PHY 132 LAB : RC time constant

Introduction

In this lab we look at the transient response of an RC circuit by digitizing the v(t) waveform and fitting it to appropriate non-linear functions, namely decaying exponential with finite asymptotes. This circuit is the basis of all electronic timing circuits and filters, such as the tone controls on a stereo amplifier. Text Reference: Wolfson 28.6.

THEORY

Consider the RC circuit shown in Fig. 1. We start with the switch S closed on B. Let us assume that the switch has been closed for a very long time (the meaning of "a very long time" will be made clear later). In this case, the capacitor will be discharged, because the potential across it must be zero. Let us assume that at t = 0 we throw S to A. The battery will effectively move electrons from the positive side of the capacitor (the side which will end up as the positive side) over to the negative side. In other words, a current is established, which charges the capacitor. Eventually the voltage across the capacitor becomes equal to the battery emf, and the current ceases to flow. We approach a static situation (maximum voltage, zero current) asymptotically.

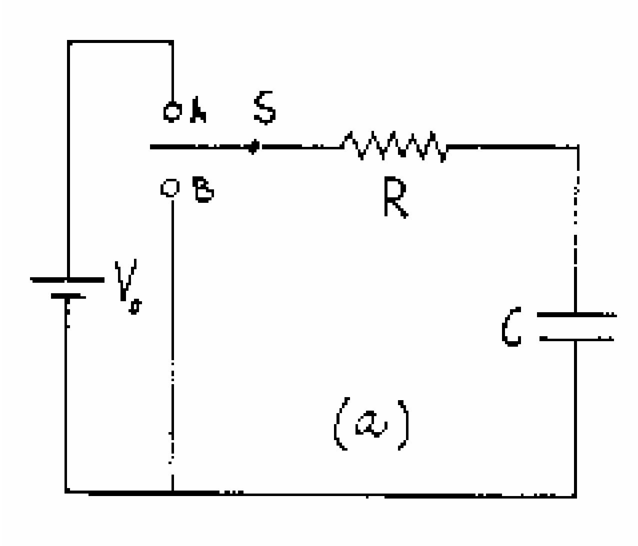

Fig. 1. Schematic of RC circuit with charge/discharge switch.

In order to study the transient response of the above circuit we start from Kirchhoff's loop rule. If we go around the circuit of Fig. 1 in a clockwise direction, we obtain the following equation

$$V_o - iR - q/C = 0 \qquad \text{eq. 1}$$

Using the fact that $i = dq/dt$ and the initial condition $q(0) = 0$, we obtain a differential equation (see phy131 textbook) with solution

$$q(t) = CV_O\,(1 - e^{-t/RC}). \qquad \text{eq. 2}$$

Then the voltage across the capacitor is given by

$$V_c(t) = q(t)/C = V_O\,(1 - e^{-t/RC}) \quad \text{(rising)} \qquad \text{eq. 3}$$

For $t >> RC$, the exponential term becomes very small, so that $V_C(t)$ tends to V_O, as expected. Let us assume that we wait for a time $t >> RC$ and throw the switch to B. In this case, the loop rule leads to

$$Ri + q/C = 0, \qquad \text{eq. 4}$$

with the initial condition $q(0) = CV_O$. The solution to this equation is

$$q(t) = CV_O\, e^{-t/RC,} \qquad \text{eq. 5}$$

so that

$$V_C(t) = V_O\, e^{-t/RC} \quad \text{(falling)} \qquad \text{eq. 6}$$

Equation 3 gives the capacitor voltage during the charging process, while Eq. 6 corresponds to the voltage across the capacitor during discharging. The value of time constant $\tau=RC$ is the same for both. In the most general form, $V(t)$ may start at some initial value V_0 and asymptotically approach some final value $V\infty$ beginning at time t_0. $V(t)$ may be either rising or falling. These cases are conveniently written as follows:

$$V(t) = (V0 - V_\infty)e\text{-}^{(t-t0)/\tau} + V\infty \text{ (falling)} \qquad \text{eq. 7}$$

This could be paraphrased as "output voltage is change times falling exponential plus final value".

$$V(t) = (V_\alpha - V0)(1 - e^{-(t-t0)/\tau}) + V0 \text{ (rising).} \qquad \text{eq. 8}$$

This could be paraphrased as "output voltage is change times rising exponential plus initial value".

