Prob 1

\[ u(x, t) = \begin{cases} 
\frac{x-t}{1+t}, & \text{if } t \leq x < 1 + 2t \\
\frac{x-2.5t}{1-0.5t}, & \text{if } 1 + 2t \leq x \leq 3 + t \\
0, & \text{otherwise}
\end{cases} \]

Finite-time blowup occurs when the second segment with \(1 + 2t \leq x \leq 3 + t\) becomes vertical, i.e., its "shadow" on the x-axis is reduced to a point. This means \(1 + 2t = 3 + t\). So, the critical time is \(t = 2\).

Plot of solution:
Prob 1, Plot of characteristics:

\[ u(x,t) = \exp(-x_0^2 - t) \]

where \( x_0 \) is determined numerically from

\[ x = x_0 + 2 \exp(-x_0^2) \left[ 1 - \exp(-t) \right]. \]

Plot:

Prob 2

\[ u(x,t) = \exp(-x_0^2 - t) \]

where \( x_0 \) is determined numerically from

\[ x = x_0 + 2 \exp(-x_0^2) \left[ 1 - \exp(-t) \right]. \]

Plot:
Prob 3
\[ u(x, y, t) = \exp[- \{(x \exp(-t))^2 + (y - t^2/2)^2\} + 2t] \]

Prob 4
\[ u(x, t) = \frac{1}{1+t} \exp\left[- \left( \frac{x}{1+t} \right)^2 + 1 \right] \]

Prob 5
\[ G(t, t') = \exp[- (3t + t^2/2) + (3t' + t'^2/2)] \]