Introduction: In this lab we use first the electrostatic force, and then the magnetic force, to accelerate electrons; doing both allows us to measure the charge-to-mass ratio of the electron. Text Reference: Serway 29.2-3, 30.1, Example 30.3.

Theory: The electron gun uses an electric field between the heated cathode and the anode to accelerate electrons to speeds which are a few hundredths of the speed of light. The electrons, of charge $-e$ and mass $m$, are accelerated across the selected high voltage $V$; energy conservation yields $eV = (1/2)mv^2$, or $e/m = v^2/(2V)$, where $v$ is the resulting speed of the electrons in the beam. Since we are using this experiment to measure the ratio $e/m$, the variables $e$, $m$, and $v$ are each unknown.

Current in a set of Helmholtz coils is used to create a magnetic field which provides a second acceleration of the electron beam; this second acceleration only changes the direction of the beam, not the speed. The figure below shows both a photograph and a schematic diagram of the apparatus. The electron gun is at the far right of the evacuated bulb which is at the center of the coils. Since the applied magnetic field $\vec{B}$ is perpendicular to the velocity $\vec{v}$ of the electron beam, application of $\vec{F}_B = q\vec{v} \times \vec{B}$ results in a magnetic force of magnitude $evB$ which causes the electron beam to travel in a circular path. The
radius \( r \) of the electron beam path is easily determined from the apparatus. The magnetic field strength \( B \) is calculated from the measured current \( I \) in the Helmholtz coils. A second expression for \( v \) is then straightforwardly determined from Newton’s Second Law.

\[
\Sigma F = ma \quad \Rightarrow \quad evB = m\frac{v^2}{r} \quad \Rightarrow \quad v = \left(\frac{e}{m}\right)rB
\]

Again each of the variables \( e, m, \) and \( v \) is unknown. Using this expression to eliminate \( v \) within \( e/m = v^2/(2V) \) results in

\[
\frac{e}{m} = \frac{2V}{r^2B^2}, \tag{1}
\]

an expression for \( e/m \) in terms of the selected high voltage \( V \), the measured beam radius \( r \), and the calculated magnetic field strength \( B \). To calculate \( B \), use the following result for the field strength at the center of Helmholtz coils of radius \( a \) with \( N \) turns carrying current \( I \),

\[
B = \left(\frac{\mu_0NI}{a}\right)\left(\frac{4}{5}\right)^{\frac{3}{2}}. \tag{2}
\]

For our coils, \( N \) is 132; you will have to measure the coil radius \( a \).

Procedure: The apparatus is self-contained, and so only has to be plugged in to a wall socket. After turning the apparatus on, allow 5-10 minutes of warm-up time before taking measurements. While the apparatus is warming-up, measure the internal and external diameter of the Helmholtz coils on several axes. The coils may not be perfectly circular and the two coils may not be exactly the same. Find the mean internal diameter and the mean external diameter, both with proper uncertainty, then average the internal and external diameters and determine a value of coil radius \( a \), with proper uncertainty. Evaluate the coefficient of \( I \) in equation (2), with proper uncertainty, so that you may rapidly calculate \( B \) from the measured \( I \)’s once you begin taking measurements.

Part (1): Set the accelerating voltage \( V \) at 300 V. You should see a bluish beam traveling almost straight downwards until it hits the glass wall. The tube contains a low-density helium gas; the blue light is created when the fast moving electrons strike helium atoms. Also, the numbers on the centimeter scale within the tube flouresce when struck by the beam. Vary the coil current \( I \) so as to bend the beam into a circle of varying diameter. Vary the diameter from 11 cm to 5 cm in 1 cm increments, and then back to 11 cm, thereby taking two current readings at each selected diameter (go a little beyond 5 cm on the inward trip).

Part (2): Set the coil current to 1.5 A. Vary the accelerating voltage so that, once again, the beam is bent into circles of diameters between 5 and 11 cm; take two voltage readings at each beam diameter.
Finally, slowly and carefully rotate the entire apparatus by 180 degrees, paying careful attention to the positioning of the power cord. Repeat parts (1) and (2). This repetition helps to average out any possible effects from the ambient magnetic field; however, you should find that those effects are small.

Analysis: For part (1), average the results from the two orientations of the apparatus, and find $e/m$ from the slope of a linearized plot of your magnetic field versus radius data ($i.e. B$ versus $1/r$). For part (2), average the results from the two orientations, and find $e/m$ from an appropriate fit to a plot of $V$ versus $r^2$. In each case, report your value of $e/m$ with proper uncertainty and compare with the accepted value.

Credits: Photo and drawings courtesy of the Daedalon Corporation.
1. Find the magnetic field strength for a current of 2.0 A in the Helmholtz coils specified in this handout, assuming a coil radius of 15 cm.

2. An electron is accelerated through an electric potential of 100 V, then enters a 10 Gauss magnetic field in a direction at a right angle to the field. Find the radius of the subsequent circular path for this electron.