ENERGY EFFICIENT TURBO BASED SPACE-TIME CODER

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ABSTRACT

Recent studies have shown that Space-Time code is an effective approach to increasing the data rate over wireless channels. Turbo-based Space-Time codes take advantage of both the diversity techniques of Space-Time systems and the randomness of the Turbo codes. In this paper, we compare two Turbo-based Space-Time Codes and their approximate versions with respect to energy consumption and performance. The approximations are aimed at reducing the computational complexity and include reduction in the number of paths, number of iterations and datapath computations. Our analysis shows that the Space-Time Turbo code based versions should be used in applications where low energy consumption is the primary design objective, and Turbo trellis coded modulation space-time block code based versions should be used where performance is the primary design objective.

1. INTRODUCTION

The recent increase in the demand of high-speed and robust wireless data transmission has generated a lot of interest in development of Space-Time(ST) based systems. Space-Time codes incorporate both spatial and temporal diversities and was introduced as an exciting technique to overcome the severe fading in wireless channels [1, 2, 3]. While Space-Time block codes and Space-Time trellis codes achieve full spatial diversity on fading channels, they do not take advantage of temporal diversity in a time varying channel. Temporal diversity can be provided by concatenating the Space-Time code with a powerful outer code such as a Turbo code, and is the system under investigation here.

Turbo based Space-Time codes are very computation intensive, making it difficult to implement on wireless devices. In order to reduce the energy consumption, we have applied a combination of one or more approximations to the MAP based coder. Approximations include the popular Max-Log-Map technique, reduction in paths, and reduction in the number of iterations. We have considered two Turbo-based Space-Time systems: Space-Time(ST) Turbo code and Turbo Trellis Coded Modulation Space-Time Block Code (TTCM-STBC). For both the systems, we have developed a suite of algorithms with energy-quality trade-offs. This enables us to choose the most suitable algorithm depending on the energy and quality requirements. A comparison of the two systems reveals that the ST-Turbo based system is more energy-efficient and should be used for high SNR($\geq$ 16dB). The TTCM-STBC system, on the other hand, should be used when performance is the primary design constraint (SNR <16dB).

The rest of the paper is organized as follows. Section 2 briefly introduces Turbo based space time code systems. Section 3 compares the two systems with respect to performance and energy consumption. Section 4 describes the experimental setup and the effect of various approximation algorithms on the performance and energy consumption. Section 5 describes the performance and energy trade-offs in the two systems. Section 6 concludes the paper.

2. TURBO BASED SPACE-TIME CODE SYSTEMS

There are two types of Space-Time codes: Space-Time block code(STBC) and Space-Time trellis code(STTC). Space-time trellis codes achieve a full spatial diversity of TR in a T transmit antenna, R receive antenna system and also offer substantial coding gain. The disadvantage of this approach is that the decoding complexity grows exponentially as a function of both bandwidth efficiency and diversity order with fixed number of antennas [2]. The Space-Time block code [1],[3] on the other hand, has the advantage of simplicity in encoding and decoding and the disadvantage of not providing significant coding gain. While block-based and trellis-based schemes guarantee to achieve full spatial diversity on fading channel, they do not take full advantage of temporal diversity in a time varying channel. Temporal diversity can be provided through powerful channel coding and interleaving. Thus a concatenation of Space-Time codes with powerful outer codes such as Turbo codes can significantly enhance the performance [4], [5], [6]. In this paper, we consider two such systems: the ST Turbo code and TTCM-STBC.

For small frame sizes, random-like structure of Turbo code offers little advantage over other highly structured concatenation schemes. However, for large frame sizes in a quasi-static fading channel, performance of these structured codes drops as all the channels in the multiple antennas stay

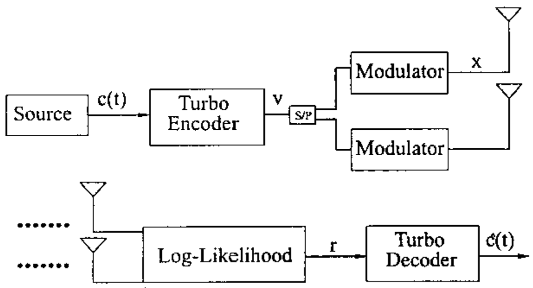

Fig. 1. Space-Time(ST) Turbo Code System.

in a deep fade. Turbo codes, on the other hand, can maintain the same performance due to the temporal diversity provided by the interleavers [4].

