# Models, Complexity and Algorithms for the Design of Multifiber WDM Networks

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***Abstract*—In this paper, we study multifiber optical networks with Wavelength Division Multiplexing (WDM). Assuming that the lightpaths use the same wavelength from source to destination, we extend the definition of the well-known Wavelength Assignment Problem (WAP), to the case where there are $k$ fibers per link, and $w$ wavelengths per fiber are available: This generalization is called the $(k, w)$-WAP. We develop a new model for the $(k, w)$-WAP, based on *conflict hypergraphs*: Conflict hypergraphs more accurately capture the lightpath interdependencies, generalizing the conflict graphs used for single-fiber networks. By relating the $(k, w)$-WAP with the hypergraph coloring problem, we prove that the former is $\mathcal{NP}$-complete, and present further results with respect to the complexity of that problem. We consider the two natural optimization problems that arise from the $(k, w)$-WAP: the problem of minimizing $k$ given $w$, and that of minimizing $w$ given $k$. We develop and analyze the practical performances of two methodologies based on hypergraph coloring, one for each of the two optimization problems, on existing backbone networks in Europe and in the USA. The first methodology relies on two heuristics based on a randomized approximation algorithm and the second consists on an integer programming formulation.**

***Index Terms*— optical networks, wavelength division multiplexing, network design, wavelength assignment problem, hypergraph coloring, integer programming, heuristics.**

## I. INTRODUCTION

Wavelength Division Multiplexing (WDM) is currently the most promising existing optical network technology, since it allows for efficient use of the high bandwidth offered by optical networks. Under WDM, wavelengths are used to implement fixed end-to-end connections — called lightpaths in this context — in the network. The major constraint imposed by this technology is that different lightpaths cannot share the same wavelength over the same link.

Our work focuses on studying WDM networks in real-life scenarios, from both theoretical and practical perspectives. From the telecommunications operator viewpoint, one of the largest costs incurred while deploying an optical network stems from physically trench-digging to bury the optical fibers. Hence, it is usual to have many fibers deployed between any two points of the network, giving rise to *multifiber* WDM *networks* (or MWNs for short).

Minimizing the cost of such a network leads to the design problem known as the *wavelength assignment problem* (WAP) [1–4]. The off-line version of the WAP can be define as follows: Given a WDM network $\mathcal{N}$ and a set of lightpaths satisfying traffic requests, assign wavelengths to the lightpaths so that any two paths that cross the same link are assigned different wavelengths.

Unfortunately, the existing work on single-fiber network design cannot be extended to MWNs in a straightforward manner. For instance, the model used for the WAP on single-fiber networks fails to fully capture the benefits of having more fibers per link when minimizing the total number of wavelengths used in the network in MWNs. The addition of multiple fibers to the network incur an extra degree of freedom in choosing the path wavelengths which was not present in single-fiber networks. Note that using $k$ fibers per link immediately allows for reducing the number of wavelengths by a factor of $k$. In fact, multifibers may reduce the number of wavelengths required even further. For example, adding just one fiber to a single-fiber network can decrease the number of wavelengths required to route $n$ lightpaths from $n$ to 1 [5, 6]. Unfortunately, results of this flavor, which specifically determine the impact of having multifibers either hold for very specific networks (as in [5, 6]) or are very preliminary as far as modeling is concerned [7–9].

In this paper, we generalize the WAP to the case where there are $k$ fibers per link, and $w$ wavelengths per fiber are available — this generalization is called the $(k, w)$-WAP. Two optimization problems naturally arise from the $(k, w)$-WAP: the problem of minimizing the number of wavelengths used, given $k$, and that of minimizing the number of fibers $k$ if we are given $w$.

In order to build a general framework around the $(k, w)$-WAP, we propose a new tool for modeling conflicts arising in wavelength utilization in MWNs, based on hypergraphs. The *conflict hypergraph*, formally defined in Section III, is a generalization of the popular conflict graph, used for the WAP on single-fiber networks. We validate the concepts proposed in this work by considering both optimization problems (that of minimizing $k$ with fixed $c$, and that of minimizing $w$ with fixed $c$) in two backbone networks: the European COST 239 and the pan-american backbone network.

