

## A LITTLE BIT OF LOGIC

### DEFINITIONS

- An **argument** is a set of two or more statements, where one purports to follow from the others. The one that supposedly follows from the others is called the **conclusion**, and the statements that purport to provide support for the conclusion are called **premises**.
- A **statement** is a sentence that has a truth-value, that is, a sentence that is either true or false. A sentence is a statement if and only if it has a truth value. So, for instance, 'All dogs have four legs' is a statement, while 'What time is it?' is not.
- A **proposition** is a truth or a falsehood that can often be expressed by a number of different, but logically equivalent, statements. Two statements express the same proposition if and only if they have exactly the same meaning. For instance, 'John Smith is married' and 'John Smith has a spouse' both express the same proposition. But 'John Smith is married' and 'John Smith has a wife' do not necessarily express the same proposition, since John Smith could be married to a man.
- A **valid** argument is one where the truth of the premises entails (i.e., necessitates or guarantees) the truth of the conclusion. An argument is valid if and only if, in no possible world, are its premises all true and its conclusion false. Arguments that are not valid are **invalid**. (Note that only arguments, and not statements or propositions, can be properly called valid or invalid. By contrast, only statements and propositions, not arguments, can be properly called true or false.)
- An argument is **sound** if and only if *both* of the following are true: (1) it is valid, and (2) all of its premises are true. Arguments that are not sound are **unsound**.
- A **deductive argument** is one in which the truth of its premises purports to establish that its conclusion is certainly true.
- An **inductive argument** is one in which the truth of its premises purports to establish that its conclusion is probably true.
- An inductive argument is **strong** if and only if the truth of its premises makes its conclusion more likely true than false. The more probable it makes the truth of its conclusion, the stronger the inductive argument is. Inductive arguments that are not strong are **weak**.
- An inductive argument is **cogent** if and only if *both* of the following is true: (1) it is strong, and (2) all of its premises are true.

- A **well-crafted version** of an argument stated in ordinary English is one that has been restated so as to make its logical features explicit by employing uniform language (e.g., using the same statement consistently to refer to the same proposition), excluding excess verbiage, and making each step of the argument explicit by numbering the premises and conclusions.
- A **class** refers to a set of things, such as the set of things that are humans or the set of things that are politicians, or even the set of things that are George W. Bush.
- A **term** is a word or a phrase that refers to some class (e.g., 'humans', 'politicians', 'George W. Bush', etc.).
- An **argument form** is a pattern of reasoning that a number of different arguments can share. The form of an argument is obtained by taking a well-crafted version of it and uniformly replacing a distinct capital letter for each distinct statement (or term) throughout the entire argument, making the logical connections (which are expressed by words such as 'if', 'or', 'all', 'and', 'not', 'some', and 'only if') between these statements (or terms) perspicuous where necessary.
- An argument that results from uniformly replacing the capital letters in an argument form with statements (or terms) is called a **substitution instance** of that form.
- A **counterexample** to an argument is a substitution instance of its form where the premises are all true and the conclusion is false. Since the validity and invalidity is a matter of form and since only an invalid argument can have true premises and a false conclusion, a counterexample to an argument proves both that it and its form are invalid.

### SOME FAMOUS ARGUMENT FORMS

Let 'P' and 'Q' stand for propositions in propositional arguments. And let 'S', 'M', 'P' stand for terms in categorical arguments.

#### Some Common Valid Argument Forms:

- *Modus Ponens*  
P1: If P, then Q.  
P2: P.  
C: Therefore, Q.
- *Modus Tollens*

P1: If P, then Q.  
P2:  $\sim$ Q. (Let ' $\sim$ Q' stand for 'it is not the case that Q'.)  
C: Therefore,  $\sim$ P.

- *Disjunctive Syllogism*  
P1: Either P or Q.  
P2:  $\sim$ P.  
C: Therefore, Q.
- *Hypothetical Syllogism*  
P1: If P, then Q.  
P2: If Q, then R.  
C: Therefore, if P, then R.
- *Constructive Dilemma*  
P1: If P, then R.  
P2: If Q, then R.  
P3: P or Q  
C: Therefore, R.
- *Categorical Syllogism AAA-4*  
P1: All S are M.  
P2: All M are P.  
C: All S are P.
- *Categorical Syllogism OAO-3*  
P1: Some M are not P.  
P2: All S are M.  
C: Some S are not P.