2.1. Space Time Turbo Codes

Figure 1 shows how the information bits are transmitted through multiple antennas in a ST Turbo system. The information $c(t)$ is first encoded in a Turbo encoder. The encoded bits v then pass through a serial to parallel converter(s/p) and mapped to a particular signal constellation. At each time t, the output from the modulator of each transmitting antenna is $x_{t,i}$, where i is the index of the transmitter. The signal at a receive antenna is a noisy superposition of the transmitted signals perturbed by Rayleigh fading. The wireless channel is modeled as a quasi-static channel, where the path gain is a constant within a frame and vary from one frame to anther. If one receive antenna is used and the two transmit antennas are far apart, the received signal can be safely assumed to be a linear superposition of the two signals from two different transmitters. A Log-likelihood of the transmitted bits are computed and used prior to the decoding process. The Log-likelihood probabilities are de-interleaved and fed to the concatenated Turbo decoder.

2.2. Turbo Trellis Coded Modulation Space-Time Block Codes

An alternative system model of concatenated Space-Time block code with two transmit and one receive antenna has been proposed in [5] and is shown in Figure 2. The information bit $c(t)$ is first encoded by a Turbo encoder, then passed to a Trellis Coded Modulation(TCM) encoder, where the Ungerböck code is used as the component code. The data are interleaved and fed into the Space-Time block code(STBC) encoder. STBC encoder takes advantage of the time diversity to send the symbols and the signed symbol conjugates in two consecutive time slots [1]. In each time slot, the symbols are transmitted through two transmit antennas. For example, at a given symbol period, symbol S_0 is transmitted from antenna I and symbol S_1 is transmitted from antenna II. In the next symbol period, antenna I sends $-S_1^*$

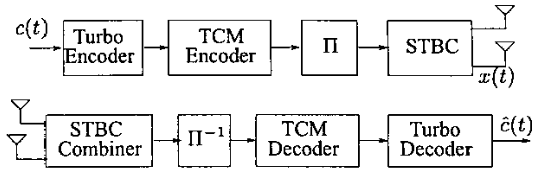

Fig. 2. Turbo Trellis Coded Modulation Space-Time Block Code(TTCM-STBC) System.

and antenna II sends S_0^*. After passing through a quasi-static Rayleigh fading channel, the corrupted symbols and the signed symbol conjugates are combined in the STBC combiner to reduce the fading effect. The combined symbols are then de-interleaved and passed to the TCM decoder. In the TCM decoder, MAP algorithm is carried out symbol by symbol to generate soft information of symbols. The Turbo decoder takes the soft output from the TCM decoder and makes hard decisions of the information bits. Inside the Turbo decoder, the Log-likelihood of the transmission bits are first computed before invoking the binary Turbo decoding.

2.3. Iterative Turbo Decoder

Both ST models proposed in the previous sections include a Turbo decoder in their systems. The iterative Turbo decoder shown in Figure 3 consists of two decoders that are serially concatenated. The component decoders are based on *maximum a posteriori*(MAP) probability that generates a soft estimate of the input sequence. The inner MAP decoder takes the received information sequence r_0 and the received parity sequence r_1 and produces a soft output Λ_{1e} that is interleaved and used as the *a priori* probability for the outer decoder. The inputs for the outer decoder are the parity sequence from inner decoder and the interleaved received information sequence r_0. The outer decoder produces *a priori* probability Λ_{1e} to improve the estimate for the information sequence at the inner decoder. This process is repeated multiple times; in the last stage of decoding, a hard decision is made after de-interleaving. While such an iterative decoding system significantly improves the error correcting abilities, it comes at a price of very high computational complexity.

The MAP algorithm minimizes the symbol(or bit) error probablity[7].

