CURRENT BALANCE

Introduction: In this lab, we balance a known gravitational force against the repulsive magnetic force between antiparallel currents. We use our results to calculate an experimental value for $\mu_0$, the permeability of free space. You may ignore errors for this lab. Text Reference: Serway 29.4 and 30.1-2.

SPECIAL CONCERNS FOR CURRENT BALANCE LAB:
1. This lab requires careful manipulation of a sensitive balancing device. Excessive movement in the room can cause problems for all groups. Please keep movement to a minimum, and when you must move around, move as softly as possible.
2. The large currents used in this experiment can cause excessive heating; PLEASE LEAVE CURRENTS ON FOR ONLY 60 SECONDS OR LESS. Thanks.

Procedure: A sketch of the current balance apparatus is shown below. The current $I$ can be passed in opposite directions through the horizontal bars. The lower bar is fixed. The upper bar is moveable and can be brought to a within a few millimeters of the fixed bar by means of an adjustable counterweight (not shown). With the current off, the gap between the bars should be approximately 2.0 mm when the upper bar is in its equilibrium position; we will eventually measure the actual gap using an “optical lever”. During the experiment, a series of small masses will be placed in the pan attached to the upper bar. To compensate for this added weight, $I$ will slowly be increased until the moveable bar returns to its original position, at which time the repulsive magnetic force will be equal to the weight of the added mass. This “measured” magnetic force could then be compared with a calculated magnetic force $F_B$ using

$$F_B = \frac{\mu_0 LI^2}{2\pi r}$$  \hspace{1cm} (1)

where $L$ is the length over which the currents are antiparallel and $r$ is the separation distance between the two currents. However, instead of comparing the forces directly, we will use our measured currents to calculate an experimental value of $\mu_0$; we will then compare with the defined value of $\mu_0$, $4\pi \times 10^{-7}$ N/A$^2$. 

![Diagram of current balance apparatus]
The distance $r$ between currents, i.e. the center-to-center distance between the two bars, will be determined with an optical lever; the geometry of this device is shown in the sketch to the right. An observer from your group will look through the scope at the mirror, and thus, by reflection, at some location on the ruler. By recording two observed ruler locations, $x_t$ when the two bars are touching and $x_{eq}$ when the upper bar is in its equilibrium position, we will be able to calculate the distance $r$ from

$$r = \text{gap} + 3.18 \text{ mm} = \frac{a}{2b} |x_{eq} - x_t| + 3.18 \text{ mm}$$

where 3.18 mm is the diameter of each bar. Equation (2) is explained by the double figure at the bottom of this page. The left-hand figure shows the mirror position and a reflected light ray when the bars are touching; $\phi$ is the angle of incidence and/or reflection for the reflected light ray. When the bars are touching, the center-to-center distance between the bars is 3.18 mm; at the equilibrium position, the center-to-center distance is of course $r$. The right-hand figure shows the mirror position and a reflected light ray when the upper bar is in its equilibrium position; the mirror has been tilted by an angle $\theta$ with respect to its initial position and as a result, because the scope hasn’t moved, the angles of incidence and reflection have both been reduced by $\theta$. The original light ray from $x_t$ is also drawn in the right-hand figure; the angle between the two light rays, one coming from $x_t$ and the other from $x_{eq}$, is $2\theta$. Our calculation for $r$ relies on the fact that $2\theta$ is a small angle, so that we can ignore the difference between a triangle containing the angle $2\theta$ (or $\theta$) and a piece of a circle containing the same small angle. Therefore, for $\theta$ in radians,

$$\theta = \frac{\text{gap}}{a} = \frac{(r - 3.18 \text{ mm})}{a} \quad \text{and} \quad 2\theta = \frac{|x_{eq} - x_t|}{b}.$$  

Eliminating $\theta$ and solving for $r$ yields equation (2).
You need to make a few preliminary measurements. The value of $a$ is 21.7 cm. Carefully measure the values of $b$ and $L$. Next, someone in your group has to adjust the telescope for his or her eyes. Only this person will then be able to use the telescope. Adjust for sharp crosshairs first by moving the part of the eyepiece nearest your eye. Once the crosshairs are focused, adjust the length of the scope (pull or push on the tube that emerges from the body of the scope) to get a sharp image of the ruler. Your adjustment is correct when MOVING YOUR EYE UP OR DOWN DOESN'T CHANGE THE RELATIVE POSITION OF CROSSHAIR AND RULER (less than 0.5 mm is acceptable). Record $x_{eq}$, the scale reading when the upper bar is at its equilibrium position. Then, pile a few weights on the pan until the bars touch and record $x_t$. Finally, remove the weights to insure that the upper bar returns to its equilibrium position. Calculate $r$ to insure that you have a reasonable value.

You are now ready to begin the formal experiment. Add masses to the pan starting with 20 mg and going to 120 mg in units of 20 mg. For each mass, adjust $I$ so that the upper bar returns to its equilibrium position. Record $m$ and $I$. Now remove the masses in units of 20 mg, once more recording $m$ and the necessary $I$ for equilibrium. REMEMBER TO LEAVE CURRENTS ON FOR LESS THAN A MINUTE. When all masses are removed, check that your equilibrium position has not been changed by the action of adding and removing masses. Your results will be affected by the magnetic field of the Earth (more properly, the ambient magnetic field at your location in the lab room); to reduce this effect, use the two-way knife-edge switch to reverse the current direction (so the current direction will be reversed in each of the two bars) and repeat the entire experiment. Note that you can save time by doing both experiments at once; in other words, for each added mass, record the value of $I$ for both forward currents and then for reverse currents – but you have to be VERY careful not to get the two current directions confused. You will compute a value of $\mu_0$ for each experiment and average the two values. In your discussion, be sure to explain why reversing the currents and averaging partially compensates for the effect of the Earth’s field; this explanation must include free-body diagrams for the moveable bar for both forward and reverse currents.

To determine the values of $\mu_0$ from your two experiments, graph $I^2$ versus $m$, first for the forward current direction, and then for the reverse current direction. From the slopes of these two graphs, determine two values of $\mu_0$. Average these two values to get your final experimental value for $\mu_0$. Compare with $\mu_0$’s defined value of $4\pi \times 10^{-7}$ N/A$^2$; agreement to within 10% is reasonable. Is equation (1) a good model for the force between antiparallel currents?
1. Calculate the force between two wires each 0.5 m long, carrying 10 A current in opposite directions and separated by 3 mm, ignoring Earth’s magnetic field.

2. Find the magnitude of the magnetic force on one wire due to the Earth’s magnetic field (ignore the other wire), assuming that the Earth’s field is 0.5 Gauss oriented perpendicular to the wire.