Eyeball Method

One can eyeball the value of τ by estimating the time at which 1/e (~0.368) ~ 1/3 of the total change remains, or equivalently the time when ($1-e^{-1}$) ~ .632 ~ 2/3 of the total change has occurred. These cases are illustrated in Fig. 2.

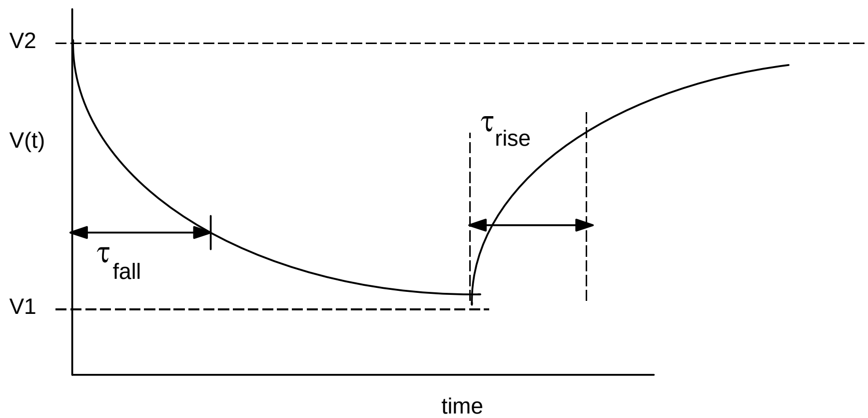

Fig. 2 Illustration of eyeball method to extract time constant for either falling or rising exponential relaxation. The vertical axis is the voltage across the capacitor.

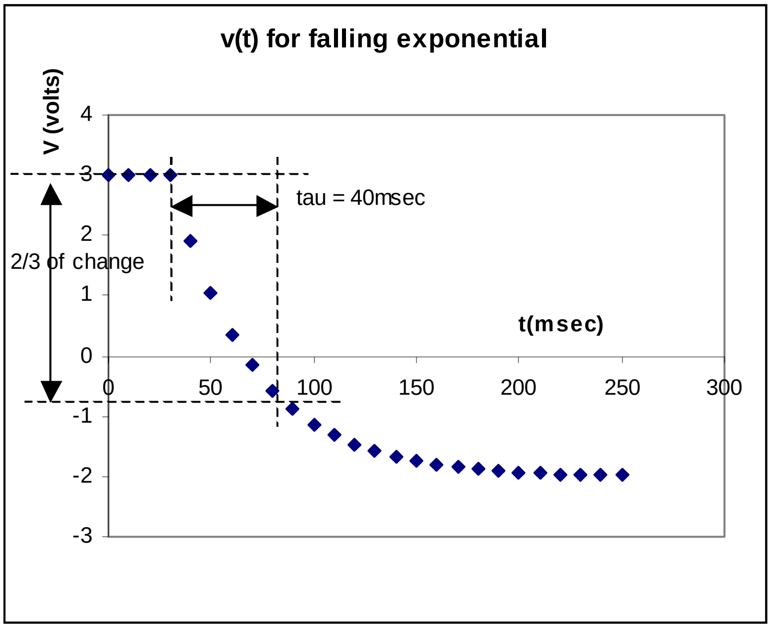

Fig. 3 Quantitative example showing v(t) exponentially falling from +3 to -2 volts with a time constant of 40msec, starting at t0=30msec.