It computes the log-likelihood ratio Λ, which can be expressed as a combination of forward recursion, backward recursion and branch metrics, denoted as α, β, and γ respectively.

$$\Lambda(c_t) = \log \frac{\sum_{(l',l)\in B_t^1} \alpha_{t-1}(l')\gamma_t^1(l')\beta_t(l)}{\sum_{(l',l)\in B_t^0} \alpha_{t-1}(l')\gamma_t^0(l')\beta_t(l)} \quad (1)$$

for $0 \leq t \leq \tau$, where τ is the received sequence length, B_t^i

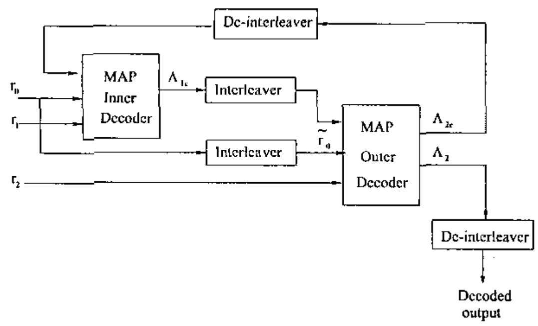

Fig. 3. An Iterative Turbo Decoder based on MAP Algorithm.

is the set of transitions $S_{t-1} = l' \rightarrow S_t = l$ that are caused by the input bit $c_t = i, i \in (0, 1)$.

3. SYSTEM COMPARISON

In this paper, we consider the BER performance and energy consumption of a ST Turbo code based system and a TTCM-STBC based system. In all the simulations, code rate 1/3 is used for the Turbo codes. The channel is a block Rayleigh fading channel with block length 260, in which the path gain remains constant for duration of 260 symbols. ST Turbo code based system employs pseudo-random interleaver with interleaver size of 1300 and 2600, and a modulation scheme with 4-PSK modulation. TTCM-STBC based system employs S-random interleaver with interleaver size of 1300 and 2600, and a modulation scheme with 8-State 8-PSK trellis code. Both systems use two transmit antennas and one receive antenna.

To calculate the energy, we use Simple-Scalar based Wattch [8], a framework for architectural level power analysis. The processor in Wattch represents a state of the art general purpose processor with the following default parameters: issue width 4, window size 16, data-path width 64, L-1 data cache size 128:32:4 (128K is the data cache size, 32K is the line size and 4 is the set associativity). L-2 data cache 1024:64:1, L-1 instruction cache 512:32:1 and L-2 instruction cache similar to the L-2 data cache.

Effect of interleaver size: Figure 4 shows the BER performance of the two systems with respect to different interleaver size. The performance is distinctly better as interleaver size increases. TTCM-STBC based systems outperform the ST Turbo based systems with the same interleaver size. At low SNR, the BER performance of ST Turbo system with N=2600 is comparable with the TTCM-STBC system with N=1300. However, at high SNR(e.g 16dB), the ST Turbo system with N=2600 achieves lower BER.

Figure 5 shows the energy consumption for the two systems with respect to different interleaver sizes. The numbers are normalized according to ST Turbo code with interleaver size $N = 1300$. The energy consumption of the TTCM-STBC scheme is about 63% more than ST Turbo code scheme with the same interleaver size, and this ratio does not change much when the interleaver size increases to 2600. In fact, the energy consumption of each component(data-path, memory, clock+bus) increases proportionately as the interleaver size increases. An interesting fact is that, at SNR=16dB, the ST Turbo system with N=2600 has better performance and lower energy consumption compared with TTCM-STBC system with N=1300.

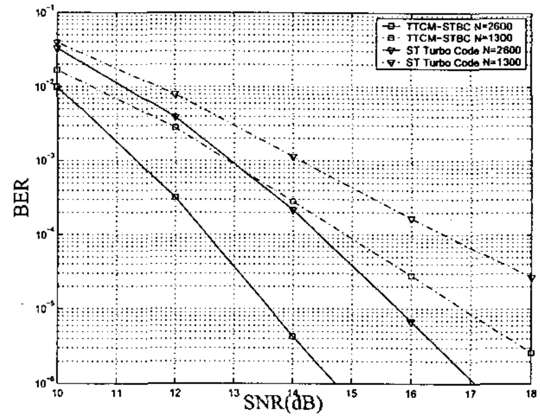

Fig. 4. BER performance of ST Turbo Code and TTCM-STBC.