The main contributions of this work can be summa-

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rized as follows:

- We formally define the $(k, w)$-WAP for MWNs, and develop a new model with the notion of a *conflict hypergraph* which generalizes the concept of conflict graphs used for single-fiber networks and captures more accurately the lightpath interdependencies in multifiber networks. With this model, we build a bridge between coloring results for hypergraphs in the literature and the $(k, w)$-WAP.
- We analyze the complexity of the $(k, w)$-WAP in MWNs. In fact, we prove that minimizing the number of wavelengths is $\mathcal{NP}$-complete, even in the case where the number of fibers is fixed in advance, answering the open question with respect to the exact complexity of this problem. We also prove some other related results.
- We analyze the practical performances of two methodologies based on hypergraph coloring on existing backbone networks in Europe and in the USA. The first relies on two heuristics based on a randomized approximation algorithm and the second consists on an integer programming formulation. We analyze the feasibility of solving real-world $(k, w)$-WAP with existing LP/IP solvers.

The remainder of this paper is organized as follows. First, we present an overview of related work in Section II. In Section III, we present the problem formulation and the proposed hypergraph model. Then, in Section IV, we prove that the $(k, w)$-WAP is $\mathcal{NP}$-complete, and presenting other results with respect to the complexity of the problems. In Section V, we address the actual problem of designing a multifiber network, with respect to the optimization of the number of fibers of wavelengths per fiber. Section VI discusses our prototypes and their performance evaluation. Finally, we conclude and present some future work in Section VII.

## II. RELATED WORK

Motivated by the very large costs of deploying WDM networks, a large volume of research has targeted design issues on these networks in the past.

In single-fiber networks, it is usual to assume that two nodes are connected by one fiber of unlimited capacity (i.e. able to carry any number of wavelengths). Hence the $(1, w)$-WAP (formerly known simply as WAP) is exactly the *path coloring problem* in standard graphs[10], which has been proven equivalent to the general vertex coloring problem. Thus, there exists a fixed $\delta > 0$ such that no approximation within $n^{\delta}$ is possible unless $\mathcal{P} = \mathcal{NP}$ [11].

Therefore, a large amount of work concentrated on specific topologies and line networks, rings, trees, meshes, and so on. Specific communication patterns have also been studied like All-to-All and multicast.

The design of multifiber networks has recently been studied under different models and traffic assumptions [5–9]. For instance the $(1, w)-$WAP is $\mathcal{NP}$-complete on undirected stars but becomes polynomial and fits the lower-bound if 2 fibers are available on each link [5, 6].

Fig. 1. A ring network and the correspondig conflict hypergraph.

Dynamic traffic — which means that lightpaths have to be established and released dynamically — has been studied in [7], where multifiber networks were shown to be more efficient than single-fiber networks with the same capacity[1] per link. Using multifiber links has also been shown to lead to performances equivalent to those provided by limited wavelength conversion.

In [8], an integer program and heuristics that solve the static problem are discussed. They consider path length constrained routing, wavelength assignment, wavelength conversion, and link failure restoration. The objective is to minimize the total number of fibers used in the network. Two meta-heuristic (simulated annealing and taboo-search) for MWNs design are proposed in [9]. Both papers show that adding fibers could improve the network efficiency.

Some theoretical properties of MWNs have been studied in [5, 6]. For instance, it was proven that increasing the number of fibers per link often simplifies the optical routing problem: For all $k$ and $w$, there exist a network and a set of communication requests such that exactly $w$ wavelengths are necessary to solve the problem with $k$ fibers per link while 1 wavelength is enough with $k + 1$ fibers.

