Prob 1 (4 points)
For \( u(x,t) \) defined on the domain of \(-\infty < x < \infty\) and \( t \geq 0 \), find the solution of the PDE,

\[
\frac{\partial u}{\partial t} + (1+u) \frac{\partial u}{\partial x} = 0 ,
\]

with the boundary condition,

\[
u(x,0) = P(x) ,
\]

where

\[
P(x) = \begin{cases} x & \text{if } 0 \leq x < 1 \\ (3-x)/2 & \text{if } 1 \leq x \leq 3 \\ 0 & \text{otherwise} . \end{cases}
\]

Plot the solution, \( u(x,t) \), as a function of \( x \) at \( t = 0, 0.5, 1, \) and 1.5. (You may simply use the given \( P(x) \) to make the plot for \( t = 0 \).) Make a plot of selected characteristics in the \( x-t \) plane. Can finite-time blowup occur in the solution of this system? If so, what is the critical time, \( t = t_c \), beyond which multiple solutions emerge?

Prob 2 (4 points)
For \( u(x,t) \) defined on the domain of \(-\infty < x < \infty\) and \( t \geq 0 \), find the solution of the PDE,

\[
\frac{\partial u}{\partial t} + 2u \frac{\partial u}{\partial x} = -u ,
\]

with the boundary condition,

\[
u(x,0) = \exp(-x^2) .
\]

Plot the solution as a function of \( x \) at \( t = 0, 0.2, \) and 0.5. (You may simply use the given boundary condition at \( t = 0 \) to make the plot for \( t = 0 \).) We do not expect a closed-form solution for this problem. It suffices to express the solution as a procedure that can be used to systematically evaluate \( u(x,t) \) with given \( x \) and \( t \). The plot can be made with the assistance of a numerical procedure such as bisection or Newton's method.

Prob 3 (1.5 point)
For \( u(x, y, t) \) defined on the domain of \(-\infty < x < \infty, -\infty < y < \infty, \) and \( t \geq 0 \), find the solution of the PDE,

\[
\frac{\partial u}{\partial t} + x \frac{\partial u}{\partial x} + t \frac{\partial u}{\partial y} = 2u ,
\]

with the boundary condition,

\[
u(x, y, 0) = \exp[-(x^2 + y^2)] .
\]
**Prob 4** (1.5 point)
For $u(x, t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, find the solution of the PDE,

$$
(1+t) \frac{\partial u}{\partial t} + x \frac{\partial u}{\partial x} = tu
$$

with the boundary condition,

$$
u(x, 0) = \exp(-x^2).
$$

**Prob 5** (3 points)
Consider the following PDE for $u(x, t)$ defined on the infinite domain of $-\infty < x < \infty$ and $t \geq 0$,

$$
\frac{\partial u}{\partial t} = -(3+t)u + Q(t)
$$

with the boundary condition,

$$
u(x, 0) = P(x).
$$

Find the Green's function, $G(t, t')$, such that for any given $Q(t)$ and $P(x)$ the solution of the system can be expressed as

$$
u(x, t) = G(t, 0)P(x) + \int_0^t G(t, t')Q(t')dt'.
$$