# Nonclassical rotational inertia in solid helium-four

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Physics 636

May 4, 2007

**Abstract**

Since 2004, when Kim and Chan announced the discovery of nonclassical rotational inertia in solid helium, there has been a race to understand this mysterious phenomenon. A combination of experimental and theoretical work has constrained the possible mechanisms, but a definitive explanation has not yet been found. Recent findings suggest that imperfections such as grain boundaries play an important role. This paper reviews the history of the subject, laying out the various theories and discussing their plausibility in light of recent experimental data.

# 1 Introduction

Rotating a container of liquid helium suddenly becomes easier when it is cooled below its superfluid transition temperature, since the superfluid can flow against the walls of the container without dissipation. Strange as this effect is, it would seem even stranger to see the same behavior in a solid. Yet this is exactly what researchers at Penn State discovered in 2004 [4]—solid helium-four also displays the effect of nonclassical rotational inertia (NCRI). Understanding how NCRI can occur in a solid is an active area of research, and the question has still not been resolved.

In this paper, we will first examine the history of the experimental observations and early theoretical speculations involving NCRI. Then, we will explore the newest theories, including effects of crystal disorder and the possibility of a new vortex phase, discussing how they may explain the observed facets of this intriguing phenomenon.

# 2 Experimental observation of NCRI in solid helium

When helium is cooled to low temperatures, it usually forms a liquid, even in the limit of $T \to 0$. The helium atoms cannot readily form a solid structure due to an intrinsically weak interatomic potential. Under high enough pressure, however (about 25 bar at 1.3 K), helium-four does form a hexagonal close packed (hcp) structure [1].

In 1970, A. J. Leggett proposed an intriguing experiment involving solid helium [2], extending an idea that came from studies of low-temperature helium in the liquid phase. It had been known for some time that liquid helium becomes a superfluid at low temperatures: just as a superconductor carries current without resistance, superfluid helium flows without dissipation. Furthermore, it was known that a simple experiment could be performed to test the percentage of the helium that had become superfluid. The liquid is placed in an annular or toroidal container, which is attached to the end of a torsional pendulum. Starting at a temperature too high for superfluidity, the pendulum is allowed to oscillate at its natural frequency, and the resonant period $\tau$ is measured—this measures the moment of inertia $I$ of the setup via the equation

$$\tau = 2\pi\sqrt{\frac{I}{G}}, \tag{1}$$

where $G$ is the torsional spring constant. The sample is then cooled below the critical superfluid temperature, and the resonant period is again measured. This time, the period is significantly shorter—the moment of inertia has decreased. The explanation is that the superfluid is no longer entrained to the walls of its container, and can sit still in the lab frame while the rest of the fluid flows with the container. The physics involved is Bose condensation: a macroscopic portion of the helium-four atoms (bosons) have condensed into the same single-particle zero-momentum state. The fraction of fluid that has condensed into the superfluid state, denoted $\rho_s/\rho$, is equal to the fraction of the moment of inertia of the fluid that has decoupled. This is measured experimentally using the resonant periods of the torsional oscillator when empty ($\tau_{empty}$), when filled with normal liquid helium ($\tau_{normal}$), and when filled with superfluid helium ($\tau_s$):

$$\rho_s/\rho = \frac{\tau_{normal} - \tau_s}{\tau_{normal} - \tau_{empty}}. \tag{2}$$

As long as the walls of the container are not moving too fast[1], and assuming perfect Bose condensation, $\rho_s/\rho \to 1$ as the temperature $T \to 0$. This phenomenon is known as "nonclassical rotational inertia" (NCRI).

Leggett's bold claim was that it could be possible to see NCRI in a solid, namely crystalline helium. With the localization of atoms that one usually finds in a solid, any sort of long-range coherent behavior seemed unlikely, but Leggett argued that, with the exchange processes that are present in low-temperature helium, superflow might occur. He admitted that the effect would be small (setting a tentative upper limit of $\rho_s/\rho \lesssim 3 \times 10^{-4}$) but encouraged experimentalists to search for the phenomenon nonetheless [2].

