1. AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY (ARCH)

[1] EMPIRICAL REGULARITIES

[See Bollerslev, Engle and Nelsen, Handbook of Econometrics, 4]

(1) Thick tails: Thicker than those of iid normal dist.

ARCH-1
(2) Volatility Clustering: Large changes tend to be followed by large changes.

(3) Leverage effects: Changes in stock prices tend to be negatively related with changes in stock volatility.

ARCH-2
(4) Non-trading periods effects:
Information that accumulates when financial markets are closed is reflected in prices after the market opens.

(5) Forecastable events effects:
Forecastable releases of important information (e.g., earnings announcement) are associated with high ex ante volatility.

(6) Volatility and serial correlation
Inverse relation between volatility and serial correlation for US stock indices.

(7) Co-movements in volatilities
Commonality in volatility changes across stocks.

(8) Macroeconomic variables and volatility
Positive relation between macro uncertainty and stock market volatility.
[2] MODEL, ESTIMATION AND SPECIFICATION TESTS

(1) BASIC MODEL

\[ y_t = x_t \sigma + u_t, \quad t = 1, 2, \ldots, T; \quad u_t = \sqrt{h_t} v_t, \]

- \( v_t \) iid \( N(0,1) \): Need not be normal.
  - Can be t-distribution with \( \text{df} = 8 \).
- \( h_t = E(u_t^2|S_{t-1}) = \text{var}(u_t|S_{t-1}) = h_t(\sigma, B), \)
  where \( B \) is a parameter vector determining volatility in \( y_t \).
- Conditional variance of \( u_t \) given \( S_{t-1} \).
- \( h_t \) could be a function of some regressors, say, \( z_t \).
- \( S_{t-1} \): Information set at time \( t-1 \)
  \[ S_{t-1} = \{ x_{t-1}, x_{t-2}, \ldots, y_{t-1}, y_{t-2}, \ldots \}. \]
- \( F^2 = E(u_t^2) = \text{var}(u_t) = \text{Unconditional variance of} \ u_t \)
  \[ = \text{Unconditional mean of volatility}. \]

NOTE:

- \( u_t \) should be serially uncorrelated.
- \( u_t^2 \) could be serially correlated.
(2) ESTIMATION

1) MLE

- Define the conditional pdf by \( f(y_t|x_t, S_{t-1}, 2) \), where \( 2 = (\theta^\top, \sigma^2) \)
- Assume \( v_t \) iid \( N(0,1) \),

\[
f(y_t|x_t, S_{t-1}, 2) = \frac{1}{\sqrt{2\pi \sigma^2}} \exp \left[ -\frac{(y_t - x_t \cdot \theta)^2}{2\sigma^2} \right].
\]

- \( l_T(2) = E_t \ln[f(y_t|x_t, S_{t-1}, 2)] \)
  \[= -(T/2)\ln(2\pi\sigma^2) - \frac{1}{2}E_t \ln(h_t) - \frac{1}{2}E_t(y_t - x_t \cdot \theta)^2/h_t.\]
- \( s_t = M_t \ln[f(y_t|x_t, S_{t-1}, 2)]/M^2_t \)
- \( H_T(2) = M_t l_T(2)/M^2_t M^2_t; B_T(2) = E_t s_t(2)/M_t(2) \)
- MLE: \( \hat{2} \cdot N[2, (-H_T(\hat{2})){^{-1}}] \cdot N[2, (B_T(\hat{2})){^{-1}}] \)
- MLE with non-Gaussian \( v_t \): See Hamilton (661-662).

2) MLE when normality assumption is violated

[QLME: Bollerslev and Wooldridge, ER, 1992]
- MLE based on the normality assumption is still consistent.
- \( \hat{2} \cdot N[0, (H_T(\hat{2})){^{-1}}B_T(\hat{2})(H_T(\hat{2})){^{-1}}]. \)
- QMLE with other distributional assumptions
  [See Newey and Steigerwald (ECON, 1997)]
3) GMM

- Moment conditions:
  \[ E(x_t \delta_t) = 0 \ ; \ E[(u_t^2 - h_t)u_{t+j}] = 0, \ j = 1, 2, \ldots \]
- Not successful (Andersen and Sørensen, 1996, JBES).

(3) SPECIFICATION TESTING

1) LM test (Engle, 1982, ECONOMETRICA)

   \( H_0: \) No CH.

