The First Law of Thermodynamics as Applied to a Mechanical System

**Goal:** Do work \( W \) on an aluminum cylinder by rotating it against a known tangential frictional force. Determine the change \( \Delta U \) in internal energy of the cylinder by measuring its rise in temperature. Compare the measured values of \( W \) and \( \Delta U \) to see if energy is conserved. Correct for heat loss to the ambient surroundings.

**Equipment:** PASCO apparatus, incl. ohmmeter and 10 kg masses and stopwatch or watch. Nominal physical parameters:

1. Mass of Al cylinder: 200 ± 1.5 g
2. Diameter of Al cylinder: 4.763 ± 0.02 cm
3. Diameter of Al cylinder plus wrapped nylon tape: 4.94 ± 0.05 cm
4. Specific heat of Al: 0.220 cal g\(^{-1}\) C\(^{-1}\) = 0.920 J g\(^{-1}\) C\(^{-1}\)
5. Thermistor calibration curve: \( T(K) = \frac{10^4}{6.3379 + 5.4484 \log_{10} R} \)

**Reference:** Wolfson and Pasachoff, Section 19-5 and Chapter 21.

**Background:** Prior to about 1800, thermal energy was often thought of as a physical liquid ("caloric") that could flow from one object to another. Were that so, it would be possible to write a "conservation of caloric" equation. Benjamin Thompson (Lord Rumford) realized while watching cannon being bored that any such conservation equation would be inconsistent with reality: Depending on how blunt the boring tool was, different amounts of heat were generated in the boring process. With a very blunt tool, an essentially unlimited amount of heat was generated, implying the unlimited production of "caloric" and suggesting that thermal energy was not a fluid. Careful experiments by James Prescott Joule (Lord Kelvin) around 1850 confirmed this appraisal, leading to the first law of thermodynamics and to our present day association of thermal energy with the random thermal motion of the atoms in a material.

**Physics:** In this laboratory experiment, we carry out measurements similar to those of Joule, but modernized. Our "system" is a cylindrical aluminum spool, around which a nylon cord is wrapped several times. A known force is applied to this cord and thereby, tangentially, to the spool. The spool is rotated with a crank in such a manner that the cord slips on the surface of the spool. Rotating the spool against an opposing force does work \( W \) on the spool and the value of \( W \) is easily computed from the torque and the total angular displacement of the cylinder,

\[
W = -\tau \Delta \theta
\]

where \( \tau \) is the torque applied to the cylinder by the rope, \( \Delta \theta \) is the angle through which the cylinder has been rotated by the crank, and the negative sign denotes this as work done ON the system by the surroundings. In order to have a well-defined torque, the tangential force applied to the cylinder is provided by hanging a known mass \( M \) from the rope wrapped around the cylinder. The force \( Mg \) acts at a radius

\[
R = \frac{D_{\text{Cyl}} + D_{\text{Cyl+Rope}}}{4}
\]

where \( D_{\text{Cyl}} \) is the diameter of the cylinder and \( D_{\text{Cyl+Rope}} \) is the diameter of the cylinder including the rope tightly wrapped around it. Thus

\[
W = -MgR \Delta \theta = -2\pi M gR N
\]
Just as when you rub your hands together to heat them on a cold day, the sliding motion of the cord on the surface of the cylinder generates thermal energy through friction. As we have discussed in class, thermal energy that flows into or out of a system during a thermodynamic process (i.e., that crosses the boundary between the system and its surroundings) is what we refer to as heat $Q$. Thermal energy that is generated within the system leads to an increase in the internal energy of the system. In a constant volume process, the exact differential of the internal energy can be expressed as

$$dU(T,V) = \left. \frac{\partial U}{\partial T} \right|_V dT + \left. \frac{\partial U}{\partial V} \right|_T dV = C_V \, dT + \left. \frac{\partial U}{\partial V} \right|_T dV = C_V \, dT \quad (0) = C_V \, dT \quad (4)$$

