This assignment is for bonus only. The rules of collaboration are the same as those for the final project.

1. **Bonus question.** For \( u(x,t) \) and \( h(x,t) \) defined on the domain of \( 0 \leq x \leq 10 \) and \( t \geq 0 \), consider the following system of equations,

\[
\frac{\partial u}{\partial t} = 2 \frac{\partial h}{\partial x}, \quad (1)
\]

\[
\frac{\partial h}{\partial t} = 0.5 \frac{\partial u}{\partial x}, \quad (2)
\]

with the boundary conditions,

\[
u(0, t) = 0,
\]

\[
u(10, t) = 0,
\]

and the initial conditions,

\[
h(x,0) \equiv 0,
\]

\[
u(x,0) = P(x),
\]

where

\[
P(x) \equiv \cos[0.5 \pi (x - 5)] , \text{ if } 4 \leq x \leq 6
\]

\[
\equiv 0 \quad , \text{ otherwise.}
\]

(Note that the \( P(x) \) here is identical to that used in HW1.) Solve the system numerically by using a staggered grid system as illustrated in Fig. 1, and the forward-in-time and central-in-space finite difference scheme for both equations. Use \( \Delta x = 0.01 \) and your own choice of \( \Delta t \). Plot the solution at \( t = 0.75 \) and 2.5, along with the initial state at \( t = 0 \). See Fig. 1 for the definition of \( \Delta x \). For this problem, the two boundary points should be the grid points for \( u \) (see Fig. 1), such that there is no need to impose the boundary conditions for \( h \). As usual, submit your code. Only high-quality solutions will receive bonus.

![Fig. 1](image-url)