1. Solve the boundary value problem,

\[
\frac{d^2 u}{dx^2} + (15 + 8x) u = 0 ,
\]

\[u'(0) = 0.5 , \quad u(1) = 0.3 ,
\]

for \(u(x)\) over the interval of \(0 \leq x \leq 1\). Use the 3-point central difference scheme (9th formula from top in p. 260) to represent \(u''\) in the differential equation and 2-point forward finite difference scheme (1st formula in Table 6-1 in p. 259) to represent the \(u'\) in the first boundary condition. Choose \(h = 0.1\). Plot your solution. (4 points)

2. (a) Solve the partial differential equation,

\[
\frac{\partial u}{\partial t} = 0.5 \frac{\partial u}{\partial x} - 0.4 u ,
\]

defined on the semi-infinite domain, \(-\infty < x < \infty\) and \(0 \leq t < \infty\), with the boundary condition given at \(t = 0\) as

\[u(x, 0) = 1 \quad \text{if} \quad 3.5 \leq x \leq 4
\]

\[= 0 \quad \text{otherwise} .
\]

Use the 2-point forward difference scheme (1st formula in Table 6-1 in p. 259) to discretize both \(\partial u/\partial t\) and \(\partial u/\partial x\). Choose \(\Delta x = 0.1\) and \(\Delta t = 0.1\). Integrate your system forward in \(t\) to find the solution, \(u(x,t)\), at \(t = 0.5, 1, \text{ and } 2\). Plot these solutions (as a function of \(x\)) along with the "initial" state, \(u(x,0)\), over the interval of \(0 \leq x \leq 5\). (3 points)

3. Find the general solution of the following PDEs by the method of separation of variables. (2 points)

(a) \[
\frac{\partial^2 u}{\partial x \partial y} - xyu = 0
\]

(b) \[
xy \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} + yu = 0
\]