The first experimental test of NCRI in solid helium was made by Bishop *et al.* in 1981 [3]. Following Leggett's suggestion, they set up a 1 cm diameter spherical container mounted on a torsional oscillator, and filled it with helium at high pressure and low temperature to form a solid. The oscillator was driven into resonance, with a resonant period of about 1500 $\mu$s. But as they lowered the temperature all the way down to 25 mK, and with average velocities down to 5 $\mu$m/s, there was no decrease in the resonant period that would be the hallmark of NCRI. Considering their experiment's precision, they concluded that $\rho_s/\rho$ must be less than $5 \times 10^{-6}$, unless the phenomenon happened only for a lower temperature or lower velocity than they measured. With these strict bounds, the search for NCRI was put on hold for many years.

It was not until 2004 that the field was jumpstarted with surprising results from the lab of E. Kim and M. H. W. Chan at Penn State [4, 5]. Kim and Chan also used a similar torsional oscillator setup. In their first experiment [4], they grew the solid helium in a slab of porous Vycor glass, which contains "tortuous" hollow tunnels on a nanometer scale. And, indeed, when the temperature was lowered below 175 mK, they saw a drop in the resonant period of the oscillator, of a magnitude that indicated $\rho_s/\rho$ of about 1%. Kim and Chan tentatively guessed that their very different geometry explained the null results of previous experiments, but soon also found NCRI in an annular channel with thickness 0.63 mm [5]: even bulk helium was displaying the phenomenon.

[1] Analogously to a superconductor's critical current, a superfluid has a critical velocity $v_c$ above which it cannot support superflow.

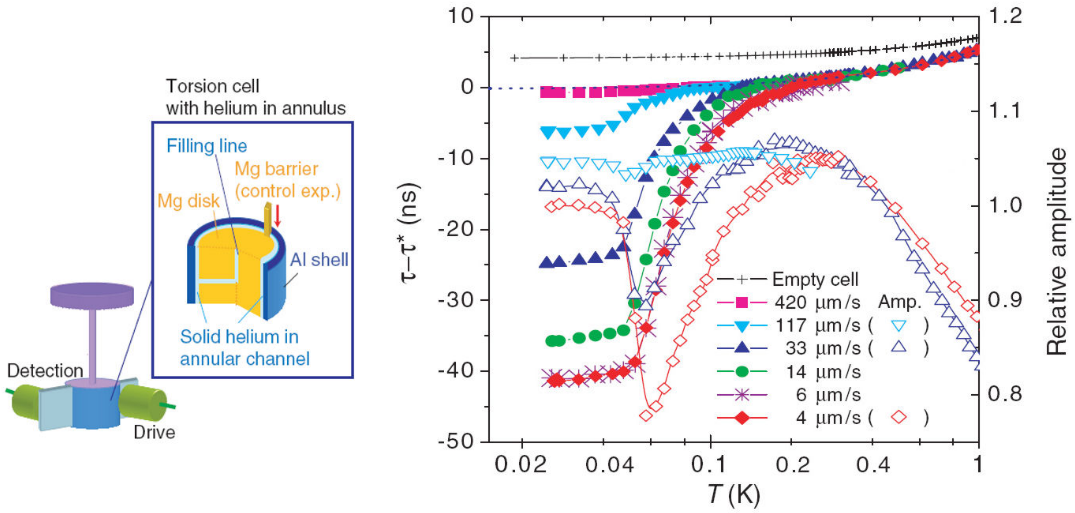


Figure 1: Kim and Chan's observation of NCRI in bulk solid helium 4, from Ref. [5]. The diagram on the left shows the torsional oscillator setup. In the plot, the solid symbols show the drop in resonant period $\tau$ as the temperature $T$ is lowered, which indicates that a portion of the solid helium's rotational inertia has decoupled from the container. The open symbols show the relative amplitude of oscillation, giving a measure of the dissipation involved. Different colors indicate different maximum velocities of the container.

Figure 1 shows the key data from Kim and Chan's second experiment. The important results are:

1. The drop in resonant period (solid symbols in Fig. 1), and therefore the inferred superfluid density, is dependent on the maximum speed in the sample. The critical velocity $v_c$, above which superflow is destroyed, is found to be on the order of 10 $\mu$m/s. This is much smaller than $v_c$ for liquid helium, which is on the order of 1 m/s. Interestingly, the solid helium $v_c$ is only slightly larger than the velocity liquid helium would be expected to have with a single quantum of circulation. This suggests that the loss of the NCRI effect is associated with the appearance of vortices of only a few units of circulation in the flow [5].