   **STEP 1:** Do OLS on \( y_t = x_t \beta + u_t \), and get \( \hat{u}_t \).

   **STEP 2:** Regress \( \hat{u}_t^2 \) on one, \( \hat{u}_{t\delta d}^2 \), \( \hat{u}_{t\delta 2}^2 \), \ldots, \( \hat{u}_{t\delta q}^2 \), and get \( R^2 \).

   **STEP 3:** \( LM_t = TR^2 \delta d P^2(q) \), under \( H_0: \) No CH.

2) Specification Tests based on standardized errors

   - \( H_0: \) Chosen model specification is correctly specified
   - Let \( e_t = \frac{\hat{u}_t}{\sqrt{\hat{h}_t}} \) (standardized residuals).
   - If your choice of \( h_t \) is correctly specified, then \( e_t \) should be roughly iid.
• Testing autocorrelation of $e_t$ or $e_t^2$:

$$\frac{E_{t'1}^{T\&j} (e_t \bar{e})(e_{t+j} \bar{e})/T}{E_{t'1}^T (e_t \bar{e})/T} = r_j/r_o,$$

where $r_j = \frac{E_{t'1}^{T\&j} (e_t \bar{e})(e_{t+j} \bar{e})/T}{E_{t'1}^T (e_t \bar{e})/T}$ and $r_o = \frac{E_{t'1}^{T\&j} (e_t \bar{e})^2/T}{E_{t'1}^T (e_t \bar{e})^2/T}$.

• PAC (partial AC):
  • Regress $e_t$ on one, $e_{t-1}$, $e_{t-2}$, ..., $e_{t-j}$.
  • PAC of $e_t$ and $e_{t+j}$
    = coefficient of $e_{t+j}$ in this regression of $e_t$.
  • Under the hypothesis that $e_t$ and $e_{t+j}$ are uncorrelated,
    \[ \sqrt{T} \cdot \text{PAC} \cdot N(0,1) \]

• Ljung-Box Q-statistic:

$$Q_{LB} = T(T+2) \frac{\sum_{j=1}^{P} r_j^2/r_o^2}{T! j}.$$  

6d $P^2(p)$, under $H_0$: no autocorrelation in $e_t$.

• Normality test (Jarque-Bera):

See Greene.
Note: (Ahn's worry)

- Is $Q_{LB}$ a relevant test?
- In fact, there is no reason to believe that $Q_{LB}$ is $P^2(p)$.
- Bollerslev and Mikkelsen's ad hoc solution:
  
  
  - Use $P^2(p - \# \text{ of ARCH and GARCH parameters})$.
  - Not theoretically relevant, but it works!!!
ARCH(q) MODEL [Engle, 1982, ECON]

(1) Specification:

- \( h_t = \sigma^2 + \sigma_1 u_{t-1}^2 + \ldots + \sigma_q u_{t-q}^2. \)
- If (i) \( \sigma_j > 0; \) (ii) \( \sigma_1 + \ldots + \sigma_q < 1, \) the model is stationary
  [Sufficient, but not necessary]
- \( \Sigma^2 / \text{var}(u_t) = \sigma^2 / (1 - \sigma_1 - \ldots - \sigma_q) \) under (i) and (ii).
  Even if \( \sigma_1 + \ldots + \sigma_q = 1, \) the model could be still stationary
  and ergodic (so MLE is consistent and efficient).
- For MLE or QMLE,
  \[ h_t = \sigma^2 + \sigma_1 (y_{t-1} - x_{t-1})^2 + \ldots + \sigma_q (y_{t-q} - x_{t-q})^2. \]
  \[ 2 = (\sum T, \sigma_1, \ldots, \sigma_q) N \]
  Set \( u_0^2 = \ldots = u_{1-q}^2 = T^{-1} E_t (y_t - x_t)^2. \)
- Can introduce some regressors in \( h_t; \) e.g.,
  \( h_t = \sigma^2 + \sigma_1 u_{t-1}^2 + \ldots + \sigma_q u_{t-q}^2 + d_t, \) where \( d_t \) is a dummy
  variable for Mondays.

