Instructions: Attempt to do all problems before looking at the solutions. Do NOT turn in your answers.

Part 1. Linear Kinematics Problems

1. Two teams are competing in swimming relay races. Swimmer A pushes off two seconds ahead of swimmer B with 50 m left to go. If swimmer A swims at an average speed of 1.9 m/s and swimmer B swims at an average speed of 2.1 m/s, who wins the race? How far ahead in seconds and meters is the winner?

2. Arlene lands from a jump and reduces the downward speed of her center of mass (CM) from 6 m/s to 4 m/s during the first 0.05 seconds after impact. If the upward direction is positive, what is the average acceleration of her CM?

3. The distance-time graph of a car traveling along a road is shown in Fig. 1. Make a graph of its speed versus time.

4. A cyclist completes a 100 km race in 2:25:30 (hr:min:sec). The race course proceeds 50 km north, then 25 km east, then 25 km south. Her split times were 1:06:05 at the 50 km point and 1:49:19 at the 75 km point.
   
   Draw a diagram of the race course and answer the following questions:

   a. What were her average speeds over the three intervals (0-50 km, 50-75 km, and 75-100 km)? How do these speeds compare to her average speed over the whole race? If we assume "even splitting" (keeping roughly a constant speed) is physiologically desirable, what would be your recommendations for improvement?

   b. What is her displacement from start to finish (both in magnitude and direction)? Draw this vector on your diagram.

   c. What is her average velocity from start to finish (both in magnitude and direction)? Is this a meaningful quantity? Explain.

   d. At the start of the race the cyclist accelerates from 0 to 15 m/s (north) in 7 s. What is her average acceleration (both in magnitude and direction)?
Solutions to Linear Kinematics Problems

1. Known: \( \overline{v}_A = 1.9 \text{ m/s} \) \( \overline{v}_B = 2.1 \text{ m/s} \) \( \Delta d_A = \Delta d_B = 50 \text{ m} \)

First, find \( \Delta t_A \) and \( \Delta t_B \) using equation \( \overline{v} = \frac{d}{\Delta t} \Rightarrow \Delta t = \frac{d}{\overline{v}} \)

\[
A: \Delta t_A = \frac{\Delta d_A}{\overline{v}_A} = \frac{50 \text{ m}}{1.9 \text{ m/s}} = 26.315789 \text{ s} \approx 26.3 \text{ s}
\]

\[
B: \Delta t_B = \frac{\Delta d_B}{\overline{v}_B} = \frac{50 \text{ m}}{2.1 \text{ m/s}} = 23.809524 \text{ s} \approx 23.8 \text{ s}
\]

These are times to complete 50 m. (don't round off yet)

However swimmer A has a 2 s "head start" so let's subtract 2 s from his time: \( \Delta t_A' = \Delta t_A - 2 = 24.315789 \text{ s} \). Note that \( \Delta t_B < \Delta t_A' \) so swimmer B wins by \( \Delta t_B - \Delta t_A' = 0.506265 \text{ s} = 0.51 \text{ s} \)

How far behind is swimmer A when swimmer B touches?

To determine this we must determine how far swimmer A moves during the last 0.506265 s of his swim. \( \overline{v} = \frac{d}{\Delta t} \Rightarrow d = \overline{v} \cdot \Delta t \)

\[
d = \overline{v}_A \cdot \Delta t = 1.9 \text{ m/s}(0.506265 \text{ s}) = 0.961904 \text{ m} \approx 0.96 \text{ m}
\]

2. Known: \( v_i = -6 \text{ m/s} \) \( v_f = -4 \text{ m/s} \) \( \Delta t = 0.05 \text{ s} \)

Find \( \overline{a} = \frac{v_f - v_i}{\Delta t} = \frac{-4 \text{ m/s} - (-6 \text{ m/s})}{0.05 \text{ s}} = 40 \text{ m/s}^2 \)

3. \( \overline{v}_1 = \frac{d_1}{\Delta t_1} = \frac{15 \text{ mi}}{0.5 \text{ hr}} = 30 \text{ mi/hr} \)

