Brief Announcement: Parameterized Maximum and Average Degree Approximation in Topic-based Publish-Subscribe Overlay Network Design

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ABSTRACT

Designing an overlay network for publish/subscribe communication in a system where nodes may subscribe to many different topics of interest is of fundamental importance. For scalability and efficiency, it is important to keep the degree of the nodes in the publish/subscribe system low. It is only natural then to formalize the following problem: Given a collection of nodes and their topic subscriptions connect the nodes into a graph which has low average and maximum degree and in such a way that for each topic t, the graph induced by the nodes interested in t is connected. We present the first polynomial time parameterized sublinear approximation algorithm for this problem.

We also considered a variation of the problem where each topic-connected overlay network is required to be of constant diameter, while still keeping the average degree low. We present two novel heuristics for this problem, which guarantee that each topic-connected overlay network will be of diameter 2 and which aim at keeping the overall average node degree low, validated by experimental results.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design—*Network Topology*; G.2.2 [**Mathematics of Computing**]: Graph Theory—*Network Problems*

General Terms

Algorithms, Theory, Experimentation

Keywords

pub/sub, overlay networks, peer-to-peer, multicast, optimization

1. INTRODUCTION

In the publish/subscribe (pub/sub) communication paradigm, publishers and subscribers interact in a decoupled fashion. Publishers publish their messages through logical channels and subscribers receive the messages they are interested in by subscribing to the appropriate services, which deliver messages through these channels.

In this paper, we will design a (peer-to-peer) overlay network for each pub/sub topic, in the sense that for each topic t, the subgraph induced by the nodes interested in t will be connected. This translates into a *fully decentralized* topic-based pub/sub system since any given topic-based overlay network will be connected and thus nodes subscribed to a given topic do not need to rely on other nodes (agents) for forwarding their messages. Such an overlay network is called *topic-connected.*

The node degrees and number of edges required by a topic-connected overlay network will be low if the node subscriptions are well-correlated. In this case, by connecting two nodes with many coincident topics, one can satisfy connectivity of many topics for those two nodes with just one edge. Several recent empirical studies suggest that correlated workloads are indeed common in practice.

In this work, we consider the problem of devising topic-based pub/sub overlay networks with low node degrees. One could argue for keeping the maximum degree of a node low or for keeping the overall average node degree low, since both are important and relevant measures of the complexity and scalability of a system. Unfortunately, previous attempts at minimizing either one of these degree measures alone [1, 2] resulted in a linear explosion for the other measure (see Table 1).

Hence it is only natural to consider the following problem: *Low Degree Topic-Connected Overlay (Low-TCO) Problem:* Given a collection of nodes V, a set of topics T, and the node interest assignment I, connect the nodes in V into a topic-connected overlay network G which has low average and maximum degree.

We present a parameterized sublinear approximation algorithm (Low-ODA) for this problem which approximates both the average and the maximum degree well (See Table 1). To the best of our knowledge, this is the first overlay network design algorithm that achieves sublinear approximations on both the average and maximum degrees (e.g., for $k = \sqrt{n}$). The Low-ODA algorithm is a greedy algorithm which relies on repeatedly evaluating the trade-off of greedily adding an edge that would not increase the maximum degree versus greedily adding an edge that would lead to a small number of total edges in the final overlay network. The main contribution of this work is therefore to show that such a greedy

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	Avg Degree	Max Degree
Chockler et. al. [1]	$O(\log(n*t))$	$\theta(n)$
Onus and Richa [2]	$\theta(n)$	$O(\log(n*t))$
This Paper	$O(k*\log(n*t))$	$O((n/k)*\log(n*t))$
Lower Bound	$\Omega(c)$	$\Omega(c)$

Table 1: Summary of known results on overlay network construction for publish/subscribe communication (n: number of nodes, t: number of topics, c: constant, k is any parameter between 1 and n)

approach can work and indeed leads to non-trivial sublinear approximation on both the average and maximum degree. We expect that the greedy parameterized template introduced by our algorithm will lead to applications in other network design domains where scalability is a key issue.