2. The amplitude of oscillation (unfilled symbols in Fig. 1) drops significantly at the transition temperature when NCRI is present. This implies a peak in dissipation at the transition.

3. When an obstruction is inserted into the annulus to block the flow (data not shown in this figure), there is no observation of NCRI. This rules out possible local kinetic or mass redistribution explanations [1]—whatever is causing the NCRI involves flow around the entire macroscopic sample.

The goal, then, is to explain these results in terms of some sort of microscopic theory. No theory has yet been definitively verified, and concentrated efforts are currently underway to understand the phenomenon.

## 3 Intrinsic "supersolid" theories

The history of contemplating superflow in solids reaches back to the 1950s, when superfluids were beginning to be described rigorously in terms of Bose-Einstein condensation. During this time, Penrose and Onsager claimed that a perfect solid at zero temperature, with a helium atom sitting on every lattice site, has no freedom to support a superflow [6]. It is still believed to be true that a perfect solid cannot Bose condense [1].

To see that a perfect crystal cannot support superfluidity, we consider the accepted measure of coherence in a sample, the single-particle density matrix $n(\mathbf{r}, \mathbf{r}')$. At $T = 0$,

$$n(\mathbf{r}, \mathbf{r}') \equiv \int \cdots \int d\mathbf{r}_2 \cdots d\mathbf{r}_N \Psi(\mathbf{r}, \mathbf{r}_2, \ldots, \mathbf{r}_N) \Psi(\mathbf{r}', \mathbf{r}_2, \ldots, \mathbf{r}_N), \tag{3}$$

where $\Psi$ is the $N$-dimensional wavefunction of the system, and the $\mathbf{r}$s represent the locations of $N$ particles. In second quantized notation,

$$n(\mathbf{r}, \mathbf{r}') \equiv \langle \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}(\mathbf{r}') \rangle, \tag{4}$$

where $\hat{\psi}^\dagger$ and $\hat{\psi}$ are boson creation and annihilation operators. In this notation we see that $n(\mathbf{r}, \mathbf{r}')$ represents the "overlap" of removing a boson at position $\mathbf{r}'$ and creating one at $\mathbf{r}$ [7]. In a normal material, there is no coherence between these two actions when $\mathbf{r}'$ is far away from $\mathbf{r}$, and $n$ decays exponentially for large $|\mathbf{r} - \mathbf{r}'|$. But with Bose condensation, this is not the case; since a macroscopic number of the bosons are in the same single-particle quantum state, $n$ instead approaches a constant at large $|\mathbf{r} - \mathbf{r}'|$. This type of coherence between distant parts of the material is called "off-diagonal long-range order," and has become the standard test for whether superflow is possible.

We can now look at a simple model of solid helium to see how off-diagonal long range order is impossible in a perfect crystal.[2] For a zero temperature perfect helium crystal, the system is in its ground state. We assume that the atoms sit in orbitals $\phi(\mathbf{r})$ that are localized, such that $\phi(\mathbf{r}) \to 0$ quickly on the scale of the lattice spacing. The ground state is then

$$\Psi_G = \sqrt{\frac{1}{N!}} \sum_P \prod_{j=1}^{N} \phi(\mathbf{R}_j - \mathbf{r}_{P_j}); \tag{5}$$

this is a product of orbitals localized at the $N$ lattice sites $\mathbf{R}_j$, with a sum over permutations

[2] The following explanation comes from Ref. [1].

of particle labels. Inserting this into Eq. (3), we obtain

$$n(\mathbf{r}, \mathbf{r}') = \frac{1}{N} \sum_i \phi(\mathbf{R}_i - \mathbf{r}) \phi(\mathbf{R}_i - \mathbf{r}'). \tag{6}$$

Since $\phi(\mathbf{R}_i - \mathbf{r})$ has appreciable magnitude only when $\mathbf{r}$ is near $\mathbf{R}_i$, then as $|\mathbf{r} - \mathbf{r}'| \to \infty$, the density matrix $n \to 0$. In second-quantized language, removing a boson from a certain lattice position will overlap only with putting it back near the same position, since all of the other lattice sites are already filled. With no possibility for long-range phase coherence, Bose condensation is therefore impossible in a perfect solid.[3]

