Prob 1

The solution is

\[ u(x, t) = \sum_{n=1}^{\infty} a_n \sin(n\pi x) \cos(n\pi t) \]

where

\[ a_n = \frac{\int_0^{0.8} -x \sin(n\pi x) \, dx + \int_{0.8}^1 4(x-1) \sin(n\pi x) \, dx}{\int_0^1 [\sin(n\pi x)]^2 \, dx}. \]

Plot:
Prob 2

(a) Setting \( u(x,y) = G(x)H(t) \), the outcome of separation of variables are

\[
\frac{dH}{dt} = c \ H ,
\]

and

\[
\frac{d}{dx} \left[ \left( 0.5 \exp \left( x^2 \right) \right) \frac{dG}{dx} \right] + x^2 \ G = c \ G , \quad G(2) = 3 \ G'(2) , \quad G(5) = 0 .
\]

The system in the \( x \)-direction satisfies the standard Sturm-Liouville form.

(b) The solution can be written as

\[
u(x,t) = \sum_{n=1}^{\infty} a_n \ G_n(x) \exp(c_n t),
\]

where

\[
a_n = \frac{\int_{2}^{5} P(x) G_n(x) \, dx}{\int_{2}^{5} [G_n(x)]^2 \, dx}.
\]