(2) Alternative representation

- Let \( w_t = u_t^2 - h_t; \)
  \( u_t^2 = h_t + w_t = \sigma^2 + \sigma_1 u_{t-1}^2 + \ldots + \sigma_q u_{t-q}^2 + w_t. \)
  \( Y u_t^2 - \text{AR}(q). \)
(3) Forecast of volatility

1) Let \( u_{t|\delta t}^2 = E(u_{t|\delta t}^2, u_{t|\delta+1}^2, \ldots) \): Optimal Predictor of future \( h_t \).

\[
\begin{align*}
2) \quad u_{t|\delta t}^2 &= T + u_{1|\delta t}^2 + u_{2|\delta t}^2 + \ldots + u_{q|\delta t}^2; \\
\end{align*}
\]

\[
\begin{align*}
22) \quad u_{t|\delta+1 t}^2 &= T + u_{1|\delta+1 t}^2 + u_{2|\delta+1 t}^2 + \ldots + u_{q|\delta+1 t}^2; \\
\end{align*}
\]

\[
\begin{align*}
222) \quad u_{t|\delta+2 t}^2 &= T + u_{1|\delta+2 t}^2 + u_{2|\delta+2 t}^2 + u_{3|\delta+2 t}^2 + u_{q|\delta+2 t}^2; \\
\end{align*}
\]

\[
\begin{align*}
\vdots \\
\end{align*}
\]

\[
\begin{align*}
2222) \quad u_{t|\delta+q t}^2 &= T + u_{1|\delta+q t}^2 + u_{2|\delta+q t}^2 + \ldots + u_{q|\delta+q t}^2. \\
\end{align*}
\]

\[
\begin{align*}
u_{t|\delta t}^2 &= \frac{p}{T/(1 ! u_{1 t}^2 ! \ldots ! u_{q t}^2)}, \text{ as } s 6 4.
\end{align*}
\]

ARCH-11
[Called GARCH(p,q)]

(1) Motivation

• q is usually too large when ARCH(q) is used.
• Need a parsimonious model.

(2) Specification

\[ h_t = \tau + \sum_{i=1}^{p} \alpha_i h_{t-i} + \sum_{j=1}^{q} \beta_j u_{t-j}^2 + \epsilon_i u_{t-1}^2 + \ldots + \epsilon_q u_{t-q}^2. \]

• For MLE or QMLE,

\[ 2 = (\sum_{i=1}^{\tau} \epsilon_i, \sum_{i=1}^{\tau} \epsilon_i u_i, \ldots, \sum_{i=1}^{\tau} \epsilon_i u_i^n) N \]

Set \( h_0 = \ldots = h_{1-p} = u_0^2 = \ldots = u_{1-q}^2 = T^{-1} E_t((y_t-x_t)^2) \).

• Let \( r = \max\{p,q\} \); and set \( \epsilon_{q+1} = \ldots = \epsilon_r = 0 \) or \( \epsilon_{p+1} = \ldots = \epsilon_r = 0 \).

\[ h_t = \tau + \sum_{i=1}^{r} \alpha_i h_{t-i} + \sum_{j=1}^{q} \beta_j u_{t-j}^2 + \epsilon_i u_{t-1}^2 + \ldots + \epsilon_r u_{t-r}^2. \]

• If (i) \( T, \epsilon_j, \epsilon_j > 0 \) and (ii) \( E_{j=1}^{r} (\epsilon_j + \epsilon_j) < 1 \), the model is stationary. [Sufficient, but not necessary.]

• \( \sigma^2 = \text{var}(u_t^2) = T/\left[1-(\epsilon_1 + \epsilon_1)\ldots-(\epsilon_r + \epsilon_r)\right] \) (if (i) and (ii) hold).
(3) GARCH(1,1)

1) \( h_t = \sigma_t^2 \)

\[
\begin{align*}
\sigma_t^2 &= \sigma_{t-1}^2 + \epsilon_t^2 \\epsilon_{t-1}^2 \\
&= (1 + \epsilon_t^2) T + \epsilon_t^2 \sigma_{t-1} + \epsilon_t^2 \epsilon_{t-1}^2 \\
&= T / (1 - \epsilon_t^2) + \epsilon_t^2 E_{\epsilon_t^2} \sigma_{t-1}^2. \quad \text{[ARCH(4)]}
\end{align*}
\]

- Let \( w_t = u_t^2 - \sigma_t^2 \): unforecastable volatility.

