Optimization of Channel Sensing Time and Order for Cognitive Radios

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***Abstract*—In this paper we consider a single cognitive radio seeking a transmission opportunity by sequentially sensing a number of statistically independent primary channels. We study the joint optimization of the time spent to sense a channel, the decision threshold to determine whether the channel is free or busy, together with the order with which the channels are sensed. The sensing time and decision threshold are assumed to be the same for all channels. The design objective is to maximize the expected secondary throughput taking sensing errors into account and penalizing for collisions that may disrupt the primary transmission. Motivated by the computational complexity of the problem, we propose suboptimal solutions that significantly reduce the complexity without sacrificing accuracy. Our results reveal a fundamental trade-off between minimizing the probability of collision with the primary user via reducing the sensing errors, which favors a longer sensing time, and increasing the secondary user's throughput, which favors shorter sensing time. The suboptimal approach, for plausible simulation scenarios, is found to reduce the computational complexity by more than 89%, while maintaining a near-optimal throughput within 0.28% of the optimal performance.**[1]

I. Introduction

The rapid growth in wireless communications and networking, evidenced by the wide proliferation of mobile devices, variety of standards and new use cases, gives rise to the problem of spectrum scarcity. However, recent spectrum measurement studies [1] have revealed that the assigned spectrum is used sporadically and, hence, remains largely under-utilized. This, in turn, has led to the concept of Dynamic Spectrum Access (DSA), or alternatively cognitive radios (CRs), which received recent attention in an attempt to remedy this problem. The notion of DSA hinges on the assumption that unlicensed users (secondary users) could access the spectrum, opportunistically, at times and locations where licensed users (primary users) are inactive.

Spectrum sensing is a major building block in such systems in order to enable the secondary users to detect the inactivity of primary users on a particular frequency channel (i.e., to discover spectrum holes). In this paper, we consider a secondary terminal that senses a specific number of primary channels, one at a time, to find a transmission opportunity as quickly and reliably as possible. The secondary transmitter relies on conventional energy detection to determine the occupancy state of a certain primary channel. Accordingly, the cognitive transmitter decides, jointly, the optimal order at which channels has to be sensed, the optimal sensing time, and the optimal sensing energy threshold based on which the channel is determined to be busy or free.

The problem of optimal channel selection in cognitive radio networks has received recent attention in the literature, e.g., [2]–[6]. In [3], given different channel availability probabilities and sensing times, the optimal sensing sequence is to sort the channels in ascending order of the ratio of the channel sensing time to the channel availability probability. This result is for the homogeneous channel capacity case, where all the channels have the exact same capacity. Note that in [3], the sensing times are fixed for each channel, i.e., they are not considered as optimization variables. In addition, the channel capacities are not considered to be time-varying. For heterogeneous channel capacities, the problem becomes NP-hard and is solved via exhaustive search.

In [4], the authors investigate the optimal channel selection problem assuming error-free sensing. Given a set of channels and their probing costs, the sender's objective is to choose the strategy that maximizes transmission reward minus the sensing costs. The strategies available to the cognitive terminal are either to transmit on a free channel, or to transmit on a previously found free channel, or to randomly select an unprobed channel and use it for transmission. The authors of [5] build upon the work in [4] by incorporating the possibility of sensing errors. Although the optimization of the sensing time is included in [5], the order by which the channels are sensed is random. When a cognitive radio wishes to transmit, it starts scanning the channels in a sequential manner. It randomly picks a channel and samples it for some time, and then decides whether the channel is idle or busy. If it is busy, another channel is randomly selected, otherwise the channel is probed to determine its quality. Depending on this quality, the channel is either used for transmission or is skipped in search for a better opportunity.