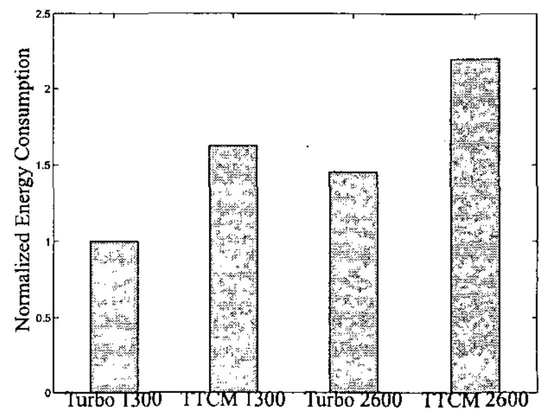

Fig. 5. Energy consumptions of two systems with different interleaver sizes at SNR=16dB.

From Figure 4 and 5, we conclude that, in general TTCM-STBC scheme should be used when performance has higher priority and ST Turbo code scheme should be used when reduction in energy consumption has higher priority.

Scaling of Extrinsic Information: The performance of the MAP algorithm improves when a scaling factor is applied to the iterative decoding procedure. The extrinsic and intrinsic information passing among iterations become more and more correlated. The scaling factor is used as a trade-off between the need to provide a decoder with full-scale extrinsic information and the concern to minimize correlation effects [9]. This scaling factor is achieved by trial and error.

The ST-Turbo system obtains 0.6dB performance gain without any energy degradation if the extrinsic information at each iteration is scaled by a factor of 0.68. TTCM-STBC system gains 0.2dB when a scaling factor of 0.7 is applied. Figure 6 shows the scaled version for ST-Turbo N=1300 and TTCM-STBC N=2600. In the rest of this paper, we assume that scaling has been applied to both the ST-Turbo and the TTCM-STBC systems.

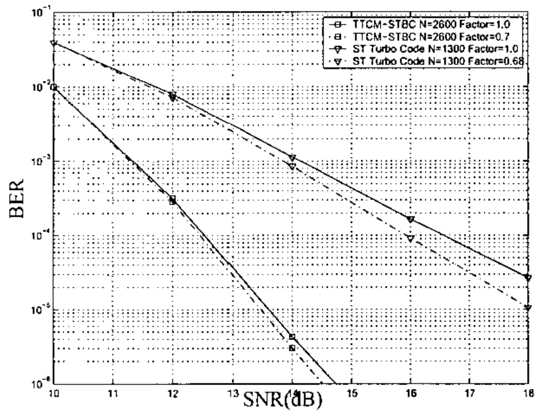

Fig. 6. Effect of Scaling of Extrinsic Information

4. APPROXIMATIONS

In this section, we investigate the effect of application of different approximations on the energy and quality for ST-Turbo codes and TTCM-STBC. The approximations include reduction in path, stopping criteria, and loop transformation.

4.1. Max Log Map

The MAP decoding algorithm requires a large number of exponentiations and multiplications. One popular way to simplify the computations is to substitute the logorithm of the summation of a series of exponetiations by selecting the largest exponent [7]. The resulting algorithm is called the Max Log Map algorithm. α and β can also be simplified in a similar way. Both α and β approximations can be applied separately from Max-Log-Map, and result in very little performance degradation (compared to the MAP algorithm).

4.2. Reduction in Paths

Pruning algorithms, such as T and M algorithms [10], that have been proposed for the classical Viterbi algorithm can be also used for Turbo-based decoder. T algorithm retains only limited path information in the trellis, if a threshold value is exceeded. At each point on the trellis, the path metrics are compared with the threshold value; if the path metric exceeds the threshold value, the path is discarded. The threshold value increases with the trellis depth.

4.3. Stopping Criteria

Stopping criteria have historically been one of the most effective ways to reduce computation complexity while retaining good performance. We have considered two early termination methods. One is cross entropy (CE) [11], which stops the iterations based on the cross entropy between distributions in each iteration. The other method is log-likelihood and extrinsic information (LLR) [12], which uses the sign changes in the log-likelihood and extrinsic information to determine the termination point.

4.4. Loop Transformation

The MAP algorithm in the serially concatenated Turbo decoder consists of many nested loops. Here we have applied loop unrolling to minimize the energy due to memory accesses. One big advantage of loop unrolling is that it gives no performance degradation.