## III. PROBLEM FORMULATION

In this section, we formally define the $(k, w)$-WAP, the conflict hypergraph, and some other concepts related to those. Let $\mathcal{N}$ be an instance of a MWN, where the set of nodes is represented by $N$ and the set of links by $L$. Assume, without loss of generality, that every link in the network contains the same number of fibers, denoted by $k$, and that a set of communication paths $\mathcal{P}$ is given. A solution to the $(k, w)$-WAP is an assignment of one out of $w$ wavelengths to each path, satisfying the constraints that for every link in the network, no more than $k$ paths using the link are assigned the same wavelength. In order to model these constraints, we define the *conflict hypergraph* $H(\mathcal{N}, \mathcal{P})$ of the set of paths $\mathcal{P}$ in $\mathcal{N}$, as follows:

*Definition 1:* The *conflict hypergraph* $H = (V, E)$ of the paths $\mathcal{P}$ in $\mathcal{N}$ is the hypergraph with a vertex $v \in V$ for each path $p \in \mathcal{P}$, and an hyperedge $e \in E$ for every link $\ell \in L$. An hyperedge of $H$ contains the vertices corresponding to all the paths going through the corresponding link$\ell$. □

[1] The capacity of a link is the sum of the capacities of each fiber in the link.

A vertex coloring of the conflict hypergraph induces a feasible wavelength assignment to the paths if and only if no hyperedge contains more than $k$ vertices with the same color. This motivates the following definition.

*Definition 2:* Given a hypergraph $H = (V, E)$ and a set of colors $\mathcal{C} = \{1 \dots c\}$, a mapping $f : V \to \mathcal{C}$ is a $(k, c)$*-coloring* if and only if no hyperedge contains more than $k$ vertices with the same color, that is, $\forall e \in E,\ \forall q \in \mathcal{C},\ |\{v \in e : f(v) = q\}| \leq k$. □

The four main parameters of the hypergraph $H = (V, E)$ can be expressed in terms of $\mathcal{N}$ and $\mathcal{P}$:

- the *number of vertices* $n \equiv |V| = |\mathcal{P}|$,
- the *number of hyperedges* $m \equiv |E| = |L|$,
- the *rank* $t \equiv \max_{\ell \in L} |\{P \in \mathcal{P} : \ell \in P\}|$,
- the *maximum degree* $\Delta \equiv \max_{v \in V} |\{e \in E : e \ni v\}|$. Note that $\Delta \leq max_{p \in \mathcal{P}}$ length$(p)$, which is equal to the diameter of the routing.

The *load* of $\mathcal{P}$ is equal to the maximum number of paths passing through any one edge in the network, and is thus equal to the rank of $H$.

It is easy to see from Definitions 1 and 2, that there is a one-to-one correspondence between the $(k, c)$-colorings of the conflict hypergraph of $\mathcal{P}$ and the feasible wavelength assignments to these paths. Since we can build the conflict hypergraph $H$ in polynomial time on $\mathcal{N}$, there is a polynomial time reduction from the $(k, w)$-WAP to the $(k, c)$-coloring problem. Thus the $(k, c)$-coloring problem is at least as difficult as the $(k, w)$-WAP. It is not trivial, however, that the converse is also true, since the hypergraph coloring problem may seem at first to be a more general (and harder) problem than the WAP. In fact, we prove the equivalence of these two problems in the next section. In the sequel, we will use colors and wavelengths interchangeably.

## IV. COMPLEXITY OF WAVELENGTH ASSIGNMENT IN MWNS

In this section, we prove the equivalence between the $(k, w)$-WAP and $(k, c)$-coloring, by proving that the $(k, w)$-WAP is $\mathcal{NP}$-complete even in the case where $k$ is fixed, and present a lower bound on the number of colors needed in a $(k, c)$-coloring of a (hyper)clique.

*Theorem 1:* The $(k, c)$-coloring problem is polynomially equivalent to the $(k, w)$-WAP on MWNs.

*Proof:* It is enough to prove that any hypergraph $H$ is the conflict hypergraph of a set of paths $\mathcal{P}$ on a network $\mathcal{N}$, where the sizes of both $\mathcal{P}$ and $\mathcal{N}$ are polynomial on the size of $H$.