\[
\begin{align*}
\epsilon_t^2 &= (u_t^2 - \sigma_t^2) + \sigma_t^2 = w_t + T + \sigma_t^2 + \epsilon_t^2 \sigma_{t-1}^2 \\
&= w_t + T + \epsilon_t^2 \sigma_{t-1}^2 - \epsilon_t^2 \epsilon_{t-1}^2 \\
&= w_t + T + \epsilon_t^2 \sigma_{t-1}^2 - w_{t-1} \quad \text{[Like ARMA(1,1)]}
\end{align*}
\]

- \( \sigma_t = \sigma_{t-1} + \epsilon_t^2 \epsilon_{t-1}^2 \quad \text{[ARCH(4)]} \)

\[
\begin{align*}
\sigma_t &= T + \epsilon_t^2 \sigma_{t-1} + \epsilon_t^2 \epsilon_{t-1}^2 \\
&= T[1 + E_{\epsilon_t^2} \epsilon_{t-1}^2 (\epsilon_t^2 \epsilon_{t-1}^2)]
\end{align*}
\]

Theorem: (Nelson, 1990, ECON)

GARCH(1,1) is stationary and ergodic iff \( \frac{\lambda}{1 - \lambda} < 4 \). And,

\[
\frac{T}{(1 - \lambda)} \# F^2 < 4.
\]

Implication:

- Even if \( \lambda = 1 \), GARCH(1,1) can be stationary.
- MLE is still consistent and efficient.
2) Forecasting:
\[ u_t^2 = T + (*+)u_{t-1}^2 + w_t - *w_{t-1}. \]

i) Let \( u_{t+s}^2 / E(u_{t+s}^2, u_{t+1}, ..., w_t^2, w_{t+1}, ...) \):

Optimal Predictor of \( h_{t+s} \) at time \( t \).
\[ w_{t+s} / E(w_{t+s}^2, ..., w_t, ...) = 0. \]

ii) \( u_{t+s}^2 = T + (*+)u_t^2 + w_{t+1} - *w_t = T + (*+)u_t^2 - *w_t; \)

\[ u_{t+s}^2 = T + (*+)u_t^2; \]

\[ u_{t+s}^2 = T + (*+)u_t^2; \]

\[ \vdots \]

\[ u_{t+s}^2 \overset{p}{\thicksim} T/(1 - *), \text{ as } s \geq 4. \]

3) Regarding \( H_0: * = 0. \)

- Not possible to test for this hypothesis by Wald tests. Under \( H_0 \),
the model is not identified. [Under GARCH(1,0) with the
stationarity conditions (i) and (ii), \( h_t = h_{t-1} = ... = F^2 = T/(1-*) \).]
- See Andrews and Ploberger (1994, ECON) and B. Hansen
(1996, ECON).

ARCH-14
(4) \textbf{GARCH(p,q)}

\[ h_t = T + h_{t-1} + \ldots + h_{t-p} + u_{t-1}^2 + \ldots + u_{t-q}^2. \]

- Let \( r = \max\{p,q\} \); and set \( h_{p+1} = \ldots = h_r = 0 \) or \( h_{q+1} = \ldots = h_r = 0 \).
- \( h_t = T + h_{t-1} + \ldots + h_{t-r} + u_{t-1}^2 + \ldots + u_{t-r}^2. \)
- \( u_t^2 = (u_t^2 - h_t) + h_t \) (let \( w_t = u_t^2 - h_t \))

\[ w_t + [T + h_{t-1} + \ldots + h_{t-r} + u_{t-1}^2 + \ldots + u_{t-r}^2] \]

\[ = w_t + T + (h_{t-1} + \ldots + h_{t-r})u_{t-1}^2 + \ldots + (h_{t-r} + \ldots + h_{t-r})u_{t-r}^2 \]

\[ - h_{t-1} - \ldots - h_{t-r} \]

\[ = T + (h_{t-1} + \ldots + h_{t-r})u_{t-1}^2 + (h_{t-r} + \ldots + h_{t-r})u_{t-r}^2 \]

\[ + w_t - h_{t-1} - \ldots - w_{t-r}. \] (Like ARMA(r,p))
[5] INTEGRATED GARCH(p,q) [Bollerslev and Engle, ER, 1986]

[Called IGARCH]

- \( h_t = T_1 + h_{t-1} + \ldots + h_{t-p} + u^2_{t-1} + u^2_{t-2} + \ldots + u^2_{t-q} \),
  
  with \( \sum_j E_{j}^{*} \sum_{j} E_{j}'' = 1. \)

- Looks like nonstationary. But it could be stationary.

- Can test IGARCH(1,1) using a Wald statistic.