4.5. Approximation results on ST-Turbo Codes

All the simulations presented in this section are for pseudo-random interleaver of size 1300. The energy saving is calculated at SNR = 16dB. The performance degradation is calculated with respect to MAP-based decoder with scaled extrinsic information at BER = 10^{-4}.

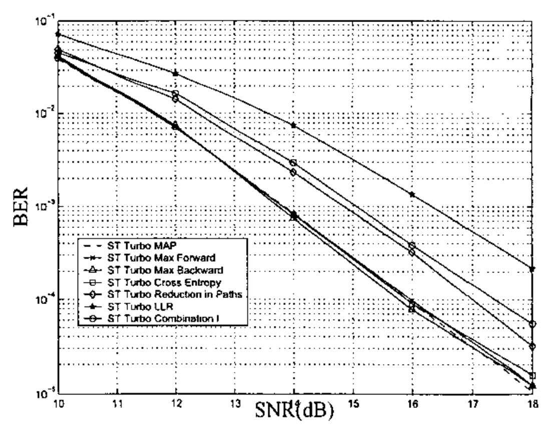

Fig. 7. BER performance for different approximations in ST-Turbo system

Figure 7 shows the BER performance for different approximations and Table 1 shows the energy savings and performance drops corresponding to each approximation. When

Table 1. Quality and Energy Savings for various approximations on ST-Turbo Codes at BER = 10^{-4}

Approximation	Performance Drop	Energy Saving
MaxLogMap on backward recursion	0.1 dB	4.73%
MaxLogMap on forward recursion	0.2 dB	5.75%
Reduction in Paths	1.2 dB	8.1%
Cross Entropy	0.2 dB	3.7%
LLR	2.4 dB	8.4%
Loop Unrolling	-	4.2%
Combination I	1.4 dB	44.8%

Max Log Map is applied on forward and backward recursions we obtained around 5% energy savings with a small SNR drop. When reduction in paths approximation is applied to both decoders, we obtained 8.1% energy savings with 1.2dB SNR drop at a BER of 10^{-4}. Cross Entropy achieved an energy savings of only 3.7% compared to LLR, which achieved an energy savings of 8.4%. In terms of quality, Cross Entropy showed almost 2 dB performance increase compared to LLR (see Figure 7). We applied loop unrolling by a factor of 2 to the entire code and a factor of 3 to the decoding loop to obtain 4.2% energy savings. Unrolling the entire code by a factor of 3 resulted in a very small increase in the energy savings. A combination of Max-Log-Map in both backward and forward recursion, reduction in paths, cross entropy and loop unrolling (Combination I) results in large energy savings of 44.8% with reasonable performance degradation(1.4dB drop) compared to a MAP with scaled extrinsic based ST-Turbo system at BER=10^{-4}.

4.6. Approximation results on TTCM-STBC

The simulations of TTCM-STBC are based on rate 1/3 Turbo code with S-random interleaver of size N=2600. The energy savings are calculated at SNR=16dB; the performance degradation is calculated with respect to MAP-based decoder with scaled extrinsic information at BER= 10^{-5}.

Table 2 lists the energy saving and performance degradation due to different approximations. Path reduction approximation offers a 10.3% energy saving at the price of 1.4dB drop. LLR stopping criteria achieved an energy saving of 11.2% with 1.8dB drop in SNR. Applying loop unrolling by factor of two saved about 4% energy without sacrificing performance. A combination of Max-Log-Map on backward and forward recursion, reduction in paths, LLR and loop-unrolling(Combination II) results in very large energy saving(48.9%), with 3.9dB performance degradation compared to MAP with scaled extrinsic based TTCM-STBC system at BER=10^{-5}.