Let $H = (\{v_1, \dots, v_n\}, \{e_1, \dots, e_m\})$ be a hypergraph, where $e_i = \{v_{j_1^i}, \dots, v_{j_{r_i}^i}\}$, for all $i$. Let $r_i = |e_i|$, for all $i$. For every hyperedge $e_i$, let $\mathcal{N}_i(V_i, E_i)$ be the network (depicted in Figure 2) containing

- $n$ nodes $x_j^i,\ j = 1 \dots n$, and $n$ nodes $z_j^i,\ j = 1 \dots n$,
- two special nodes $Y_i$ et $Y_i'$,
- an edge $Y_i \to Y_i'$,
- $\forall j \in \{j_1^i, \dots, j_{r_i}^i\}$,
  - an edge $x_j^i \to Y_i$, and
  - an edge $Y_i' \to z_j^i$,
- $\forall j \notin \{j_1^i, \dots, j_{r_i}^i\}$, an edge $x_j^i \to z_j^i$.

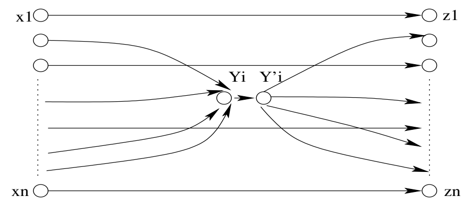


Fig. 2. Network $\mathcal{N}_i$ of Theorem 1.

Let $\mathcal{N}$ be the network composed of the union of the subnetworks $\mathcal{N}_i$ connected by the edges $z_j^i \to x_j^{i+1},\ \forall i = 1 \dots n-1,\ \forall j = 1 \dots n$. Clearly, the size of $\mathcal{N}$ is $O(nm)$ and $H$ is the conflict hypergraph of the set of the unique shortest paths between $x_j^1$ and $z_j^n$, for all $j$, in $\mathcal{N}$. ■

The $(k, c)$-coloring problem is clearly $\mathcal{NP}$-complete for an arbitrary $k$, since it generalizes the graph coloring decision problem when $k = 1$. Therefore,

*Corollary 1:* The $(k, w)$-WAP on a MWNs is $\mathcal{NP}$-complete for an arbitrary $k$.

Moreover —and not too surprisingly— it can be proved that the problem remains difficult even when $k$ is fixed. The proof is included in the full version of this extended abstract.

*Theorem 2:* The $(k, c)$-coloring problem is $\mathcal{NP}$-complete for any fixed $k$. Therefore, the $(k, w)$-WAP on a MWNs is $\mathcal{NP}$-complete for any fixed $k$.

### *A lower bound*

Extending the notion of cliques in graphs, we can give a lower bound on the number of colors needed in a $(k, c)$-coloring, by using (hyper)cliques, as follows. Recall that $K_{n,t}$ is a hypergraph with $n$ nodes that contains all the possible hyperedges of rank $t$. The following holds.

*Lemma 1:* A $(k, c)$-coloring of $K_{n,t}$ is feasible if and only if

$$c \geq \begin{cases} \lceil \frac{n}{k} \rceil & \text{if } t > k, \\ 1 & \text{otherwise.} \end{cases}$$

*Proof:* The case where $t \leq k$ is trivial because the hyperedges do not impose any restriction on the coloring and therefore one color is enough. For the case where $t > k$, suppose $K_{n,t}$ can be colored with $c$ colors. Then, since the hypergraph is symmetric, every color will be repeated $\lceil \frac{n}{c} \rceil$ or $\lfloor \frac{n}{c} \rfloor$ times in some hyperedge. Since we assumed that $c$ colors were feasible, we must have that $\lceil \frac{n}{c} \rceil \leq k$. The minimum number of colors that satisfies that condition is $\lceil \frac{n}{k} \rceil$ and it is easy to see that any permutation of the colors is feasible. ■

The lemma above bounds the number of colors required to color any hypergraph that contains $K_{n,t}$, yielding the following generalization of the fact that the chromatic number of a graph is larger than the size of its maximum clique (just make $t = 2$ and $k = 1$).

*Corollary 2:* Let $H$ be a hypergraph containing $K_{n,t}$. If $H$ can be $(k,c)$-colored, with $k < t$, then $c \geq \lceil n/k \rceil$.