Perhaps the closest to our work is [2], where a dynamic programming approach is adopted for finding the optimal order due to the sequential decision nature of the problem at hand. In [2], the time incurred for sensing a primary channel is assumed given and fixed. Although the authors tackle the issue of imperfect sensing via incorporating the probabilities of false alarm and misdetection, their formulation does not

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link the probabilities of false alarm and misdetection to the sensing time, despite the fact that both probabilities are closely linked to the sensing time variable. Moreover, the model therein does not incorporate the sensing detection threshold or put a collision penalty on the secondary user in case of a misdetection. Instead, the collision probability is simply kept below an arbitrary value.

Our contribution in this paper is three-fold: i) joint optimization of sensing variables with channel sensing order, ii) extending the problem formulation in [2] and modifying the design objective function to explicitly account for the negative impacts of sensing errors on both the primary and secondary links, and iii) providing suboptimal, yet efficient, solutions that yield accurate results with a slight degradation in performance.

The rest of this paper is organized as follows. Section II introduces the underlying assumptions and system model. Afterwards, our problem formulation and solution approach are discussed in Section III. Simulation results and discussion are presented in Section IV. Finally, conclusions are drawn and potential directions for future work are pointed out in Section V.

II. System Model

We consider one secondary link with one transmitter-receiver pair where the secondary transmitter attempts to access one of M frequency channels of a primary network. The primary users' activity follows a time slotted structure. During a slot duration T, a particular channel is either used by the primary network or is vacant. The primary activity over a channel is assumed to be uncorrelated with the activity over the other channels. Moreover, the activity over a certain channel is independent from one slot to another. We denote the probability of channel i being idle as θ_i. The values of θ_i $(1 \leq i \leq M)$ are assumed to be known a priori to the secondary user (SU). In this work, we assume that the SU employs an energy detector in order to sense the channels and determine their state (free or busy). This is implemented by averaging the received energy over a number of consecutive samples and comparing the average to a pre-specified energy threshold (E_T). Define N as the number of samples taken by this energy detector to sense one channel. If the energy detector's sampling frequency is $f_{\rm s}$, thus the time taken to sense a channel is $\tau = N/f_{\rm s}$. Suppose that the SU senses the channels according to the sequence $S = (s_1, ...s_j, ...s_M)$ which is a permutation of the set $\{1, 2....M\}$. If channel s_j is sensed to be busy, it is skipped to sense channel s_{j+1}, otherwise, transmission begins and the SU's throughput would be proportional to $c_j = 1 - j\tau/T$. This factor reflects the penalty for increasing τ (or N) to get a reliable estimate of channel occupancy at the expense of the time left for transmission. Decreasing τ, on the other hand, may increase secondary throughput but with an increased risk of colliding with the primary or missing spectrum opportunities due to reduced detection reliability. This fundamental trade-off between reliability and throughput is incorporated in our joint optimization problem to obtain the optimal sensing order and sensing duration.

We assume an in-phase/quadrature (I/Q) receiver [7] where the noise variance is V. The PDF of $\mathcal{E}_k$, defined as the energy of the k'th sample, is exponential when the channel is vacant and is given by,

$$f_{\circ}^{1}(\mathcal{E}_k) = \frac{1}{V} \exp\left(-\frac{\mathcal{E}_k}{V}\right) \tag{1}$$

where the subscript denotes a vacant channel while the superscript denotes one sample. If the energies of N samples are averaged to obtain the sensing metric $\mathcal{E} = \frac{1}{N}\sum_{k=1}^{N} \mathcal{E}_k$, then the PDF of $\mathcal{E}$ when the primary users are not using the channel is given by,

$$f_{\circ}^{N}(\mathcal{E}) = \left(\frac{N}{V}\right)^{N} \frac{\mathcal{E}^{N-1}}{(N-1)!} \exp\left(-\frac{N\mathcal{E}}{V}\right) \tag{2}$$

where the superscript represents averaging over N samples. In order to obtain the PDF of $\mathcal{E}_k$ when the channel is busy, we consider two models. The first model is a fixed sensing channel gain and, consequently, a fixed received energy ϵ at the secondary transmitter's spectrum sensor. In this case, the required PDF is given by the non-central chi-square distribution,