Table 2. Quality and Energy Savings for various approximations on TTCM-STBC at BER=10^{-5}

Approximation	Performance Drop	Energy Saving
Max Log Map on forward recursion	0.2 dB	6.3%
Max Log Map on backward recursion	0.1 dB	5.2%
Reduction in Paths	1.4 dB	10.3%
Cross Entropy	0.3 dB	4.3%
LLR	1.8dB	11.2%
Loop Unrolling	-	4.0%
Combination II	3.9 dB	48.9%

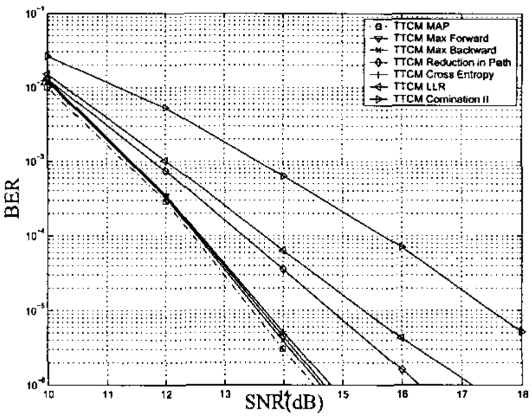

Fig. 8. BER performance for different approximations in TTCM-STBC system

5. ENERGY-PERFORMANCE TRADE-OFFS

In this section, we analyze the two systems, ST-Turbo and TTCM-STBC, with the objective of determining the conditions in which each system should be invoked. Figure 9 shows the performance curves for the two Turbo based systems. The solid lines indicate the original system performance curves using MAP algorithm with scaled extrinsic; the dotted lines are the performance curves using combinations of approximations. Note that the TTCM system with N=2600 (case A) achieves the best performance. Table 3 lists the energy comparisons and performance drops with respect to case A.

The energy-performance trade-off space is bounded one side by TTCM, N=2600 (case A) which has the best performance and on the other side by ST-Turbo Combination I with N=1300 (case B+) which has the lowest energy. For the same performance, the ST Turbo based system has lower energy compared to the TTCM based system. For example, a combination of approximations on ST-Turbo with N=2600 (case C+) consumes 41% energy with 3.7dB SNR drop while

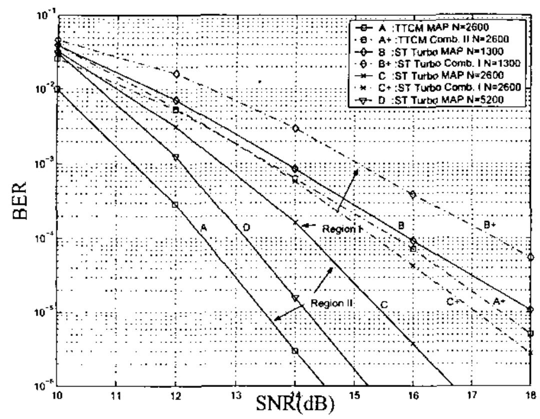

Fig. 9. BER Performance of the two systems.

a combination of approximations on TTCM with N=2600 (case A+) consumes 51% energy with larger drop (4.0dB) in SNR. Thus TTCM system should only be used when ST-Turbo based systems are unable to achieve the desired performance.

In order to determine the region of operation of TTCM-based system, we increased the size of the interleaver of ST-Turbo since this enhances the performance. However, when N increases from 2600 to 5200, the performance gain did not scale at the same rate as the increase in energy consumption. In fact, ST Turbo with N=5200 (case D) has higher energy consumption and 0.8 dB drop in SNR and is clearly not a desired choice.

From the above analysis, we conclude that the TTCM-based system should operate in Region II bounded by curves A and C which translate to SNR of 13.4-15.5 dB. The ST-Turbo system should operate in Region I which corresponds to an SNR of >15.5 dB.

Table 3. Energy Consumption and SNR Drop at BER = 10^{-5}

Case Setup	Energy Consumption	SNR Drop
A	100%	0.0 dB
A+	51%	4.0 dB
B	43%	4.5 dB
B+	25%	5.6 dB
C	66%	1.9 dB
C+	41%	3.7 dB
D	120%	0.8 dB

6. CONCLUSION

In this paper, we have studied two Turbo based Space-Time systems, ST Turbo system and TTCM-STBC system. Various approximations have been applied to these two systems to reduce computational complexity. The approximation includes reduction in the number of paths, number of iterations and data-path computation. We propose two regions of operation for the two systems. ST Turbo system operates in region I where SNR is high(15.5-20.4dB) and power is the primary design constraint; TTCM-STBC system works in region II where SNR is low(13.4 -15.5dB) and performance is a more important design objective.

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