## V. TOOLS FOR DESIGNING MWNS

In this section, we will present two scenarios in the design of multifiber networks. The equivalence between solving the WAP for $\mathcal{P}$ and computing $(k,c)$-colorings of $H$ allows us to concentrate on the latter. For instance, the problems we consider are the problems of finding the minimum $k$ (respectively, $c$) such that there is a feasible $(k,c)$-coloring of $H$ with $c$ (respectively, $k$) given. We address these two problems in Section V-A and V-B, respectively.

### A. Minimizing the number of fibers

We consider first the problem of minimizing the number of fibers when the number of colors is given. This problem can be formulated as a Minimax Integer Program [12]. For instance, we define $(0,1)$-integer variables $x_{ij}$, for all $i \in V$ and $1 \leq j \leq c$, such that $x_{ij} = 1$ if and only if node $i$ is colored with color $j$ and $x_{ij} = 0$ otherwise. The variable $k$ is a common upper bound for the constraints defined by each hyperedge. The optimal number of fibers can be found by solving the following IP.

*Integer Program 1:*

$$\begin{array}{llcll} \text{minimize} & k & & & \text{(minimize \# of fibers)} \\ \text{s.t.} & \sum_{c} x_{ic} & = & 1 & \forall \text{ node } i \\ & \sum_{i \in H} x_{ic} & \leq & k & \forall \text{ color } c,\ \forall \text{ hyperedge } H \\ & k \geq 0,\ x_{ic} & \in & \{0,1\} & \forall \text{ color } c,\ \forall \text{ node } i. \end{array}$$

Srinivasan showed that if the optimal solution of the LP relaxation is rounded randomly, with positive probability, a solution that is feasible and not too large can be encountered [12]. A simple randomized algorithm, discussed by Lu [13], computes a solution within an approximation ratio not too far from the one given by Srinivasan. This algorithm proceeds by successive recoloring phases.

Another algorithm, recently published by Leighton et al. [14], which is based on a specific kind of randomized rounding and recoloring techniques proposed in Lu's algorithm [13]. This algorithm achieves the same theoretical approximation ratio than the best known existential result [12]. Furthermore, we shall show in Section VI that the recoloring techniques are not used in practical situations since the randomized rounding yields solutions that are close to optimality.

### B. Minimizing the number of wavelengths

Given the number of fibers $k$, we now would like to minimize the number of colors $c$ such that a valid $(k,c)$-coloring of the hypergraph exists. We present an IP formulation for this problem below. We define a variable $x_{ic}$ for each node and each color: $x_{ic} = 1$ if node $i$ is colored with color $c$, and 0 otherwise. We have seen that the number of colors is bounded by $\lceil n/k \rceil$ (this bound is tight if the graph is a clique).

*Integer Program 2:*

$$\begin{array}{lrcll} \min & \sum_{c} y_c & & & \text{(minimize \# of colors)} \\ & \sum_{c} x_{ic} & = & 1 & \forall \text{ node } i \\ & \sum_{i \in H} x_{ic} & \leq & k & \forall \text{ color } c, \text{ hyperedge } H \\ & x_{ic} & \leq & y_c & \forall \text{ color } c, \text{ node } i \\ & x_{ic}, y_c & \in & \{0,1\} & \forall \text{ color } c, \text{ node } i \end{array}$$

There are $O(n^2)$ variables and $O(n^2m)$ constraints (we could reduce the number of constraints to $O(nm)$ if the solver generates cuts automatically).

The drawback of this IP formulations is that it is not symmetric and thus Branch-and-Bound will waste a lot of time iterating trough similar solutions [15]. The problem arises because after a variable is constrained by the algorithm, a permutation of the set of variables may still give a feasible solution. This issue can be addressed by automatic pruning techniques, as described in [16].

## VI. IMPLEMENTATION AND PERFORMANCE EVALUATION

To computationally evaluate the problems, we implemented the two integer programs described in Section V, Lu's recoloring algorithm [13] and Leighton et al. randomized rounding based algorithm [14]. This allowed us to evaluate the tradeoff between the performance and the running time of the exact version and the two approximations. We also report our findings in the experience of solving the problem of minimizing $c$. During our tests we realized that the randomized rounding performed in Leighton et al. algorithm produces very good solutions such that the recoloring procedures of the algorithm are never used. On the other hand, Lu's algorithm is always close to its theoretical approximation ratio.

