1. For each of the following parametric families of vector fields:

   i show that it is conservative;

   ii calculate its potential;

   iii calculate its equilibrium set and bifurcation set;

   iv calculate its catastrophe set;

   v classify each point of the catastrophe set in accordance with Thom’s list of seven elementary catastrophes.

(a) 
\[ F(x, y) = \left( \begin{array}{c} 3x^2 - 4y + 2u_1x + u_2 \\ -4x + 8y \end{array} \right) \]

(b) 
\[ F(x, y) = \left( \begin{array}{c} u_1 \cdot x + u_2 \cdot y \\ u_2 \cdot x + u_1 \cdot y \end{array} \right) \]

(c) 
\[ F(x, y) = \left( \begin{array}{c} x^2 + u_1 \cdot y + u_2 \\ y^2 + u_1 \cdot x + u_3 \end{array} \right) \]
(d) \[ F(x, y) = \left( \begin{array}{c} 2 \cdot x \cdot y + u_1 \cdot x + u_3 \\ y^3 + x^2 + u_2 \cdot y + u_4 \end{array} \right) \]

2. Find all fixed points and classify their stability for all values of the parameter \( \mu \),
\[ x_{n+1} = \mu + x_n^2 \]

3. Consider
\[ x_{n+1} = \mu + x_n^2 \]
For which values of \( \mu \) there is a stable 2-cycle \( x_0, x_1 \)?
\[ x_1 = \mu + x_0^2 \]
\[ x_0 = \mu + x_1^2 \]
Calculate the values \( x_0, x_1 \) for 2-cycle. When is the 2-cycle superstable?

4. Consider the logistic equation
\[ x_{n+1} = r \cdot x_n (1 - x_n) \]
Find all fixed points and analyze their stability versus \( r \).
(a) For which values of \( r \) there is a stable 2-cycle
\[ x_1 = r \cdot x_0 (1 - x_0) \]
\[ x_0 = r \cdot x_1 (1 - x_1) \]
(b) Show that \(x_0, x_1\) are the roots of the quadratic equation

\[x^2 - \left(1 + \frac{1}{r}\right) \cdot x + \frac{1}{r^2} + \frac{1}{r} = 0.\]

5. Consider

\[x_{n+1} = r \cdot x_n - x_n^3\]

(a) For which values of \(r\) there exist fixed points? Classify their stability.

(b) In order to calculate the 2–cycle

\[
\begin{align*}
x_1 &= r \cdot x_0 - x_0^3 \\
x_0 &= r \cdot x_1 - x_1^3
\end{align*}
\]

show that \(x_0, x_1\) are the roots of

\[x \cdot (x^2 - r + 1)(x^2 - r - 1)(x^4 - rx^2 + 1) = 0\]

(c) Determine the stability of the 2–cycle as a function of \(r\).