Q1. (15 pts.) Download a data file called “tsls.txt” from my website. The data contains 4 variables: y, x, z1 and z2. The number of observations is 100. The regression model you wish to estimate is given:

\[ y = \beta_1 + \beta_2 x + \epsilon. \]

Make a gauss program that can do the followings. Report both your program and output files. Estimate the above model by LIML. Report variable names, estimates, standard errors, and t-statistics. Also, report R\(^2\). Also test whether your instruments are exogenous or not.

Q2. (20 pts.) Use panel_1.txt (This data set contains 4 variables, y, x1, x2, and z; and N = 30 and T = 5.) You wish to estimate:

\[ y_{it} = \beta_1 x_{1it} + \beta_2 x_{2it} + \gamma_1 + \gamma_2 z_i + (\alpha_i + \epsilon_{it}), \]

where the \( \alpha_i \) are iid \((0, \sigma^2_\alpha)\) over different i, and the \( \epsilon_{it} \) are iid over i but heteroskedastic and autocorrelated over time. Use Gauss to answer the following questions.

(1) (10 pts.) Estimate the above model by Kiefer’s within GLS. Report variable names, estimates, standard errors, and t-statistics. Also, report R\(^2\).

(2) (10 pts.) Assuming that the effects \( \alpha_i \) are random and uncorrelated with \( x_{1it} \). Estimate the model by AM-MGIV. Report variable names, estimates, standard errors, and t-statistics. Also, report R\(^2\). Test whether your AM-MGIV estimator is consistent or not by the Hausman test.

Q3. (10 pts.) Consider the following SUR model:

\[ y_{1t} = \alpha_1 + \alpha_2 x_{1t} + \alpha_3 x_{2t} + \epsilon_{1t}; \]
\[ y_{2t} = \gamma_1 + \gamma_2 x_{3t} + \gamma_3 x_{4t} + \epsilon_{2t}. \]

To estimate this model, use the data set named “sur.db”. The data set contains 30 observations on 6 variables (y1, y2, x1, x2, x3 and x4). Using this data set, construct a GAUSS program that estimates the model by two-step feasible GLS. Report the variable names, the estimated coefficients, standard errors, and t statistics. Report both your program and output files.

Hint 1: For a p×p identity matrix, use “eye(p)”.

Hint 2: For the Kronecker product of two matrices A and B, use “A.*. B”
Q4. (10 pts.) Consider the following two-equation system:

\[ y_1 = \beta_1 e_T + \beta_2 x_1 + \beta_3 x_2 + \epsilon_1; \]
\[ y_2 = \beta_1 e_T + \beta_2 x_3 + \beta_4 x_4 + \epsilon_2, \]

where \( e_T \) is a T×1 vector of ones, all of \( y_1, y_2, x_1, x_2, x_3, x_4, \epsilon_1 \) and \( \epsilon_2 \) are T×1 vectors. Using the data set “sur.db”, construct a GAUSS program that can estimate \( \beta_1, \beta_2, \beta_3 \) and \( \beta_4 \) by two-step GLS. Report both your program and output files. Report the variables name, estimated coefficients, and standard errors. Compute standard errors under two different assumptions: (i) the errors are iid over time, (ii) the errors are not autocorrelated but heteroskedastic over time.

Q5. (20 pts.) Consider the following regression model:

\[ y = X_1 \beta_1 + X_2 \beta_2 + \varepsilon = X \beta + \varepsilon, \]

where \( y, X_1, X_2 \) and \( \varepsilon \) are T×1, T×k, T×k and T×1 respectively. Assume that \( Z = [H, X_2] \) is not correlated with \( \varepsilon \), where \( H \) is T×g. Let \( W = (y, X) \), \( W_1 = (y, X_1) \), \( \xi_1 = (-1, \beta')' \) and \( \xi_2 = \beta_2 \).

1. (6 pts.) Show that all of the eigenvalues of \( (WW)W(ZW)W \) are positive. Show this using the definition of eigenvalues and eigenvectors.
2. (6 pts.) Let \( \lambda_1 \) be the smallest eigenvalue of \( (WW)W(ZW)W \). Show that if \( g = k_1, \lambda_1 = 0 \). Show this using the fact that if a matrix \( A \) is not of full column, than there exists a non-zero vector \( c \) such that \( Ac = 0 \).
3. (8 pts.) Show that if \( g = k_1 \), \( \text{LIML} = 2\text{SLS} \) numerically.

Q6. (10 pts.) Consider the two-equation system:

\[ y_1 = \beta_1 x_1 + \epsilon_1 \]
\[ y_2 = \beta_2 x_2 + \beta_3 x_3 + \epsilon_2, \]

where \( x_1, x_2 \) and \( x_3 \) are T×1 nonrandom exogenous variables and \( \epsilon_1 \) and \( \epsilon_2 \) are T×1 vectors of errors. Assume that \( \epsilon_t = (\epsilon_{t1}, \epsilon_{t2})' \) are iid N(0,\( \Sigma_{2×2} \)), where \( \Sigma = [\sigma_{ij}] \) is known. Suppose that the analyst of this model applies GLS but erroneously omits \( x_3 \) from the second equation. Is the GLS estimator of \( \beta_1 \) unbiased? Justify your answer.

Q7. (15 pts.; 10 on each.) Consider a following translog cost function

\[ \log(c) = \alpha + \Sigma_{i=1}^2 \beta_i \log(p_i) + .5 \Sigma_{i=1}^2 \Sigma_{j=1}^2 \delta_{ij} \log(p_i) \log(p_j) \]
\[ + \Sigma_{i=1}^2 \gamma_{y,i} \log(p_i) \log(y) + \theta_y \log(y) + .5 \theta_{yy} \log^2(y), \]

with two input share functions:
\[ s_1 = \beta_1 + \Sigma_{i=1}^2 \delta_{i1} \log(p_i) + \gamma_{y,1} \log(y) + \epsilon_1; \]
\[ s_2 = \beta_2 + \Sigma_{i=1}^2 \delta_{i2} \log(p_i) + \gamma_{y,2} \log(y) + \epsilon_2. \]

Here, the subscripts \( i = 1 \) and \( 2 \) index inputs, capital (\( k \)) and labor (\( l \)), respectively. Also, \( \delta_{ij} = \delta_{ji} \). The symbols \( c \), \( y \) and \( p_i \) denote total costs, quantity of output, and input prices, respectively. The symbol \( s_i \) denotes the cost share of input \( i \) (e.g., \( s_1 = s_k = p_k/c \)).

(1) (7 pts.) The three input shares must add identically to one. What parametric restrictions this requirement place on the unknown parameters? Explain why.

(2) (8 pts.) Show that your restrictions obtained from (1) can be imposed directly on the model by specifying the translog cost function and the share functions in \( c/p_2 \), \( p_1/p_2 \), and \( y \) (instead of \( c \), \( p_1 \), \( p_2 \), and \( y \)), and dropping the second share equation.