$$f_{1a}^{1}\left(\mathcal{E}_k\right) = \frac{1}{V} \exp\left(-\frac{\mathcal{E}_k + \epsilon}{V}\right) I_{\circ}\left(\frac{2}{V}\sqrt{\epsilon\mathcal{E}_k}\right) \tag{3}$$

where the subscript $1a$ denotes the fixed-gain model when the primary is on. Averaging over N samples, the PDF of $\mathcal{E}$ becomes,

$$f_{1a}^{N}\left(\mathcal{E}\right) = \frac{N}{V}\left(\frac{\mathcal{E}}{\epsilon}\right)^{\frac{N-1}{2}} \exp\left(\frac{-N(\mathcal{E}+\epsilon)}{V}\right) I_{N-1}\left(\frac{2N\sqrt{\epsilon\mathcal{E}}}{V}\right) \tag{4}$$

Our second model for the sensing channel between the primary transmitter and the secondary transmitter is a Rayleigh-fading channel. This means that ϵ is an exponential random variable. Hence, the PDF of $\mathcal{E}_k$ is (3) averaged over ϵ,

$$\begin{aligned} f_{1b}^{1}\left(\mathcal{E}_k\right) &= \int_0^{\infty} f_{1a}^{1}\left(\mathcal{E}_k\right) \frac{1}{\bar{\epsilon}} \exp\left(\frac{-\epsilon}{\bar{\epsilon}}\right) d\epsilon \\ &= \frac{1}{V+\bar{\epsilon}} \exp\left(-\frac{\mathcal{E}_k}{V+\bar{\epsilon}}\right) \end{aligned} \tag{5}$$

where $\bar{\epsilon}$ is the average energy received by the secondary user, and the subscript $1b$ denotes the Rayleigh channel model for the sensing channel when the primary is on. Averaging over N samples, the PDF of $\mathcal{E}$ becomes,

$$f_{1b}^{N}\left(\mathcal{E}\right) = \left(\frac{N}{V+\bar{\epsilon}}\right)^{N} \frac{\mathcal{E}^{N-1}}{(N-1)!} \exp\left(-\frac{N\mathcal{E}}{V+\bar{\epsilon}}\right) \tag{6}$$

As in any classical binary hypothesis testing problem, there are two types of errors: false alarm and misdetection. A false alarm occurs if a free channel is sensed to be busy, thereby causing the secondary terminal to lose a transmission opportunity. A misdetection, on the other hand, means that a busy channel is sensed to be free. This means that if the secondary terminal decides to transmit, the primary and secondary transmissions will collide resulting in a possible packet loss for both. Misdetection events harm the primary network and degrade its throughput. Given the energy threshold E_T used for detection, the false alarm and misdetection probabilities can be determined as follows,

$$P_{\text{FA}} = \int_{E_T}^{\infty} f_{\circ}^{N}\left(\mathcal{E}\right)\, d\mathcal{E} \tag{7}$$

$$P_{\text{MD}} = \int_{0}^{E_T} f_{1}^{N}\left(\mathcal{E}\right)\, d\mathcal{E} \tag{8}$$

where f_1^N is the PDF of received energy when the primary is on given either the fixed-gain or Rayleigh sensing channel models. Eq. (7) indicates that we can decrease P_{FA} by increasing E_T. This increases P_{MD}, however. Simplifying the expression of false alarm probability using (2) and (7), we obtain,

$$P_{\text{FA}} = 1 - \Gamma_{\text{inc}}\left(\frac{NE_T}{V}, N\right) \tag{9}$$

where Γ_{inc} is the incomplete gamma function [8]. For the misdetection probability given a fixed-gain sensing channel, we get

$$P_{\text{MD}_\text{a}} = Q_{\text{Mrq}}\left(\sqrt{\frac{2N\epsilon}{V}}, \sqrt{\frac{2NE_T}{V}}, N\right) \tag{10}$$

where Q_{Mrq} is the generalized Marcum Q function [9]. For the Rayleigh fading sensing channel model, the misdetection probability is given by

$$P_{\text{MD}_\text{b}} = \Gamma_{\text{inc}}\left(\frac{NE_T}{V+\bar{\epsilon}}, N\right) \tag{11}$$

As in [2], the secondary link is a Rayleigh fading channel[2], and the secondary user can use adaptive modulation. In other words, the secondary channel capacity is known to the secondary transmitter and is used to determine the transmission rate once the channel is detected to be free. In reality, a probing phase is required to get the secondary channel gains and capacities. This phase is repeated periodically with a rate that depends on the rate of channel temporal variation.