After ruling out superflow in perfect crystals, theorists continued to search for other ways that a solid could have intrinsic "supersolid" properties. A reasonable place to start was to consider the possibility that zero-temperature solids could be incommensurate—there was no fundamental reason that the number of atoms had to equal the number of lattice sites. An incommensurate solid would have either vacancies or interstitial atoms at zero temperature, and it was argued that these point defects had the possibility of Bose condensing [8]. As a simple example, note that a state with $N$ helium atoms similar to Eq. (5), but residing on a larger number of lattice sites $N_L$, would have a density matrix [1]

$$n(\mathbf{r}, \mathbf{r}') = \frac{N_L - N}{N_L^2} \sum_{i,j=1}^{N_L} \phi(\mathbf{R}_i - \mathbf{r}) \phi(\mathbf{R}_j - \mathbf{r}'). \tag{7}$$

Since vacant sites exist at large distances away from one another, now $n$ can be appreciable even when $|\mathbf{r} - \mathbf{r}'|$ is large; removing a boson at a certain lattice site can be paired with adding one at a vacant site far away.[4] The movement of vacant sites leads to a picture of a noninteracting gas of point vacancies, which has the possibility of Bose condensing and supporting a superflow [1]. Thus, if solid helium has an incommensurate ground state at zero temperature, it is possible that it intrinsically becomes "supersolid" at low temperatures.

At the time of Kim and Chan's initial discovery, it was the condensation of intrinsic vacancies that seemed the most probable explanation for NCRI in solid helium [9]. Kim and Chan suggested that there were likely to be many more vacancies and other defects in helium confined in their porous Vycor glass sample, which could explain why previous experiments were unsuccessful [4].

Shortly thereafter, however, the vacancy condensation explanation began to fall apart. First, experimental efforts have found no evidence for intrinsic vacancies in bulk solid helium [10]. In addition, simulations showed that when point defects are introduced throughout the sample, the

[3]Liquid helium has no such constraint, since it has no long-range configurational order: its atoms are not confined to sit on a lattice.

[4]The key difference between Eq. (6) and Eq. (7) comes in the $i \neq j$ terms in Eq. (7).

defects always phase separate, leaving a commensurate crystal behind; this ruled out a possible metastable incommensurate state [11]. For these reasons,[5] evidence now points away from NCRI being an intrinsic effect of perfect crystalline solid helium.

# 4 The importance of crystal defects

Once superflow has been ruled out for perfect crystals and helium is found to have no point defects at zero temperature, the next step is to check whether larger imperfections in the helium crystal may play a role. And indeed, macroscopic defects have been found to have a large effect on the NCRI phenomenon.

Interest in the effects of crystal imperfections has been spurred by current work done at Cornell by Sophie Rittner and John Reppy [12]. In 2006, after confirming Kim and Chan's NCRI results,[6] they found that they could eliminate the nonclassical effects by melting and slowly refreezing the helium sample: this annealing presumably resulted in fewer crystal defects. Figure 2 shows Rittner and Reppy's data, where annealing can be seen to markedly affect, or even completely remove, the NCRI signal. Like Kim and Chan, they found a dissipation peak at the NCRI transition that was largest when the NCRI effect was largest. The annealed samples that produced no NCRI signal also showed no dissipation peak; in addition, they had a higher overall $Q$ value, further suggesting a lower degree of crystal imperfection. This experiment provided strong evidence that the disorder caused by imperfections is important to the NCRI effect.

Another recent experiment (which did not involve NCRI) made a strong case for the importance of grain boundaries in solid superflow. Sasaki *et al.* [13] set up a 1 cm vertical tube filled partway with solid helium and open at the bottom; solid helium also filled the region around the tube, but to a lower level than inside (see Fig. 3 for a photograph). The idea was to observe whether the inner level would be pulled down by gravity to equilibrate to the outer level, as one would usually expect for a fluid. Looking directly at the solid through a window, they watched, and amazingly found that the solid did slowly flow for a few of their samples—and the flow occurred only in the samples in which grain boundaries were visible. Assuming the same mechanism underlies both this superflow and NCRI, large defects seem essential.

In the absence of the ability to directly observe the microscopic construction and dynamics of crystal defects in solid helium, path integral Monte Carlo simulations have been useful in exploring the effects of imperfections. Since the pairwise interatomic potential for helium is well-known,

[5] Also, Monte Carlo methods found that vacancies and interstitials have activation energies around 10-20 K, making it unlikely that the NCRI effects seen in the mK range could have anything to do with thermally activated defects [11].