  [We conjecture that Wald tests can be used for IGARCH(p,q).
  But there is no formal proof.]

- \( u^2_{t=1} \) as \( \alpha_4 \).

- QMLE for IGARCH(1,1) is consistent and asymptotically normal under certain conditions (See Lumsdaine, 1996, ECON).
Exponential GARCH [Nelson, 1991, ECONOMETRICA]

[Called EGARCH]

(1) Motivation:
GARCH models do not capture leverage effects.

(2) Basic Model

\[ \ln(h_t) = T + \gamma \ln(h_{t-1}) + \ldots + \gamma_p \ln(h_{t-p}) + \gamma_1 \eta_{t-1} + \gamma_2 \eta_{t-2} + \ldots + \gamma_q \eta_{t-q}, \]

where \( \eta_t = |v_t| - E|v_t| + (v_t \text{ and } v_t \text{ follows generalized error distribution (p. 668, Hamilton, or Nelson, 1990)}) \]

- \( E|v_t| = \sqrt{2/B}. \)
- \( T, \gamma \)’s and \( \gamma \)’s do not have to be positive.
- If \( \gamma = 0, \eta_t = |v_t| - E|v_t| \).
  - Positive and negative \( v_t \) have the symmetric effects on \( h_t \).
- If \(-1 < \gamma < 0:\)
  - If \( v_t > 0, \eta_t = (1+\gamma)v_t - E|v_t| = (0 < 1+\gamma) v_t - E|v_t| \)
  - If \( v_t < 0, \eta_t = (-1+\gamma)v_t - E|v_t| = (-2 < -1+\gamma) v_t - E|v_t| \)
  - Negative \( v_t \) has greater effects than positive \( v_t \).
- If \( \gamma < -1:\)
  - If \( v_t > 0, \eta_t = (1+\gamma)v_t - E|v_t| = (1+\gamma) v_t - E|v_t| \)
  - If \( v_t < 0, \eta_t = (-1+\gamma)v_t - E|v_t| = (-1+\gamma) v_t - E|v_t| \)
  - Positive \( v_t \) reduces \( h_t \).
(3) Conditions for stationarity and ergodicity:
\[ E_j^\prime j < 4. \]

(4) Example:

- \( r_t = \) (daily return on stock) - (daily interest rate on treasury bills)
  - \( r_t = \$_1 + \$_2 r_{t-1} + 8 h_t + u_t \);
  - \( u_t = \sqrt{h_t} v_t \);

- \( \ln(h_t) - \approx = *_1 (\ln(h_{t-1}) - t-1) + *_2 (\ln(h_{t-2}) - t-2) + \approx 0_{t-1} + \approx 2 0_{t-2} \).

- \( \approx = . + \ln(1+DN_t), N_t = \# \) of nontrading days bet/w (t-1) and t.
(1) Symmetric Component GARCH (1,1).

- Reconsider GARCH(1,1):
  - 1) \( h_t = \bar{T} + \ast h_{t-1} + "u_{t-1}^2. \)
  - 2) \( h_t = \bar{T} \% * (h_{t \&} \& \bar{T}) \% " (u_{t \&}^2 \& \bar{T}), \) where \( \bar{T} = \bar{T} (1 \& " \& \ast). \)
- The equation 2) implies that in GARCH(1,1) \( h_t \) fluctuates around \( \bar{T} \) (mean reversion to \( \bar{T} \)).
- The volatility measure \( h_t \) could have transitory (short-run) and permanent (long-run) components.
- In GARCH(1,1), \( (h_t \& \bar{T}) \) is the transitory component and \( \bar{T} \) is the permanent component.

- Symmetric Component GARCH (1,1)
  - Wish to allow the permanent component fluctuate over time.
  - \( h_t \% * (h_{t \&} \& q_{t \&}) \% " (u_{t \&}^2 \& q_{t \&}); \)
  - \( q_t \% \bar{D}_t (q_{t \&} \& \bar{T}) \% N(u_{t \&}^2 \& h_{t \&}). \)
  - The transitory component \( (h_t \& q_t) \) fluctuates around zero and the long-run component fluctuates around \( \bar{T} \).
• The above two equations imply:

\[ h_t = T(\%^1 h_{t\&l} \%^2 h_{t\&l} \%'' u_{t\&l}^2 \%'' u_{t\&l}^2), \]

where

\[ T(\%^1 (1\&\%')(1\&D)T; \%^1 \%D \& N; \%^2 \& (\%D\&(\%''N); \''^1 \%N; \''^2 \& (\''D\%\%''N). \]

• Thus, component GARCH(1,1) can be viewed as a restricted GARCH(2,2).