III. Optimal Sensing Time and Threshold Energy

A. Problem Formulation

We define U_i, similar to [2], as the expected reward if the secondary user proceeds to channel s_i. In contrast to [2], we incorporate sensing imperfections into the reward function and include a negative term to penalize concurrent transmission with the primary users over the same channel as it leads to collision and possible packet loss.

We now construct the expected reward, U_i. Towards this goal, we consider all four possible cases, namely channel s_i is free and sensed to be free, is free and sensed to be busy (false alarm event), is busy and sensed to be free (misdetection event) and, finally, is busy and sensed as such. Recall that channel s_i is free with probability θ_{s_i}.

1) The probability of the event that channel s_i is free and is sensed to be free is $\theta_{s_i}\left(1 - P_{\text{FA}}\right)$. When found vacant, the *effective* channel capacity, which accounts for the remaining slot time available for transmission, is $c_i \log\left(1+\gamma\right)$, where γ is the SNR of the secondary link. The cognitive terminal compares this quantity with U_{i+1}, which is the expected reward if the channel is skipped and the next channel (s_{i+1}) is sensed. If $c_i \log\left(1+\gamma\right)$ exceeds U_{i+1}, the secondary user transmits because the transmission reward outweighs the expected reward if the terminal refrains from transmission and proceeds to channel s_{i+1}.

For this case, we have the following term in the expression of U_i,

$$\theta_{s_i}\left(1 - P_{FA}\right) c_i . \int_{0}^{\infty} \log\left(1+\gamma\right) \frac{1}{\bar{\gamma}} \exp\left(-\gamma/\bar{\gamma}\right) \mathbb{1}\left(c_i \log\left(1+\gamma\right) > U_{i+1}\right)\, d\gamma \tag{12}$$

where the secondary link's SNR is exponentially distributed, due to the Rayleigh fading assumption, with a mean value of $\bar{\gamma}$. The indicator function $\mathbb{1}\left(c_i \log\left(1+\gamma\right) > U_{i+1}\right) = 1$ if $c_i \log\left(1+\gamma\right) > U_{i+1}$ and zero otherwise. To simplify our notation, we shall denote this indicator function $\beta_i(\gamma)$. Note the averaging over channel statistics and that, without loss of generality, we assume that all the channels have the same PDF and, hence, we drop the index i from γ.

On the other hand, if U_{i+1} is greater than the effective channel capacity, then the secondary transmitter skips the channel and proceeds to s_{i+1}. The expected reward associated with this case would be,

$$\theta_{s_i}\left(1 - P_{\text{FA}}\right) U_{i+1} \int_{0}^{\infty} \frac{1}{\bar{\gamma}} \exp\left(-\gamma/\bar{\gamma}\right) \left[1 - \beta_i\left(\gamma\right)\right]\, d\gamma \tag{13}$$

2) If the channel is free and a false alarm occurs, the secondary transmitter would skip channel s_i and sense channel s_{i+1}. The expected reward in this case would be $\theta_{s_i} P_{\text{FA}} U_{i+1}$.

3) If the channel is busy and is sensed as such, then we have a similar term but which involves the probability of correct detection. That is, the expected reward would be $\left(1-\theta_{s_i}\right)\left(1-P_{\text{MD}}\right)U_{i+1}$.