where $C_V$ is the heat capacity ($J/K$, an extensive quantity) of the system at constant volume. If the volume of the system does not change during the process (i.e., $dV = 0$), the change in internal energy of the system can be directly related to the change in temperature of the system. Integrating the above equation gives

$$\Delta U = C_V \, \Delta T = m \, c \, \Delta T \quad (5)$$

where $\Delta U = U - U_0$ and $\Delta T = T - T_0$ ($U_0$ and $T_0$ being the initial values of internal energy and temperature) and where $m$ and $c$ are the mass and the specific heat ($J/kg/K$, an intensive quantity) of the system. By knowing the specific heat and mass of the cylinder, the change in the internal energy of the cylinder can be computed from a measured temperature rise.

Embedded within the cylinder is a thermistor, which is a solid state device that exhibits several decades of resistance change over a narrow temperature range, allowing temperature $T$ to be measured very accurately simply by measuring the thermistor resistance $R$. The $T(R)$ curve of a thermistor is usually quite nonlinear, but this is not a problem provided a full calibration curve or table is available. The thermal conductivity of aluminum is quite high, so that thermal energy introduced at the periphery of the cylinder rapidly diffuses throughout the cylinder. To a first approximation, this diffusion can be taken to be instantaneous so that the temperature is always uniform through the cylinder. With this assumption, changes in internal energy can be computed by means of Eqn. (2) from the measured change in temperature registered by the thermistor.

If we take the boundary of the system to be just outside of the rope layer, then the generation of thermal energy is entirely within the system. Assuming that no thermal energy escapes as heat flow to the surroundings (e.g., by conduction or convection away through the air), then $Q \equiv 0$ and the first law of thermodynamics requires that

$$\Delta U = Q - W = -W \quad (6)$$

computing $W$ and $\Delta U$ from Eqns. (3) and (5), we should therefore find them to be equal.

**Measurement:** The trick in this measurement is to wrap the nylon cord properly around the aluminum spool so that when the crank is turned at a moderate rate, the 10 kg mass (hanging from the lower end of the cord) rises a few centimeters from the floor and hangs suspended at that height. Finding this dynamic balance may require some practice in cranking and some experimentation with the number of wraps of cord around the cylinder. Three or four wraps, laid carefully side-by-side and with no twists in the cord, usually supplies the proper frictional force. It may help to place a very slight pressure on the loose end of the cord (not too much, since this extra force makes $F_f \neq mg$!). Be very careful not to raise the mass too far from the floor as you crank, since it can easily come crashing down. Also, stand back from the apparatus as you crank or be prepared to conduct a group first-aid session on crushed toes.

Once you have these specifics worked out, have one team member do the cranking while a second counts the number of cranks and announces every tenth crank (say). The third team member, who is reading the thermistor resistance, then calls out the thermistor resistance, to be recorded by the fourth team member who also observes and writes down the time of the measurement (the reason for the time measurement will become clear shortly). This will be a real team effort! Try to crank at a relatively constant rate.
starting the cranking, be sure to record the initial thermistor resistance. **Starting from room temperature will simply the analysis, as will become clear later.** Also work out beforehand, using the thermistor \( T(R) \) curve, the resistance that corresponds to a temperature about ten degrees higher than the starting temperature. Start cranking and recording the total number of turns, the thermistor reading, and the time of measurement until this ten degree temperature rise has been surpassed. Stop cranking at that point, but keep on recording the thermistor reading as the cylinder cools through heat transfer to the surrounding air. Probably only about one measurement per minute is needed during this cooling phase, but these measurements should be continued until the temperature has dropped by 4 or 5 degrees. This may take 20 minutes or more! (In contrast, the heating phase is relatively quick, probably only a couple of minutes).

