Prob 1 (3 points)
For \( u(x,t) \) defined on the domain of \( 0 \leq x \leq 1 \) and \( t \geq 0 \), solve the 1-D Wave equation
\[
\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2},
\]
with the boundary conditions,

(i) \( u(0, t) = 0 \)
(ii) \( u(1, t) = 0 \)
(iii) \( u(x, 0) = P(x) \)
(iv) \( u_t(x, 0) = 0 \) \((u_t \text{ is } \partial u/\partial t)\),

where
\[
P(x) = \begin{cases} 
-x & \text{if } 0 \leq x \leq 0.8 \\
4(x - 1) & \text{if } 0.8 < x \leq 1 .
\end{cases}
\]

Plot the solution at \( t = 0, 0.3, 0.5, 0.7, 1.0, \) and \( 1.8 \). Please collect all 6 curves in one figure.

Prob 2 (3 points)
For \( u(x,t) \) defined on the domain of \( 2 \leq x \leq 5 \) and \( t \geq 0 \), consider the following PDE and boundary conditions:
\[
e^{-2x} \frac{\partial u}{\partial t} = 0.5 \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + x^2 e^{-2x} u ,
\]

(i) \( u(2, t) = 3 \ u_x(2, t) \) \((u_x \text{ is } \partial u/\partial x)\)
(ii) \( u(5, t) = 0 ,\)
(iii) \( u(x,0) = P(x) ,\)

where \( P(x) \) is a well-behaved function. (The detail of \( P(x) \) is unimportant for this problem.)
(a) Perform separation of variables on the PDE and the first two boundary conditions to obtain an eigenvalue problem in the \( x \)-direction (including an ODE and two boundary conditions) and an accompanying ODE in the \( t \)-direction. Write them out clearly. Is the eigenvalue problem in the \( x \)-direction of the form of a Sturm-Liouville system?
(b) If your answer to the last question in Part (a) is "no", no need to proceed further. If the answer is "yes", and if \( c_n \) and \( G_n(x) \) denote the \( n \)-th eigenvalue and \( n \)-th eigenfunction of the eigenvalue problem in the \( x \)-direction, express the final solution, \( u(x,t) \), as an eigenfunction expansion in terms of \( c_n \) and \( G_n(x) \). Please also provide the formula for evaluating the expansion coefficients when \( P(x) \) is given.