4) Finally, we consider the occurrence of misdetection. Knowing the channel capacity, and if it is less than U_{i+1}, the secondary terminal skips the channel and the expected reward would be the same as (13) but with P_{MD} in place of $\left(1 - P_{\text{FA}}\right)$, and $\left(1-\theta_{s_i}\right)$ instead of θ_{s_i}. If a misdetection occurs and the secondary terminal decides to transmit, a collision takes place. We assume here that a collision causes packet loss and, hence, the expected reward is zero. In addition, the primary packet is lost due to interference. To account for this, we include a negative term in the reward function, that is,

$$-\alpha\left(1-\theta_{s_i}\right) P_{\text{MD}} \int_{0}^{\infty} \frac{1}{\bar{\gamma}} \exp\left(-\gamma/\bar{\gamma}\right) \beta_i\left(\gamma\right)\, d\gamma \tag{14}$$

where α is a non-negative pre-specified penalty factor representing the primary's lost reward. A large value of α gives more protection to the primary network from interference and service interruption. Thus, it determines the quality of service of the primary network in our model.

[2]We assume that the SNR in this channel is statistically independant from that of the Sensing-Channel.

Given the above cases, the expected reward function at a given stage i is given by,

$$U_i =$$
$$\theta_{s_i}\left(1-P_{\mathrm{FA}}\right)c_i\int_0^\infty \log\left(1+\gamma\right)\frac{1}{\bar{\gamma}}\exp\left(-\gamma/\bar{\gamma}\right)\beta_i\left(\gamma\right)\,d\gamma$$
$$+\,\theta_{s_i}\left(1-P_{\mathrm{FA}}\right)U_{i+1}\int_0^\infty \frac{1}{\bar{\gamma}}\exp\left(-\gamma/\bar{\gamma}\right)\left[1-\beta_i\left(\gamma\right)\right]\,d\gamma$$
$$+\,\theta_{s_i}P_{\mathrm{FA}}U_{i+1}+\left(1-\theta_{s_i}\right)\left(1-P_{\mathrm{MD}}\right)U_{i+1}$$
$$+\left(1-\theta_{s_i}\right)P_{\mathrm{MD}}U_{i+1}\int_0^\infty \frac{1}{\bar{\gamma}}\exp\left(-\gamma/\bar{\gamma}\right)\left[1-\beta_i\left(\gamma\right)\right]\,d\gamma$$
$$-\,\alpha\,\left(1-\theta_{s_i}\right)P_{\mathrm{MD}}\int_0^\infty \frac{1}{\bar{\gamma}}\exp\left(-\gamma/\bar{\gamma}\right)\beta_i\left(\gamma\right)\,d\gamma \tag{15}$$

It is worth noting that if the secondary user reaches the last channel, skipping this channel yields zero reward. If the channel is sensed to be free, the secondary user would always transmit regardless of the channel SNR (of the secondary link). The expected reward at this stage, U_M, is given by,

$$U_M=\theta_{s_M}\left(1-P_{FA}\right)c_M\int_0^\infty \log\left(1+\gamma\right)\frac{1}{\bar{\gamma}}\exp\left(-\gamma/\bar{\gamma}\right)\,d\gamma$$
$$-\,\alpha\,\left(1-\theta_{s_M}\right)P_{\mathrm{MD}} \tag{16}$$

The expected reward at the first sensing stage, denoted $U_1(N, E_T, S)$ or simply U_1, represents the expected secondary transmission rate, which depends on the sensing order of the M channels and constitutes our optimization objective. Note that the negative term in (15) may cause U_1 to be negative, especially for increasing α values. In this case, the secondary terminal does not transmit. We assume, without loss of generality, that the channels are symmetric in terms of the PDF's of the sensing metric $\mathcal{E}$. Our optimization problem is then to find the number of samples N, the energy detection threshold, E_T, and the channel sensing order that maximizes U_1, or formally,

$$\max_{N,E_T,S} U_1(N, E_T, S) \tag{17}$$

The the objective function in (17) is a highly complicated non-convex function of the variables, two of which are also discrete. Not considering the sensing parameters, the authors in [2] devise a dynamic programming approach to solve for the optimal channel sensing order only. Although dynamic programming can significantly reduce the complexity relative to brute-force search, its complexity is still exponential in the number of channels.

