MAE/MSE 502 Spring 2015, Homework #4

Prob. 1 (3 points) For \( u(x,t) \) defined on the domain of \( 0 \leq x \leq 2\pi \) and \( t \geq 0 \), solve the PDE

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^3 u}{\partial x^3} ,
\]

with the boundary conditions (the first three simply indicate that the system is periodic in \( x \)),

(i) \( u(0, t) = u(2\pi, t) \)
(ii) \( u_x(0, t) = u_x(2\pi, t) \)
(iii) \( u_{xx}(0, t) = u_{xx}(2\pi, t) \)
(iv) \( u(x, 0) = \sin(x) + \cos(2x) \).

We expect a closed-form solution without any unevaluated integral or summation of infinite series. Plot the solution as a function of \( x \) at \( t = 0, 0.1, 0.2, \) and \( 0.5 \).

Prob. 2 (3 points) For \( u(x,t) \) defined on the domain of \( 0 \leq x \leq 1 \) and \( t \geq 0 \), solve the PDE,

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \cos(2\pi x) e^{-t} + t ,
\]

with the boundary conditions

\( (i) \ u_x(0, t) = 0 \)
\( (ii) \ u_x(1, t) = 0 \)
\( (iii) \ u(x, 0) = 1 + \cos(\pi x) + \cos(2\pi x) \).

We expect a closed-form solution without any unevaluated integral or summation of infinite series.

Prob. 3 (3 points) For \( u(x,t) \) defined on the domain of \( 0 \leq x \leq 2\pi \) and \( t \geq 0 \), solve the PDE

\[
\frac{\partial u}{\partial t} = \frac{\partial^3 u}{\partial x^3} + \sin(x) e^{-t} + 1 ,
\]

with the boundary conditions (the first three simply indicate that the system is periodic in \( x \)),

(i) \( u(0, t) = u(2\pi, t) \)
(ii) \( u_x(0, t) = u_x(2\pi, t) \)
(iii) \( u_{xx}(0, t) = u_{xx}(2\pi, t) \)
(iv) \( u(x, 0) = 3 + \cos(x) \).

We expect a closed-form solution without any unevaluated integral or summation of infinite series. The solution of this problem is real. Please arrange your solution such that there is no imaginary number "\( i \)" (= \( \sqrt{-1} \)) in the final expression of \( u(x, t) \).