**Data Analysis:** From the thermistor measurements, work out the change in internal energy of the cylinder and compare it with the work done by the surroundings on the system (i.e., by the person turning the crank). How you present these data is up to you, but as always a graphical plot is preferred and you should extract a numerical value (e.g., a slope) from the data to compare with the predictions of Eqns. (3), (5), and (6).

**Refinement in Data Analysis:** From your measurements on the cylinder as it cools after the cranking has stopped, what would you conclude about the assumption that no heat is lost to the surroundings? A first order correction for this heat loss is easily obtained. Differentiating Eqn. (5) with respect to time delivers

\[
\frac{d(\Delta U)}{dt} = mc \frac{d(\Delta T)}{dt}
\]

Equating this to the time rate of work being done by the cranking gives

\[
MgR \frac{d(\Delta \theta)}{dt} = mc \frac{d(\Delta T)}{dt}
\]

or simply

\[
\frac{d(\Delta T)}{dt} = \frac{MgR \omega}{mc}
\]

assuming that you did indeed crank at a constant \( \omega \). (Since you have recorded both the number of turns and the time with each thermistor reading, you can check this.) To correct for heat flow out of the system, a heat loss term must be added to this differential equation. To a first approximation (known as Newton’s Law of Cooling), the heat loss is assumed proportional to the temperature difference between the system and its surroundings, which are at the ambient temperature \( T_{amb} \). Hence

\[
\frac{d(\Delta T)}{dt} = \frac{MgR \omega}{mc} - \alpha (T - T_{amb})
\]

where \( \alpha \) is a constant parameter having dimensions of inverse time. This equation can be integrated to find the temperature of the cylinder \( T(t) \) as corrected for heat loss to the surrounding air. If the cranking starts from room temperature, then \( T_0 = T_{amb} \) and the integration delivers simply

\[
mc \Delta T(t) = MgR (\omega / \alpha) \left( 1 - e^{-\alpha t} \right)
\]

for the temperature rise above room temperature as a function of time. Now you perceive the reason for the cooling measurements after the cranking has stopped: You should be able to fit your data curves to this improved analytical form by adjusting the parameter \( \alpha \) to fit the exponential decay in the cooling portion of your measured curves. The RHS of Eqn. (11) gives the work done while cranking at a constant angular rotation rate \( \omega \) for the time \( 1/\alpha \), less the heat flow to the surroundings. A test of the First Law is whether this is equal to the increase in internal energy, given on the LHS. Specifically, once the parameter \( \alpha \) has been obtained form fitting to the data points of the cooling portion of the \( \Delta T(t) \) curve, the fit should also pass exactly through the data points constituting the heating portion of the temperature curve.

**Report:** Analyze your data as described above and discuss how well your measurements support the First Law of Thermodynamics. If the data do not support the First Law, suggest where the experiment or analysis may have gone wrong.
**Submission:** Lab work is a team effort and only a single lab report is to be submitted by each team. However, every member of the team must preserve a copy of every lab report and all data and computer files that are part of the lab report. All of this, including files on floppy disk, must be saved by each student in a special lab binder, which will be used during hour exams and the final exam for questions pertaining explicitly to the lab work. Students who fail to save their lab reports or do not understand the lab work will fare poorly on such exam questions.

Specify the specific contributions of each team member to the lab work. These roles must change from week to week. The specific style and format of the lab report are up to the team, but the report must be neat, readable, and carefully organized. It must be self-contained, so that an intelligent reader is able to understand what was measured and why without additional information or insight. A copy of this lab handout is available on the class website. All relevant original data must be included in the lab report as an appendix and preserved on floppy disk in the lab binders of each student. Most of any spreadsheet analysis can also go into appendices, with relevant data tables and graphs copied into the main report. Do not include pages of spreadsheet numbers, but rather a graphical presentation of the data and calculations along with any mathematical formulas used in the spreadsheet. Questions posed in the lab handout must be answered in the lab report. Calculations, graphs, predictions, etc. that are requested in the handout must be presented in the report. Include sketches of the experimental apparatus to supplement any descriptions.