B. The Optimum Solution and its Complexity

In our case, the U_1 maximization problem is further expanded to incorporate optimization over two additional sensing variables, namely the number of samples N and the energy detection threshold E_T. This, in turn, adds more complexity to the problem. In order to obtain the optimal solution, we discretize E_T and then adopt an exhaustive search approach over the entire search space of N and discretized E_T. Note that when either N or E_T changes, P_{FA} and P_{MD} changes altering the expected reward and, consequently, the optimum sensing order. Therefore, for each examined (N, E_T) pair, dynamic programming is employed to obtain the maximum expected reward U_1. The optimal N and E_T are those which yield the optimal channel sensing sequence with the maximum U_1.

C. A Suboptimal Solution

Motivated by the sheer complexity of (17), aggravated by the recursive nature of the objective U_1, we resort to a suboptimal approach in order to reduce the computational complexity needed to find the optimal sensing parameters and channel sensing order. We discuss the scheme in two steps.

1) First step: The direct solution of (17) is to obtain the optimal sensing order via dynamic programming for each value of N and E_T. The combination of N and E_T that yields the maximum U_1 is the solution to (17), together with the sequence corresponding to the maximum U_1. Our proposal is to find, for each value of N, an approximation for the decision threshold, denoted by $\widehat{E_T}$, that is obtained via solving a simpler optimization problem. Specifically, we find the threshold that minimizes the sum of P_{FA} and P_{MD}. (The authors of [10] have utilized the same objective function in a different context involving cooperative sensing.) Hence, the suboptimal value of E_T becomes

$$\widehat{E_T}=\underset{E_T}{\operatorname{argmin}}\; P_{\mathrm{FA}}+P_{\mathrm{MD}} \tag{18}$$

Note that the resulting $\widehat{E_T}$ is a function of N because P_{FA} and P_{MD} are both functions of N. Minimizing the sum of P_{FA} and P_{MD} tends to reduce the sensing errors and their consequences–whether lost secondary transmission opportunities or disrupted transmissions due to collision. We do not involve the probability θ_{s_i} in our expression because we are searching for one threshold to be applied to all channels, whereas the use of θ_{s_i} would necessitate the presence of different thresholds for different channels, thereby increasing the complexity immensely. Another advantage of (18) is that $P_{\mathrm{FA}}+P_{\mathrm{MD}}$, as a function of the threshold, has one global minimum that can be easily obtained numerically (Fig. 1) [10].

In summary, for each value of N, we solve (18) to obtain $\widehat{E_T}$ which is then used to compute P_{FA} and P_{MD}. Dynamic programming is used to obtain the optimal sequence, whereas the optimal N, with its associated $\widehat{E_T}$, are those yielding the highest U_1 in value. Hence, (17) is simplified to

$$\max_{N,S} U_1\left(N, \widehat{E_T}, S\right) \tag{19}$$

2) Second Step: Our second level of simplification pertains to the search space of parameter N in (19). Equation (19) is solved as follows: for each value of N we compute, using dynamic programming, the optimal sequence $\widehat{S}$ and the corresponding reward function $U_1\left(N, \widehat{E_T}, \widehat{S}\right)$. The value of N that solves (19), denoted $\widehat{N}$, is the one that gives the maximum $U_1\left(N, \widehat{E_T}, \widehat{S}\right)$. In other words,

$$\widehat{N}=\underset{N}{\operatorname{argmax}}\; U_1\left(N, \widehat{E_T}, \widehat{S}\right) \tag{20}$$

From numerous simulations, and if N is treated as a continuous variable, we have observed that $U_1\left(N, \widehat{E_T}, \widehat{S}\right)$ as a