Chockler et al. [1] introduced the MinAv-TCO problem, which aims at minimizing the average degree alone of a topic-connected overlay network. They present an algorithm, called *GM*, which achieves a logarithmic approximation on the minimum average degree of the overlay network.

While minimizing the average degree is a step forward towards improving the scalability and practicality of the pub/sub system, their algorithm may still produce overlay networks of very uneven node degrees where the maximum degree may be unnecessarily high. In [2], it is shown that GM algorithm may produce a network with maximum degree $|V|$ while a topic-connected overlay network of constant degree exists for the same configuration of I (See Table 1).

In [2], the problem of minimizing the maximum degree of a topic-connected overlay network (MinMax-TCO) is considered, and a logarithmic approximation algorithm on the minimum maximum degree of the overlay network (MinMax-ODA) is presented.

The MinMax-ODA algorithm may produce overlay networks of very high average degree: As we will show in Section 2, this algorithm may produce a network with average degree $|V|-2$ while a topic-connected overlay network of constant average degree exists for the same configuration of I (See Table 1).

2. LOW DEGREE OVERLAY DESIGN ALGORITHM (LOW-ODA)

In this section we present our overlay design algorithm (Low-ODA) for the Low-TCO problem. The weight of an edge (u,v) is given by the reduction on the number of topic-connected components which would result from the addition of (u,v) to the current overlay network. Let $1 \leq k \leq n$. Low-ODA starts with the overlay network $G(V,\emptyset)$. At each iteration of Low-ODA, the algorithm considers two edges:

e_1 : a maximum weight edge among the ones which minimally increases maximum degree of the current graph

e_2 : a maximum weight edge

If the weight of edge e_1 is greater than the weight of e_2 divided by k, edge e_1 is added to edge set of the overlay network; otherwise edge e_2 is added.

Let $NC(V,E)$ denote total number of topic connected components in the overlay network given by (V,E).

Algorithm 1 Low Degree Overlay Design Algorithm (Low-ODA)

OverlayEdges $\leftarrow \emptyset$
$V \leftarrow$ Set of all nodes
$G'(V,E') \leftarrow$ Complete graph on V
for $\{u,v\} \in E'$ **do**
  $w\{u,v\} \leftarrow$ Number of topics that both of nodes u and v have
end for
while $G(V,$OverlayEdges$)$ is not topic-connected **do**
  Let e_1 be a maximum weight edge in $G'(V,E',w)$ among the ones which increase the maximum degree of $G(V,$OverlayEdges$)$ minimally.
  Let e_2 be a maximum weight edge in $G'(V,E',w)$
  if $w(e_1) \geq w(e_2)/k$ **then**
    $e = e_1$
  else
    $e = e_2$
  end if
  OverlayEdges = OverlayEdges $\bigcup e$
  $E' \leftarrow E' - e$
  for $\{u,v\} \in E'$ **do**
    $w\{u,v\} \leftarrow$ NC(V, OverlayEdges) - NC(V,OverlayEdges $\bigcup\{u,v\}$)
  end for
end while

At each iteration of the while loop, two edges are considered: an edge (e_1) with maximum weight among the edges in E' that increase the maximum degree of the current graph minimally and an edge (e_2) with maximum weight in all of E'. If weight of the first one (e_1) is greater than or equal to weight of the second one (e_2) over k, e_1 is added to the set of overlay edges; otherwise e_2 is added. Note that the addition of an edge to OverlayEdges can either increase the maximum degree by 1 or not increase it at all. The crux in the analysis of this algorithm is to show that each of the edges will reduce the number of connected components by a "large" amount without increasing the maximum degree by too much.

3. APPROXIMATION RATIO

THEOREM 1. *The overlay network output by Low-ODA has average node degree within a factor of* $O(k*\log(\sum_{v\in V}|\{t\in T|I(v,t)=1\}|))$ *from the minimum possible average node degree for any topic-connected overlay network on* V*.*

THEOREM 2. *The overlay network output by Low-ODA has maximum node degree within a factor of* $O((n/k)*\log(\sum_{v\in V}|\{t\in T|I(v,t)=1\}|))$ *from the minimum possible maximum node degree for any topic-connected overlay network on* V*.*

4. REFERENCES

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