Each problem is worth 5 pts of extra credit.

1 Problem

(a) Find an equation of the hyper-plane in \( \mathbb{R}^5 \) that passes through the following five points

\[
(1, 0, 1, 0, -1) \\
(1, 1, 1, 1, 1) \\
(1, 2, 1, -2, -1) \\
(0, 1, 1, 0, 0) \\
(1, 0, 3, 1, 0)
\]

(b) Calculate the distance from \((1, -2, 3, 0, 5)\) to the plane defined in (a).

2 Problem

Calculate the volume of the parallelepiped in \( \mathbb{R}^4 \) spanned by

\[
(1, 0, 1, 0, ) \\
(1, 1, 1, 1, ) \\
(1, 2, 1, -2, )
\]

3 Problem

Find the canonical form of the second order curve

\[
2x^2 + xy + 2y^2 + x + y = 0 \tag{1}
\]

Write the respective change of coordinates that brings (1) to its canonical form.

Sketch the curve in the original system of coordinates \((x, y)\).

4 Problem

Find the canonical form of the second order surface in \( \mathbb{R}^3 \)

\[
2x_1^2 + 2x_2^2 + x_3^2 + x_1 \cdot x_2 + 2x_2 \cdot x_3 + x_1 + x_2 + x_3 = 0 \tag{2}
\]

Write the respective change of coordinates that brings (2) into its canonical form.

Sketch the surface in the original system of coordinates \((x_1, x_2, x_3)\).

5 Problem

Prove that

\[
\frac{x^n + y^n}{2} \geq \left( \frac{x + y}{2} \right)^n
\]

for \( n \geq 1 \) and \( x \geq 0, \ y \geq 0 \).
6 Problem
Find all extrema and classify them as local maxima, minima, saddles or unknown.

\[ z = x^2y^3(6 - x - y) \]

7 Problem
Find the volume of the body bounded from above by

\[ z = \min \{ xy, 1 - x - y \} \]

and from below by

\[ z = 0. \]

Assume that \( x \geq 0 \) and \( y \geq 0 \).

8 Problem
Find the area of the surface defined by

\[ (x^2 + y^2)^{\frac{1}{2}} + z = 1 \text{ and } z \geq 0. \]

9 Problem
Find the volume of the body bounded by

\[ \left( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \right)^2 = \frac{x}{h} \]

10 Problem
Find the circulation

\[ \oint_C ydx + zdy + xdz \]

where the curve \( C \) is the intersection between \( x^2 + y^2 + z^2 = a^2 \) and \( x + y + z = 0 \). \( C \) is oriented counterclockwise if looking from the tip of \( z \)-axes.

11 Problem
Establish whether the vector field in \( \mathbb{R}^4 \) is conservative and if it is then find its potential.

\[ F(x_1, x_2, x_3, x_4) = \begin{pmatrix}
  x_1 + x_2x_3x_4 + x_2x_3 + x_2,
  x_2 + x_1x_3x_4 + x_1x_3 + x_1,
  x_3 + x_1x_2x_4 + x_1x_2,
  x_4 + x_1x_2x_4
\end{pmatrix} \]