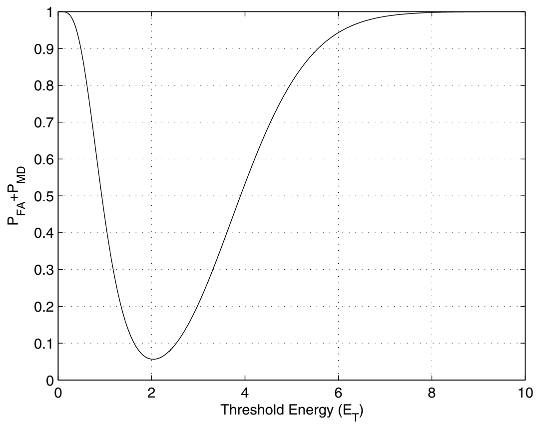

Fig. 1. $(P_{FA} + P_{MD})$ has only one global minimum which can be obtained, for instance, by gradient descent method.

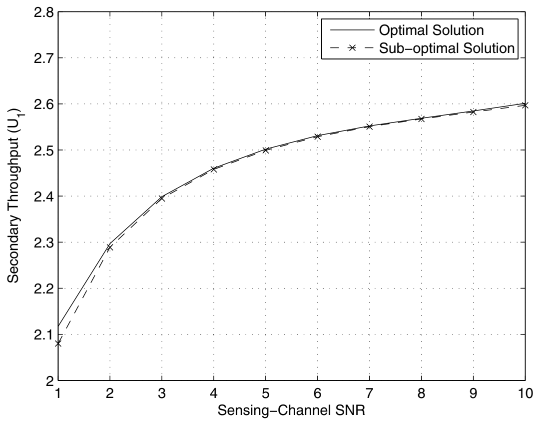

Fig. 3. The throughput obtained by the optimal and the sub-optimal solutions to problem (17). Both give close results with less than 0.2% gap difference for a fixed-gain sensing channel with SNR=10.

function of N is *unimodal* (see, for example, the plots in Fig. 2). Specifically, it is observed to be quasi-concave with one global maximum. We conjecture that this is a characteristic property of $U_1\left(N, \widehat{E_T}, \widehat{S}\right)$. (The proof is the subject of ongoing work.) This property can be exploited to reduce the search space of N where the golden section search algorithm [11] can be used to solve (20) efficiently.

We may note that the sub-optimality of our solution approach is a result of the first step. In other words, simplifying (17) to (19) results in the mentioned degradation in secondary throughput. This is due to the use of a sub-optimal value ($\widehat{E_T}$) instead of the optimal one. On the other hand, the golden search algorithm gives the same results as the exhaustive search and does not affect the solution of (20) when $U_1\left(N, \widehat{E_T}, \widehat{S}\right)$ is quasi-concave.

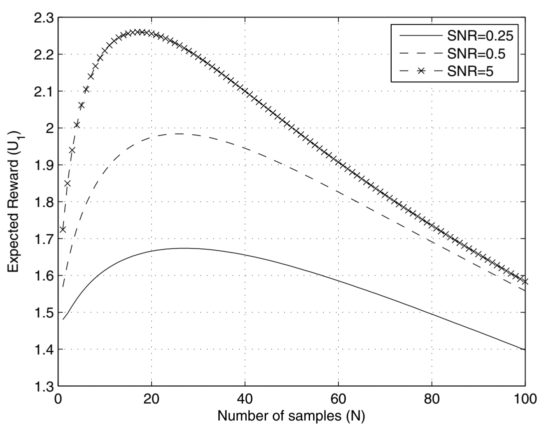

Fig. 2. The optimal expected reward U_1 as a function of N and the sensing channel SNR. The function has one peak that is the global maximum point. The peak occurs at lower N as the SNR increases. This figure is simulated at f_s=500KHz.

IV. Numerical Results and Discussion

In this section, we present some numerical results obtained using Matlab. Our prime objective is to assess the performance gap between the proposed sub-optimal solution (19) and the optimal solution of (17). In addition, we provide results demonstrating the computational savings attributed to the use of golden section search algorithm in order to find the optimal number of samples.