\( \overline{v}_2 = \frac{d_2}{\Delta t_2} = \frac{45 \text{ mi} - 15 \text{ mi}}{1.00 \text{ hr} - 0.5 \text{ hr}} = 30 \text{ mi/hr} = 60 \text{ mi/hr} \)

\[
\begin{array}{c}
\text{v} \\
30 \text{ (mi/hr)} \\
\text{0}
\end{array}
\]

\[
\begin{array}{c}
\text{t} \\
0.5 \text{ (hr)} \\
1.0 \text{ (hr)}
\end{array}
\]
4. \[ \begin{align*}
\vec{d}_1 &= 50 \text{ km north} \quad t_1 = 1:06:05 = 3965 \text{ s} \\
\vec{d}_2 &= 25 \text{ km east} \quad t_2 = 1:49:19 = 6559 \text{ s} \\
\vec{d}_3 &= 25 \text{ km south} \quad t_3 = 2:25:30 = 8730 \text{ s}
\end{align*} \]

\[ a. \quad 0-50 \text{ km}: \quad d_1 = 50,000 \text{ m}, \quad \Delta t_1 = t_1 - 0 = 3965 \text{ s} \]
\[ \bar{v}_1 = \frac{d_1}{\Delta t_1} = \frac{50,000 \text{ m}}{3965 \text{ s}} = 12.610340 \text{ m/s} \approx 12.6 \text{ m/s} \]

\[ 50-75 \text{ km}: \quad d_2 = 25,000 \text{ m}, \quad \Delta t_2 = t_2 - t_1 = 6559 - 3965 = 2594 \text{ s} \]
\[ \bar{v}_2 = \frac{d_2}{\Delta t_2} = \frac{25,000 \text{ m}}{2594 \text{ s}} = 9.6376633 \text{ m/s} \approx 9.6 \text{ m/s} \]

\[ 75-100 \text{ km}: \quad d_3 = 25,000 \text{ m}, \quad \Delta t_3 = t_3 - t_2 = 8730 - 6559 = 2171 \text{ s} \]
\[ \bar{v}_3 = \frac{d_3}{\Delta t_3} = \frac{25,000 \text{ m}}{2171 \text{ s}} = 11.515431 \text{ m/s} \approx 11.5 \text{ m/s} \]

\[ 0-100 \text{ km}: \quad d_1 + d_2 + d_3 = 100 \text{ m} \quad (\text{total distance}) \]
\[ \Delta t = t_3 = 8730 \text{ s} \quad (\text{total time}) \]

\[ \text{overall avg. speed} = \frac{\text{(total distance)}}{\text{(total time)}} = \frac{100 \text{ m}}{8730 \text{ s}} = 11.454754 \text{ m/s} \approx 11.4 \text{ m/s} \]

By comparing avg. speeds for each interval, she clearly went out "too fast", then slowed down between 50 & 75 km. She should probably go out a bit more slowly, then maintain speed during the 50-75 km interval.

\[ b. \quad \begin{align*}
\Delta d &= \sqrt{\Delta d_x^2 + \Delta d_y^2} = \sqrt{25^2 + 25^2} = 35.355339 \text{ m} \\
\theta &= \tan^{-1}\left(\frac{\Delta d_y}{\Delta d_x}\right) = \tan^{-1}\left(\frac{25}{25}\right) = 45^\circ \quad (\text{North of due east})
\end{align*} \]

\[ c. \quad |\bar{v}| = |\frac{\Delta d}{\Delta t}| = \frac{35.355339 \text{ m}}{8730 \text{ m/s}} = 4.0498670 \text{ m/s} \approx 4.05 \text{ m/s} \]
\[ \theta = 45^\circ \quad \text{North of due east} \quad \text{(same as for d)} \]

\[ d. \quad \bar{v}_i = 0 \quad \bar{v}_f = 15 \text{ m/s} \quad (\text{north is positive here}) \quad \Delta t = 7 \text{ s} \]
\[ \bar{a} = \frac{\bar{v}_f - \bar{v}_i}{\Delta t} = \frac{15 - 0 \text{ m/s}}{7 \text{ s}} = 2.142857 \text{ m/s}^2 \approx 2.14 \text{ m/s} (\text{North}) \]
1. A diver completes 3 1/2 somersaults before entering the water. He overrotates by 20 degrees. His time in the air is 1.4 s. Assuming that he is rotating in the positive direction, compute the following in rad or rad/s:

   a. angular distance
   b. angular displacement
   c. average angular speed
   d. average angular velocity

2. A cyclist hits the brakes and decelerates. His wheels were spinning at 200 rev/min initially and 10 rev/min after 3 s of deceleration.

   a. Compute the average average angular acceleration (in rad/s²) of his wheel during this 3 s period.

   b. How long does it take him (altogether) to come to a complete stop if he maintains the same acceleration?