We ran tests on several instances and will present the results we had on the pan-european network COST 239 [17] and the pan-american network.

COST 239 network interconnects 11 european capitals using 24 multifiber links as depicted in Figure 3. The demand matrix, which was provided to us by France Telecom [17], is made of 176 requests, which cover all possible pairs of cities. The maximum load of the given routing is $58$, which is also a lower bound for the number of colors in the single-fiber case.

The pan-american network is bigger and consists of 78 cities, interlinked by 102 arcs (see Figure 4). The demand matrix was generated with the well-known gravitational model, where the weights of the cities represent their importance and are proportional to the distance to 5 main population areas in the USA. Finally,

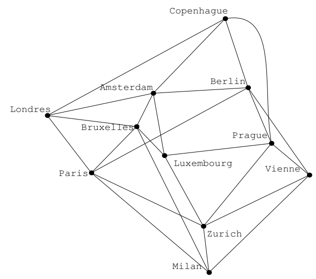


Fig. 3. *The* COST 239 *pan-european network.*

the demand between every two cities is proportional to the product of the two weights while keeping the outgoing number of requests from every city equal to the weight. Using different weights, we generated instances that were used for the benchmarks. The routing strategy took into account reliability issues and was computed through a *minimum cost disjoint paths problem* for each origin-destination pair. For each origin and destination, the demand was randomly distributed among the disjoint paths with the shortest total distance. We report on a relatively big instance with 2022 requests with load 520.

When solving the exact version of the minimization of the number of fibers, the solver found feasible solutions reasonably fast when restricted to small instances. Except for the biggest instances (the pan-american network with many colors), the solver did not have difficulties in proving optimality. It was expected, though, that when the instances grew bigger, the running time was going to degrade because the underlying problem is $\mathcal{NP}$-hard. Nevertheless, this does not seem to be an issue for the instances generated from real-world networks.

*Results*

Optimal and approximate computations of the required number of fibers, as a function of the number of colors available, are depicted in Figure 5 for COST 239. The worst results are given by Lu's algorithm, then comes the randomized rounding wich is optimal up to an additive factor of 5. IP 1 gives the optimal number of fibers. The small size of this network allows the IP to be solved even with a large number of colors. Moreover both the randomized rounding and Lu's algorithm are solved almost immediately. Therefore, running times are not plotted for COST 239.

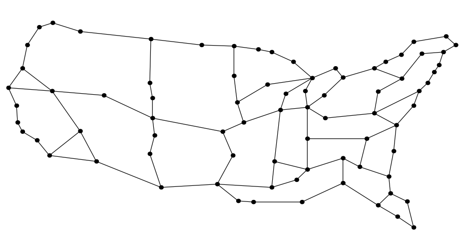

Fig. 4. *The pan-american network.*

On the other hand, Figure 6 depicts the running times of the experiments on the the pan-american network. Lu's approximate algorithm runs almost in constant time compared to the randomized rounding which solves the linear relaxation of IP 1. The time taken by IP 1 is exponential, as expected. When the number of colors grows bigger than $25$, the IP is not solvable any more by our computers (PIV 1GHz running CPLEX v7.5). Comparatively, Figure 7 shows that the approximate number of fibers given by Lu's algorithm is around $3$ times the optimal, while the randomized rounding stays close to the optimum up to an additive factor of $4$. Thus, the tradeoff between the quality of the approximation and the running time is obvious. We could use the randomized rounding to optimize a static network during an offline process, where the running time is not the main issue, while Lu's algorithm could be useful when time is an issue, in online optimization for instance. One can note that when the IP becomes too large, an exact solution is not computable, not only because of computation time but also because of memory requirements for the Branch-and-Bound solver. Therefore, even if long running times are allowed, approximation may be required and randomized rounding would be the good approach.