[6] Rittner and Reppy also used a cubical oscillator to rule out the possibility that there was simply a layer of superfluid forming along a cylindrical wall.

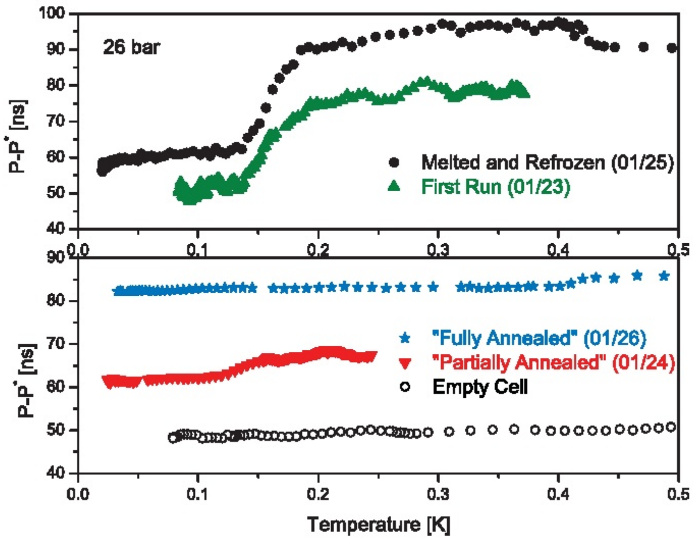


Figure 2: Rittner and Reppy's observation of NCRI [12], showing the effects of annealing. The partially and fully annealed samples have much smaller drops in the resonant period, indicating a strong dependence of NCRI on sample history and defects.

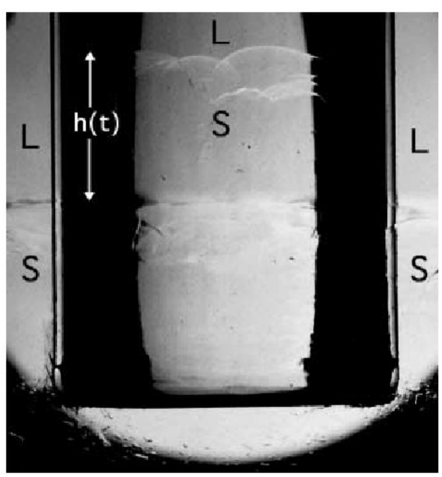


Figure 3: Sasaki *et al.*'s photograph of solid helium in a 1 cm test tube, from Ref. [13]. S denotes solid helium, L denotes liquid helium. The solid-liquid interface is higher inside the tube by $h(t)$; if (and only if) grain boundaries are present, the interface is observed to fall to the outside level.

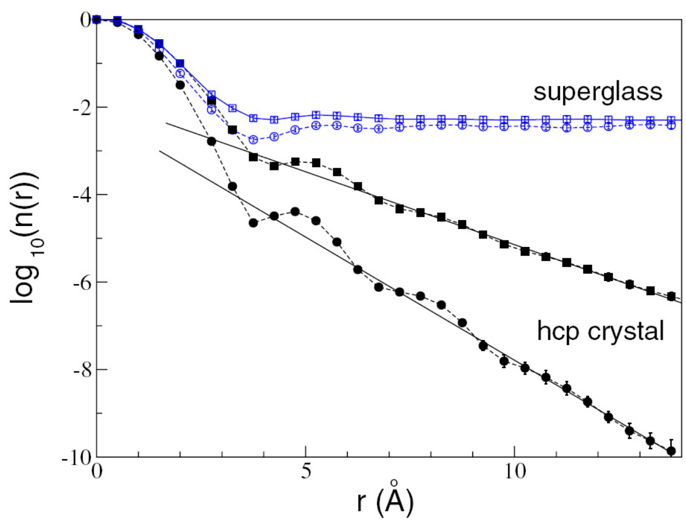


Figure 4: From [14]. The single-particle reduced density matrix $n(r)$ for the crystalline and "superglass" phases of helium 4, computed from path integral quantum Monte Carlo simulations. The perfect hcp crystal has an exponentially decaying density matrix, meaning there is no phase coherence at long distances. The superglass phase, however, displays off-diagonal long-range order.