(2) Asymmetric Component GARCH(1,1)

• \[ h_t = q_t \%^1 (h_{t\&l} \& q_{t\&l}) \%'' (u_{t\&l}^2 \& q_{t\&l}^2) \%'' (u_{t\&l}^2 \& q_{t\&l}^2) d_{t\&l}, \]

where \( d_t = 1 \) iff \( u_{t-1} < 0. \)
[7] **THRESHOLD ARCH**

[Glosten, Jagannathan and Runkle, JF, 1994; Called TARCH]

\[ h_t = \theta + *h_{t-1} + u_{t-1}^2 + (u_{t-1}^2)1(u_{t-1} < 0), \]

where \( \theta, *, " \) and \( ( > 0. \)

[8] **GARCH IN MEAN**

[Engle, Lilien and Robins, 1987, ECON; Called GARCH-M]

\[ y_t = x_t + 8h_t + u_t \text{ or } y_t = x_t + 8\sqrt{h_t} + u_t. \]

[9] **MULTIVARIATE GARCH**

(See Hamilton.)
(1) Estimation

STEP 1: Push Objects/New Object.
STEP 2: Choose Equation. Push OK button. Then, you are in Equation Specification box.
        Go to Equation Setting, and Choose ARCH.
STEP 3: In Equation Specification box, type:

        dy100 c

STEP 4: Go to Equation Setting and type:

        2 1001

STEP 5: Click the option buttom. Increase the convergence rate (0.0001) and increase maxit to 1000. Choose algorithm and Heteroskedasticity-Robust Covariance matrix (for QMLE). Once you have chosen appropriate options, click the ok buttom.

STEP 6: Choose a specification and run the program.
(2) GARCH(1,2)

\[ y_i = \sigma + u_i; \quad h_i = \tau + \alpha_1 h_{i-1} + \beta_1 u_{i-1}^2 + \beta_2 u_{i-2}^2. \]

Dependent Variable: DY100  
Method: ML - ARCH  
Sample(adjusted): 2 1001  
Included observations: 1000 after adjusting endpoints  
Convergence achieved after 42 iterations  
Bollerslev-Wooldrige robust standard errors & covariance

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<th>Prob.</th>
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Variance Equation

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R-squared -0.000051  
Adjusted R-squared -0.004071  
S.E. of regression 0.757033  
Sum squared resid 570.2337  
Log likelihood -1097.888  
Mean dependent var 0.044314  
S.D. dependent var 0.755497  
Akaike info criterion 2.205777  
Schwarz criterion 2.230316  
Durbin-Watson stat 2.137029
(3) TARCH(1,2)

\[ y_t = \delta + u_t; \ h_t = \alpha + \beta_1 h_{t-1} + \alpha_2 u_{t-1}^2 + \delta (u_{t-1} < 0) + \alpha_2 u_{t-1}^2. \]

Dependent Variable: DY100
Method: ML - ARCH
Sample(adjusted): 2 1001
Included observations: 1000 after adjusting endpoints
Convergence achieved after 28 iterations
Bollerslev-Wooldrige robust standard errors & covariance

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
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<tbody>
<tr>
<td>C</td>
<td>0.047872</td>
<td>0.021862</td>
<td>2.189744</td>
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Variance Equation

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
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<tbody>
<tr>
<td>C</td>
<td>0.007089</td>
<td>0.008815</td>
<td>0.804239</td>
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<tr>
<td>ARCH(1)</td>
<td>0.028297</td>
<td>0.034449</td>
<td>0.821422</td>
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<tr>
<td>(RESID&lt;0)*ARCH(1)</td>
<td>0.004401</td>
<td>0.017584</td>
<td>0.250280</td>
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<tr>
<td>GARCH(1)</td>
<td>1.312629</td>
<td>0.852429</td>
<td>1.539868</td>
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<tr>
<td>GARCH(2)</td>
<td>-0.356171</td>
<td>0.803179</td>
<td>-0.443452</td>
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R-squared -0.000022 Mean dependent var 0.044314
Adjusted R-squared -0.005053 S.D. dependent var 0.755497
S.E. of regression 0.757403 Akaike info criterion 2.207258
Sum squared resid 570.2172 Schwarz criterion 2.236705
Log likelihood -1097.629 Durbin-Watson stat 2.137091

