tr>
<tr>
<td>Selling drugs</td>
<td>.0573</td>
<td>1.18 (8.44)</td>
</tr>
<tr>
<td>Total</td>
<td><strong>.2444</strong></td>
<td><strong>2.83 (14.73)</strong></td>
</tr>
</tbody>
</table>
Variety score, mean=.42, s.d.=.91
# IRT Results

<table>
<thead>
<tr>
<th>Item</th>
<th>Item Discrimination</th>
<th>Response Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Destruction of property</td>
<td>2.53</td>
<td>1.74</td>
</tr>
<tr>
<td>2: Theft &lt;$50</td>
<td>2.12</td>
<td>1.86</td>
</tr>
<tr>
<td>3: Theft &gt;$50</td>
<td>3.01</td>
<td>2.19</td>
</tr>
<tr>
<td>4: Other property crime</td>
<td>3.33</td>
<td>2.20</td>
</tr>
<tr>
<td>5: Attacking to hurt</td>
<td>1.62</td>
<td>2.00</td>
</tr>
<tr>
<td>6: Selling drugs</td>
<td>1.96</td>
<td>2.14</td>
</tr>
</tbody>
</table>
Item Characteristic Curve: Theft of items over $50

Graded Response Model

Offense frequency category: 0 = Black, 1 or 2 = Blue, 3 or 4 = Magenta, 5+ = Green
Distribution of IRT criminality estimates

Frequency

latent criminality

0 0 0 0 731 320 281 209 128 73 37 15 11 5

0 0.5 1 1.5 2 2.5 3

6399
Conclusions

Prevalence
- Sometimes most appropriate scale (i.e. conviction, imprisonment, homicide)
- When multiple items are combined, most prevalent (least serious) contributes the most variation
- Easy to interpret as IV/DV
- Linear probability / Logit / Probit models

Frequency
- Multiple item scales are dominated by high frequency items
- Typically very skewed
- Easy to interpret as IV/DV
- Negative binomial/poisson models
- Weighting by seriousness makes results less interpretable
Conclusions, cont.

Variety*

- Limits contribution of less serious items
- Highly correlated with IRT estimates (.92)
- Slightly more difficult to interpret results as IV/DV
- Negative binomial/poisson models
- HH&W were right!, reasonable approximation of criminality
- Variety scales are similar to summed category scales in that they impose only two categories before summing
- As the number of categories increases in summed category scales, the influence of less serious items on the scale increases.
Conclusions, cont.

IRT

- Explicitly models relationship between criminality and offending questions
- Reveals information about people, and about behaviors
- Extra estimation step adds error to scale that requires extra work to correct
  - Recent work estimates model in single step (Osgood & Schreck, 2007)
- Complicated interpretations
- Tobit models?, need wide breadth of items
Next time:

bye week for homework

Read: Wooldridge Chapter 17, look over Bushway et al., 2007, Smith & Brame, 2003