Q1. A random variable X has the following pdf:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>b</td>
<td>2b</td>
<td>3b</td>
<td>4b</td>
<td>5b</td>
</tr>
</tbody>
</table>

1) What is the value of b? Why?
2) Find the P(X ≤ 3).
3) Find E(x).

Q2. The joint probability distribution of X and Y is given by the following table: (For example, f(4,9) = 0.)

<table>
<thead>
<tr>
<th>x \ y</th>
<th>1</th>
<th>3</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1/8</td>
<td>1/24</td>
<td>1/12</td>
</tr>
<tr>
<td>4</td>
<td>1/4</td>
<td>1/4</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1/8</td>
<td>1/24</td>
<td>1/12</td>
</tr>
</tbody>
</table>

1) Find the marginal pdfs of X and Y.
2) Find var(2x+3y).

Q3. Let X stand for the rate of return on a security (say, IBM) and Y the rate of return on another security (say, General Motors). Let \( \mu_X = \mu_Y = 0.5 \), \( \sigma_X^2 = 4 \), \( \sigma_Y^2 = 9 \) and corr(x,y) = -0.8.

1) Find E[0.5x+0.5y] and var[0.5x+0.5y]. [Hint: E[0.5x+0.5y] = 0.5•E(x)+0.5•E(y) and var[0.5x+0.5y] = (0.5)^2•var(x) + (0.5)^2•var(y) + 2•(0.5)•(0.5)•cov(x,y); cov(x,y) = corr(x,y)•\sigma_X\sigma_Y.]
2) Is it better to invest equally in the two securities (i.e., diversify) than in either security exclusively? (Hint: Investors consider both expected rate of return and risk.) Explain in detail why or why not.

Q4. Let \( Y \sim \chi^2(5) \).

1) Find a such that P( Y > a ) = 0.05.
2) Find c such that P( Y < c ) = 0.9.

Q5. Let the two random variables, \( X_1 \) and \( X_2 \), are i.i.d. with N(0,1). Find P( \( X_1^2+X_2^2 > 9.21 \) ).

Q6. Consider the three random variables, X, Y, and Z. Assume that all of them are stochastically independent. Let X be N(0,1); Y be \( \chi^2(5) \); Z be \( \chi^2(4) \).
1) Find \( \Pr\left( \frac{X}{\sqrt{Y/5}} > 2.57 \right) \).

2) Find \( \Pr\left( \frac{\sqrt{Y/5}}{Z/4} > 2.5020 \right) \).

Answers:

1.  
   1) \( b = 1/15 \), since \( \sum x f(x) = 1 \).
   2) \( \Pr(X=0) + \Pr(X=1) + \Pr(X=2) + \Pr(X=3) = 2/3 \).
   3) \( 8/3 \).

2.  
   1) \( f_x(2) = 1/4; f_x(4) = 1/2; f_x(6) = 1/4; f_y(1) = 1/2; f_y(3) = 1/3; f_y(9) = 1/6 \).
   2) \( 80 \).

3.  
   1) \( E(0.5x+0.5y) = 0.5\cdot E(x)+0.5\cdot E(y) = 0.5 \)
   \( \text{var}(0.5x+0.5y) = (0.5)^2\cdot 4+(0.5)^2\cdot 9+2\cdot(0.5)\cdot (0.5)\cdot 2\cdot 3\cdot (-0.8) = 0.85 \).
   2) Observe that \( E[(1/2)\cdot x + (1/2)\cdot y] = E(x) = E(y) = 0.5 \). Thus, the two investment strategies give you the same expected return. However, \( \text{var}(1/2\cdot x+1/2\cdot y) = 0.85 \), \( \text{var}(X) = 4 \) and \( \text{var}(Y) = 9 \). So, investing equally in the two securities is less risky than investing in one security exclusively.

4.  
   1) \( a = 11.07 \)
   2) \( \Pr(Y > c) = 1 - 0.9 = 0.1 \). \( \rightarrow c = 9.24 \).

5. \( \Pr[\chi^2(2) > 9.21] = 0.01 \). [Hint: Note that \( X_1^2+X_2^2 \) is \( \chi^2(2) \).]

6.  
   1) \( \Pr[t(5) > 2.57] = 0.025 \);
   \( \Pr\left( \sqrt{F(5,4)} > 2.5020 \right) = \Pr\left( F(5,4) > 2.5020^2 \right) 
   = \Pr(F(5,4) > 6.26) = 0.05 \).