ARCH-24
(4) EGRACH(1,2)

\[ y_t = \$ + u_t; \]
\[ \ln(h_t) = T + \ln(h_{t-1}) + \ln(v_{t-1}) + \ln(v_{t-2}) \]
\[ \ln(h_t) = T + \ln(h_{t-1}) + \ln(v_{t-1}) + \ln(v_{t-2}) \]

Dependent Variable: DY100
Method: ML - ARCH
Sample(adjusted): 2 1001
Included observations: 1000 after adjusting endpoints
Convergence achieved after 47 iterations
Bollerslev-Wooldrige robust standard errors & covariance

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<th>Coefficient</th>
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<td>0.022159</td>
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Variance Equation

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
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<tbody>
<tr>
<td>C</td>
<td>-0.040548</td>
<td>0.035358</td>
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<tr>
<td>RES/SQR<a href="1">GARCH</a></td>
<td>0.045956</td>
<td>0.039668</td>
<td>1.158513</td>
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<td>RES/SQR<a href="2">GARCH</a></td>
<td>0.004615</td>
<td>0.009793</td>
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<td>EGARCH(1)</td>
<td>1.563571</td>
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<td>EGARCH(2)</td>
<td>-0.571963</td>
<td>0.375954</td>
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R-squared -0.000010 Mean dependent var 0.044314
Adjusted R-squared -0.005040 S.D. dependent var 0.755497
S.E. of regression 0.757398 Akaike info criterion 2.209685
Sum squared resid 570.2104 Schwarz criterion 2.239132
Log likelihood -1098.843 Durbin-Watson stat 2.137116
(5) GARCH(1,2)-M with Variance
\[ y_t = \delta + \beta h_t + u_t; \]
\[ h_t = \eta + \gamma_1 h_{t-1} + \gamma_2 u_{t-1}^2 + \gamma_3 u_{t-2}^2. \]

Dependent Variable: DY100
Method: ML - ARCH
Sample(adjusted): 2 1001
Included observations: 1000 after adjusting endpoints
Convergence achieved after 46 iterations
Bollerslev-Wooldrige robust standard errors & covariance

<table>
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<td>C</td>
<td>0.155385</td>
<td>0.064306</td>
<td>2.416321</td>
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Variance Equation

<table>
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<td>-0.434450</td>
<td>0.470056</td>
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R-squared 0.005024  Mean dependent var 0.044314
Adjusted R-squared 0.000019  S.D. dependent var 0.755497
S.E. of regression 0.755490  Akaike info criterion 2.207335
Sum squared resid 567.3398  Schwarz criterion 2.236781
Log likelihood -1097.667  F-statistic 1.003836
Durbin-Watson stat 2.146356  Prob(F-statistic) 0.414162
(6) Graph for $\sqrt{h_t}$.

Go to view/conditional SD graph.

(7) Forecast: from 1002 to 1101
(8) Wald Test:

If you wish to test multiple restrictions on parameters, go to view/coefficient tests.

(9) Specification tests based on standardized residuals

1) For Specification tests, go to view/residual tests/correlogram-Q statistics.

[About v. If your model is correctly specified, then the standardized residuals should be serially uncorrelated.]

Date: 04/03/00   Time: 16:49
Sample: 2 1001
Included observations: 1000

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</table>
2) Go to **view/residual tests/correlogram squared residuals**.

   [About \( v_i^2 \). If your model is correctly specified, then squared standardized residuals should be serially uncorrelated.]

Date: 04/03/00   Time: 16:51  
Sample: 2 1001  
Included observations: 1000

<table>
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<tr>
<th>Autocorrelation</th>
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</tr>
</tbody>
</table>
3) Go to **view/residual tests/histogram normality test.**

4) Go to **view/residual tests/ARCH LM tests.** Set Lag = 4.

   [If you model is correctly specified, then standardized errors should not include any ARCH effects.]

**ARCH Test:**

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Probability</th>
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</thead>
<tbody>
<tr>
<td>F-statistic</td>
<td>0.668245</td>
<td>0.614110</td>
</tr>
<tr>
<td>Obs*R-squared</td>
<td>2.679241</td>
<td>0.612852</td>
</tr>
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</table>

ARCH-30