Some Common Invalid Argument Forms:

- *Affirming the Consequent*  
P1: If P, then Q.  
P2: Q.  
C: Therefore, P.
- *Denying the Antecedent*  
P1: If P, then Q.  
P2:  $\sim$ P.  
C: Therefore,  $\sim$ Q.

- *Categorical Syllogism AAA-3*  
P1: All M are P.  
P2: All M are S.  
C: All S are P.
  
- *Categorical Syllogism IOO-1*  
P1: Some M are P.  
P2: Some S are not M.  
C: Some S are not P.

#### THE PROCEDURE FOR CONSTRUCTING A COUNTEREXAMPLE TO AN ARGUMENT

1. Write out a well-crafted version of the argument.
2. Identify the form of the argument. Look for repeated references to the same class (or to the same proposition) and then rewrite the argument uniformly substituting a distinct capital letter for each distinct class (or proposition). In a key to your abbreviations, indicate which capital letters stand for which classes (or propositions).
3. Find English terms (or statements) that, if substituted for the capital letters in the conclusion of the argument form, produce a *well-known* falsehood. Substitute these English terms (or statements) for the relevant capital letters uniformly throughout the argument form.
4. Find additional English terms (or statements) that, if substituted uniformly for the remaining capital letters in the argument form, produce premises that are *well-known* truths.

Important Note: If you have trouble coming up with a **substitution instance** whose premises are well-known truths and whose conclusion is a well-known falsehood, then come up with a substitution instance whose premises are true and whose conclusion is false in some possible world and then describe that possible world so as to make clear how, in that world, the premises would be true and the conclusion would be false.

#### EXAMPLE 1

If God exists, then life has meaning. But there is no God. Therefore, life is meaningless.

*Step 1:*

1. If God exists, then life has meaning.
2. It is not the case that God exists.

So, 3. It is not the case that life has meaning.

*Step 2:*

G: God exists.

L: Life has meaning.

1. If G, then L.
2. It is not the case that G.

So, 3. It is not the case that L.

*Step 3:*

1. If G, then Al Gore is a U.S. citizen.
2. It is not the case that G.

So, 3. It is not the case that Al Gore is a U.S. citizen. [False]

*Step 4:*

1. If Al Gore is the President of the U.S., then Al Gore is a U.S. citizen. [True]
2. It is not the case that Al Gore is the President of the U.S. [True]

So, 3. It is not the case that Al Gore is a U.S. citizen. [False]

Thus the argument given in Example 1 is invalid.

## EXAMPLE 2

Some catholic priests are bachelors. All bachelors are single. Fred Feldman is married.  
So Fred Feldman isn't a catholic priest.

*Step 1:*

1. Some catholic priests are bachelors.
2. All bachelors are single.
3. Fred Feldman is not single.

So, 4. Fred Feldman is not a catholic priest.

*Step 2:*

C: catholic priests

B: bachelors

S: people who are single

F: people who are identical to Fred Feldman

1. Some C are B.
2. All B are S.
3. F is not S.

So, 4. F is not C.

*Step 3:*

1. Some animals are B.

2. All B are S.
3. Fido is not S.

So, 4. Fido is not an animal. [False] Imagine the possible world where Fido is a dog.

*Step 4:*

1. Some animals are lizards. [True]
  2. All lizards are reptiles. [True]
  3. Fido is not a reptile. [True]
- So, 4. Fido is not an animal. [False]

*Step 5:*

One possible world is one in which Fido is a dog and everything else is as it is in the actual world. In that possible world, the premises would be true and the conclusion would be false.

Thus, the argument given in Example 2 is invalid.

#### **SOME COMMON TYPES OF INDUCTIVE ARGUMENTS**

- *Empirical Generalization*  
P1: N percent of observed S have property x.  
C: Therefore, N percent of all S have property x.
- *Argument by Analogy*  
P1: S and P share properties a, b, c....  
P2: S has the further property z.  
C: Therefore, P also has the further property z.
- *Argument from Best Explanation*  
P1: P.  
P2: One possible explanation for P is Q.  
P3: No other possible explanation provides as good an explanation for P as Q does.  
C: Therefore, Q.