ANGULAR MOMENTUM

supplement for PHY122 lab 10

COLLISIONS IN TWO DIMENSIONS

ANGULAR MOMENTUM is created whenever any particle rotates about a fixed axis. For a particle with linear momentum $\mathbf{p}$ the magnitude of angular momentum $L$ is defined as the product of the distance $r$ from the particle to the fixed axis times the tangential component of $\mathbf{p}$, i.e. the component of $\mathbf{p}$ which is perpendicular to the vector $\mathbf{r}$. $L$ may also be found by the product of $\mathbf{p}$, the magnitude of the particle’s linear momentum, and the shortest distance between the path of the particle and the fixed axis (often denoted by $r_\perp$). The units of $L$ are \text{kg m}^2/\text{s}.

Selected fixed axis (⊥ to page)

![Diagram of particle with momentum and angular momentum](image)

\[
L = r p_\perp = r (p \sin \theta) \\
\text{or} \\
L = r_\perp p = (r \sin \theta) p
\]

Note that, according to our definition, straight-line motion can also create angular momentum. This makes sense, since straight-line motion is also a kind of rotation. In the figure above, the angle $\theta$ would go from $0^\circ$ to $180^\circ$ as the particle moved in a straight line from the extreme right to the extreme left.

For a macroscopic rotating rigid object, the magnitude of angular momentum is given by $L = I \omega$, where $I$ is the rotational inertia of the object, in \text{kg m}^2, and $\omega$ is the rate of rotation, or angular velocity, in radians/second. For a uniform disk of mass $M$ and radius $R$, $I = \frac{1}{2}MR^2$. For two uniform disks which are stuck together and are rotating about their common center-of-mass, $I = 3MR^2$.

In a collision between two isolated disks there is no external torque on the two-disk system, just as there is no external force on the two-disk system. Therefore both linear and angular momentum must be conserved.
Test conservation of angular momentum for your two collisions by computing the angular momentum of the two-disk system both before and after each collision. A convenient fixed axis is the line perpendicular to the surface and passing through the center of the stationary disk before the collision occurs. After the collision the total angular momentum is the sum of the angular momenta due to the translating centers-of-mass (there are two of these in the case of the nearly elastic collision, one of these in the case of the inelastic collision) plus the angular momenta due to the rotating rigid bodies.

Appropriate columns to add to your tables would be \( r_\perp \), \( r_\perp v \), \( \frac{1}{2} R^2 \omega \), and \( r_\perp v + \frac{1}{2} R^2 \omega \). And for the after table of the perfectly inelastic collision, \( r_\perp \), \( r_\perp v \), \( 3R^2 \omega \), and \( 2r_\perp v + 3R^2 \omega \). The uncertainties are contained in the \( r_\perp \) and in the speed \( v \).