such simulations can be used to obtain reliable results [1]. This technique has recently been used to clearly demonstrate the difference in superflow properties between a perfect commensurate crystal and one riddled with defects. Using path integral Monte Carlo simulations, Boninsegni *et al.* [14] studied a perfect hcp helium crystal along with a random arrangement of particles that was quenched to a low temperature. They study the behavior of the single-particle density matrix $n(r)$ [as in Eq. (4)], here averaged over a sphere due to rotational symmetry. Their results for $n(r)$ are shown in Fig. 4. The perfect crystal shows exponential decay of $n(r)$, indicating so-called insulating behavior, like a normal solid. The quenched system, however, shows off-diagonal long-range order, indicating the possibility for superflow. Furthermore, they found that while the quenched system retains a specific density profile over long times that breaks translational symmetry, it displays no direct long-range order. In a way reminiscent of a glass, the density of the helium atoms shows distinct clumps that remain stuck in a certain configuration, but this configuration does not have the long-range order of a crystal. This lack of direct long-range order is in essentially equivalent to having a solid with lots of crystal defects. They dubbed their new predicted phase a "superglass," and argued that the NCRI effect seen in crystalline helium with defects was likely a similar situation [14].

These experimental and computational results strongly suggest that crystal defects are fundamentally related to the effect of NCRI in solid helium. The exact mechanism involved is, however, unknown. The current most popular qualitative picture is one of grain boundaries with superfluid properties that run through the sample, along which supercurrents of helium atoms can flow. Boninsegni *et al.* imagine a "generalized superfluid grain-boundary, which may include a foam-shaped superglass network interpenetrating the polycrystal" [14]. A similar explanation can be imagined as a sort of slush, in which crystals of solid helium exist in equilibrium with superfluid liquid that fills the cracks. The former picture has recently been reinforced by Monte Carlo simulations that demonstrate grain boundaries that are mechanically stable against forming cracks, and along which phase coherence can extend [15].

# 5 A different explanation: vortex liquid

Very recently, Philip Anderson proposed a considerably different possible explanation for the experimental results [16]. Drawing an analogy to the "pseudogap" phase of high-temperature superconductors, he believes that the observed NCRI transition happens due to critical fluctuations that signify only the precursors to supersolidity, and that the supersolid phase would occur only at a much lower temperature. In Anderson's picture, the supersolid phase is an incommensurate density wave at zero temperature, forming a lattice through which the underlying "fluid" helium can freely flow. The lattice is constrained to move with the container, and carries the majority of the rotational inertia. The helium in the lattice is described using a local phase $\phi$; the velocity of the helium is given by the gradient of the phase:

$$v = \frac{\hbar}{m}\nabla\phi, \tag{8}$$

where $m$ is the mass of a helium atom. Then the helium can rotate with the lattice only through the formation of quantized vortices, each of which contributes a velocity

$$v = \hbar/mr, \tag{9}$$

where $r$ is the distance from the vortex. Notice that these vortices must move into the bulk of the material in order for the helium to move along with the lattice. Similar to Kosterlitz-Thouless behavior in thin films, the phase in which NCRI is observed would be filled with thermally activated vortex lines that viscously travel through the sample. The NCRI would be explained as follows: At low temperatures, the vortices would move too slowly, the helium would not move with the lattice, and thus a smaller moment of inertia would be observed. At higher temperatures, the vortices would move in quickly and the full inertia would be observed. And importantly, when the timescale of vortex movement matches the period of rotation, the viscous travel of the vortices through the sample would show up as a peak in the dissipation. Thus Anderson suspects that the peak in dissipation found by Kim and Chan is due to this matching of timescales, and that testing NCRI at different frequencies is an important next step to understanding whether this picture is correct.

Anderson argues that this explanation is more reasonable than one involving simply structural defects due to the otherwise unlikely coincidence that the observed critical velocity is so close to a small number of vorticity quanta. Still, there is not yet a way to reconcile this theory with the observed dependence on the degree of structural disorder.

# 6 Conclusion

It is still unclear how the puzzle of NCRI in solid helium will be resolved. The correct explanation must bring together many observed results: a macroscopic rotational inertia decoupling, a small critical velocity, and dependence on sample history, among others. Many explanations have been offered: supersolidity in the form of Bose-Einstein condensation of vacancies in an incommensurate ground state, superfluidity of extended lattice defects such as grain boundaries, and even a new form of vortex liquid state. None of these can yet stand on its own to explain NCRI. Perhaps some combination will lead to a better understanding of this curious phenomenon.

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