Table I includes the values of the variables used in our simulations unless otherwise is mentioned. The channel availabilities are generated from a uniform distribution on the interval $[0, 1]$, and the throughput results are averaged over 100 realizations. In Fig. 2, the optimal U_1 is plotted as a function of N with the SNR of a fixed-gain sensing channel as a parameter. As is evident from the figure, as the quality of the sensing channel increases, the optimal U_1 increases and is achieved using a smaller number of samples. Note also the quasi-concave nature of the curves which we have exploited to find N efficiently.

TABLE I
Simulation Variables

Variable	Value
M	10
f_s	100KHz
$\bar{\gamma}$	10
V	1
α	1
T	1msec

A. Performance Results

We solve the optimization problem (18) using the gradient descent technique which converges rapidly to the optimal point [12]. Fig. 3 contrasts the sub-optimal solution to the optimal (obtained using exhaustive search) in case of the fixed-gain sensing channel model. The relative gap between the two approaches is slightly less than 2.08% at $SNR = 1$ and decreases with SNR till reaching 0.2% at $SNR = 10$. It is evident that the two solutions experience the same trend and yield nearly the same throughput. Fig. 4 plots the SU's expected throughput with sensing channel SNR using the suboptimal solution. It demonstrates the effect of different sensing channel models. The throughput of a Rayleigh fading

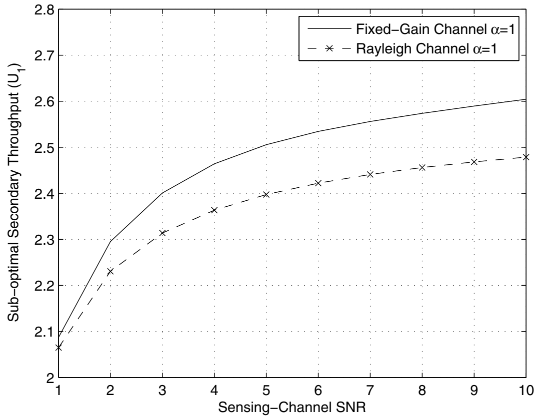

Fig. 4. Comparing the throughput of a Rayleigh fading sensing channel to that of a fixed-gain channel. The former has less throughput due to the randomness of its SNR.

sensing channel is less than that of a fixed-gain channel, which agrees with intuition. This is due to the channel gain being random in the former model causing degradation in the throughput.

B. Required Number of Samples

As pointed out earlier in Section III-C2, the golden section search algorithm can significantly reduce the number of computations to find the optimal number of samples $\widehat{N}$ relative to the brute-force search. Table II compares their computational complexity at different sampling frequencies. The golden section search needs less than 11% of the total number of iterations needed by brute-force search. This saving is achieved while both still reach the same suboptimum value of the expected reward U_1.

TABLE II
NUMBER OF ITERATIONS NEEDED IN BOTH THE BRUTE SEARCH AND THE GOLDEN SECTION SEARCH METHODS

Sampling Frequency	Brute Search	Golden Section Search
100 KHz	100	11
500 KHz	500	16
1000 KHz	1000	17
5000 KHz	5000	20
10000 KHz	10000	22

V. CONCLUSION

We have studied the problem of optimizing the channel sensing parameters and order for cognitive radio networks in the presence of sensing errors. First, we introduce a reward function which reflects the secondary throughput and incorporates the probabilities of false alarm and miss detection along with the price of colliding with the primary. Next, we formulate an optimization problem with respect to the number of samples, energy detection threshold and channel sensing order. Motivated by the sheer complexity of the problem, we resort to a simpler, yet sound, formulation that yields a near optimal solution, at a considerably less computational cost (less than 11% of the total number of needed iterations for an optimal solution). The numerical results reveal interesting insights. The throughput is near its optimal value within 0.28% relative gap. This work can be extended to include the recall option where a previously sensed channel can be used at a later stage.

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