It is important to notice that in these instances, and often with real-world networks, the number of colors equals its lower bound, that is, the load of the network divided by the number of fibers. It is known that pathological examples can be constructed, although they do not usually appear in real instances.

The biggest dependency of the running time of IP 2, which optimizes the number of colors, is on the number of variables representing the colors. Initially, we used as many colors as the number of requests, because that is an upper bound. Obviously, this did not scale well when the size of the instances increased to real-world problems. Instead, we performed a binary search for the upper bound of the colors. We relied on the observation that when the bound is too small, the IP solver returns quickly that no feasible solution exists. On the other hand, when the upper bound is not tight, it takes too long to solve the first node of the Branch-and-Bound tree because there are too many variables. With this strategy we got IPs of the correct size that could be handled by

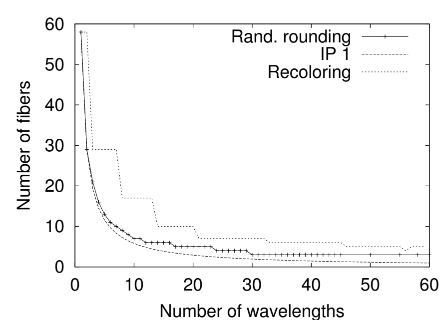


Fig. 5. *Experiments on* COST 239.

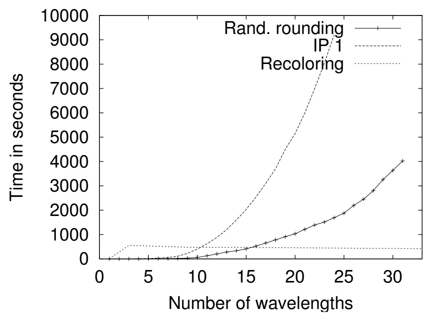


Fig. 6. *Experiments on the pan-american network : time.*

the solver. As expected though, due to the symmetry in the formulation (the labeling of the colors can be permuted without altering the solution), the enumeration of the nodes of the Branch-and-Bound tree, could not be completed in general. In any case, we had a proof of optimality. Indeed, we found that when using one less color, the LP relaxation of the problem was already not feasible. Therefore, showing a feasible solution with that many colors was enough. Indeed, it would be interesting to characterize the integrality gap of that problem.

## VII. CONCLUSION

In this paper, we have proposed a framework to model the WAP in MWNs, reducing it to a coloring problem on hypergraphs. Practically, the coloring problem appeared to be tractable when there are few colors, since its straightforward IP formulation gives optimal solutions reasonably fast. Unfortunately, this is not the case for real-world instances and the number of available wavelengths will dramatically increase with future D-WDM and UD-WDM networks.

Furthermore, the heuristics that we implemented well illustrate the tradeoff between the quality of approximation and their running time. When an exact solution cannot be computed in reasonable time and space, randomized rounding can be used to produce very good solutions. When quicker solutions are required, one would rather follow approaches based on Lu's algorithm, which runs in quasi constant-time at the cost of a multiplicative factor of $3$ on the optimal solution.

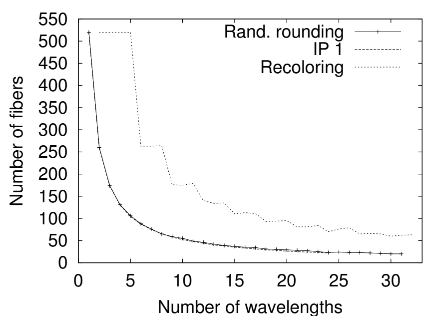


Fig. 7. *Experiments on the pan-american network : results.*

It is also interesting to note that these hypergraph coloring algorithms may be useful in the context of *radio ad-hoc* network optimization. Indeed, the hypergraph structure appears naturally when addressing capacity constrained radio coverage optimization. Further work is still required in this direction.

Another interesting research direction is to study the design of MWNs in the case where the routing is not fixed in advance. In such a case the lightpaths are not given, and one needs to design both the routing and the wavelength assignment at once. We believe that, as soon as $k$ is large enough, this problem can be practically solved to optimality [18].

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