3. A golfer swings his golf club and angularly accelerates it from rest at the peak of the back swing until it strikes the ball. At this instant the club is rotating with an angular velocity of 45 rad/s. If the down swing is considered to occur in a single plane and if the time of the downward swing is 0.15 s, what is the club's average angular acceleration if the downward swing rotational direction is considered positive? If the radius of rotation in the above problem was 1.3 m, what is the linear velocity of the clubhead at impact? What is the radial acceleration of the clubhead at impact? What was the average tangential acceleration of the clubhead during the downswing?
Solutions to Angular Kinematics Problems

1. a. \( \phi = 3.5 \text{ rev} \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) + 20^\circ \left( \frac{2\pi \text{ rad}}{360^\circ} \right) = 22.340213 \text{ rad} = 22.3 \text{ rad} \)
   
   b. \( \theta = -160^\circ \left( \frac{2\pi \text{ rad}}{360^\circ} \right) = -2.7925265 \text{ rad} = -2.79 \text{ rad} \)

   c. \( \dot{\alpha} = \frac{\phi}{\Delta t} = \frac{22.340213 \text{ rad}}{1.4 \text{ s}} = 15.957295 \text{ rad/s} = 16.0 \text{ rad/s} \)

   d. \( \ddot{\theta} = \frac{\theta}{\Delta t} = \frac{-2.7925265 \text{ rad}}{1.4 \text{ s}} = -1.9946617 \text{ rad/s} = -1.99 \text{ rad/s} \)

   ![Diagram](image)

   Note: \( \theta \) is limited on \( \theta \)

   \(-180^\circ < \theta \leq 180^\circ\)

2. \( \omega_1 = 200 \text{ rev} \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 20.94395 \text{ rad/s} \)

   \( \omega_2 = 10 \text{ rev} \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 1.0471975 \text{ rad/s} \)

   \( \omega_3 = 0 \Delta t_1 = 3 \text{ s} \)

   a. \( \ddot{\alpha} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{\omega_2 - \omega_1}{t_1} = \frac{1.0471975 - 20.94395}{3 \text{ s}} = -6.63 \text{ rad/s}^2 \)

   b. \( \ddot{\alpha} = \frac{\omega_f - \omega_i}{\Delta t} \Rightarrow \Delta t = \frac{\omega_f - \omega_i}{\ddot{\alpha}} = \frac{\omega_3 - \omega_1}{\ddot{\alpha}} = \frac{0 - 20.94395}{-6.632251 \text{ rad/s}^2} = 3.1578946 \text{ s} = 3.16 \text{ s} \)

3. \( \omega_i = 0, \omega_f = 45 \text{ rad/s}, \Delta t = 0.15 \text{ s}, \Delta r = 1.3 \text{ m}, \) (note \( v_i = 0 \) also)

   \( \ddot{a} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{45 - 0}{0.15} = 300 \text{ rad/s}^2 \)

   \( v_f = \omega_f \cdot r = 45 \text{ rad/s} \cdot (1.3 \text{ m}) = 58.5 \text{ m/s} \)

   \( a_r = \frac{\Delta v}{\Delta r} = \frac{(58.5 \text{ m/s})}{1.3 \text{ m}} = 2632.5 \text{ m/s}^2 \)

   \( a_z = \frac{v_f - v_i}{\Delta t} = \frac{58.5 \text{ m/s} - 0}{0.15} = 390 \text{ m/s}^2 \)

   \[ \text{Note: can also calculate } \ddot{a}_z = a_z = (300 \text{ rad/s}^2)(1.3 \text{ m}) = 390 \text{ m/s}^2 \]
Part 3. Kinematic Curve Analysis

Consider the following velocity vs. time curve. Qualitatively derive both the position (x) vs. time and the acceleration vs. time curves and draw them on the blank graphs below. Note: Assume $x = 0$ when $t = 0$. (The solution is presented on the next page. Don’t look at the solution until after you have tried this.)
Position and acceleration curves were derived qualitatively from a given velocity curve. Please accept the drawing limitations inherent in Microsoft Word “AutoShapes”. If you disagree or find an error in this analysis, please let Dr. Hinrichs know.

**Notes:** At $t_1$ the velocity has reached a maximum. The acceleration is zero here. The slope of the position curve is steepest here. At $t_2$ the velocity has reached its greatest negative slope. This is where the acceleration is most negative (minimum value). At $t_3$ the position reaches its maximum value at the point when velocity reaches zero for the first time (representing the end of the first forward movement phase). This represents a reversal in direction from positive to negative; hence acceleration is negative here. At $t_4$ velocity has reached its most negative value. This represents a point of zero acceleration and the steepest downward slope on the position curve. At $t_5$ the velocity crosses zero on its way from negative to positive. This represents a reversal in direction (hence a positive acceleration) and a relative minimum on the position curve. At $t_6$ the velocity reaches another relative maximum on the second forward movement phase. The comments are the same here as for $t_1$ above.