1. (10 pts.) Let $Y$ be a random variable distributed as shown in the accompanying table.

\[
\begin{array}{cccc}
 y & 1 & 2 & 3 & 4 \\
 f(y) & .1 & .2 & .3 & .4 \\
\end{array}
\]

Find $E(y)$ and $\text{var}(y)$.

2. (70 pts.) The joint probability distribution of $X$ and $Y$ is given by the following table: (For example, $f(4,9) = 0$.)

\[
\begin{array}{ccc}
 x \backslash y & 1 & 3 \\
 2 & 1/8 & 1/24 \\
 4 & 1/24 & 1/12 \\
 6 & 1/12 & 1/24 \\
\end{array}
\]

(1) Derive the marginal pdfs of $X$ and $Y$.
(2) Determine whether $X$ and $Y$ are stochastically independent or not.
(3) Compute population means of $X$ and $Y$.
(4) Compute population variances of $X$ and $Y$.
(5) Compute the correlation coefficient between $X$ and $Y$.
(6) Check whether or not $\Pr(X = 2|Y = 9) = \Pr(Y = 9|X = 2)$.
(7) Given $X = 2$, find the population mean of $Y$, i.e., $E(Y|X = 2)$ [Hint: $E(Y|X = 2) = \sum_y y f(y|x=2)$].

3. (20 pts.) Let $X \sim \text{N}(2,9)$, $Z \sim \text{N}(0,1)$, $Y \sim \chi^2(4)$ and $W \sim \chi^2(5)$. Assume that all of the random variables $X$, $Z$, $Y$ and $W$ are stochastically independent.

(1) Find $\Pr(X < 6)$.
(2) Find $\Pr(Y < 9.49)$.
(3) Find $\Pr\left(Z > 3.75\sqrt{Y/4}\right)$.
(4) Find $\Pr\left(\frac{5Y}{4W} > 5.19\right)$. 