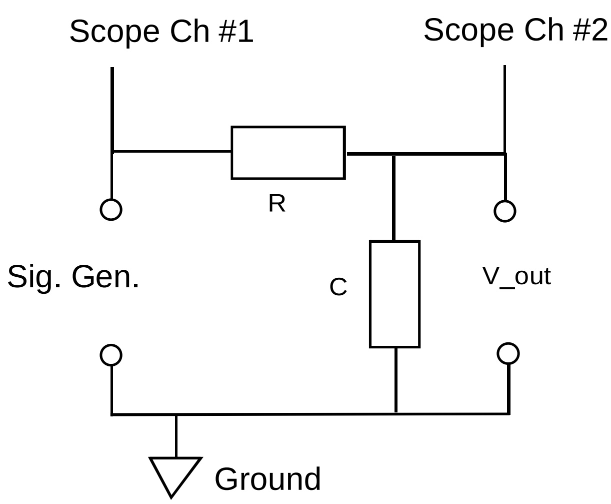

Fig. 4. Experimental setup used for the determination of the decay constant of an RC circuit.

The experimental setup is shown in Fig. 4. We use a signal generator to simulate the switching back and forth between charge and discharge. (This is not a lab on AC circuits. The voltage decays entirely before the beginning of the next cycle from the signal generator. Think of the generator as just a switch). The signal generator acts like the battery (emf) in figure 1, but it reverses polarity and back again every 1/200 second. The horizontal scan of the oscilloscope is synchronized with this frequency, so that it can be displayed in a steady fashion. (The oscilloscope beam runs across the screen from left to right taking 1/200 second to do so, and then takes negligible time to go back to the start). Each time the beam is about to start on the left, the "square-wave" voltage from the signal generator is about to start its upward rise. The effect is a bit like a strobe light on dancers doing a repeating movement - if the light comes on at the same point in the movement, they will look frozen still. The output voltage is monitored on scope channel #2, while the input voltage is on channel #1.

Procedure

1. Connect your circuit as in Fig. 4 with nominal values R = 5.0kΩ, C = 0.1 μF (microfarad) and f = 200Hz square wave. It would help to review the tips on wiring and meters.
2. Select the frequency of the square wave so that the charging and discharging cycles are clearly separated as in figure 2. Note that the input voltage may switch before the output (capacitor) voltage can fully reach its asymptotes (V1 and V2).
3. Describe the qualitative effect on the output wave form, $V_{cap}(t)$, as you adjust R, C and f. Relevant attributes are: initial slope, the period and the asymptotes.
4. Set values back to R = 5.0kΩ, C = 0.1 μF and f = 200Hz. Use the eyeball method to estimate the exponential relaxation time τ both for the rising and falling part of the wave form. Be sure the scope sensitivity and time scales are set on “cal” positions. The left hand mode switch should be on "both", and the right hand mode switch on "auto". Use the scope dials to expand and center the "trace" (picture of waveform) as needed. It may be useful to change the trigger\slope setting to view the down/up cycle. Recall that scope values are given by X(volts) = trace size (cm) * sensitivity (V/cm). Don’t forget uncertainties – these are determined by width of the line, jitter of trace, etc. Include a sketch of the waveform and procedure in your report.

5. Connect the SW data logger to V_{in} and V_{out} using black for ground. Load the SW data acquisition setup file “\\PSCF\phy132\RC.sws”. Press the “rec” button to capture data. It will autostop. Autoscale the plot. You should see about 2 cycles of the waves. See that you have nice looking data, otherwise repeat. You may over-write Run #1 or save as Run #2, etc then use the best ones for analysis. In

any case, save your file frequently (in case of crash, etc.) Copy/paste into GA for analysis. In SW, select the run (in table), copy, open GA, paste.

Analysis:

1. Find the value of τ by fitting your data to eq. 7 or 8. See notes on non-linear fits from the early handouts.
2. Present your various determinations of τ in a single neat table such as below. For errors on calculated value (τ=RC) you can assume R (1%) and C (10%).

RC time constant	Scope eyeball	GA fit	Calc (=RC)
rising			
falling			

Pre-Lab Quiz: PHY132 Lab

Your name_________________________ Section day/time____________________

Consider the following data for exponential relaxation of capacitor voltage in a simple RC circuit. The data can be downloaded as phy132Quiz10.txt

1. Find the value of τ by the 1/e eyeball method (plot the data and draw on it).
2. Fit the data to the appropriate function (eq. 7 or 8). Give the value of all constants. Ignore errors.

t(msec)	V(volts)
0	2.00
10	2.00
20	2.00
30	2.00
40	2.62
50	3.27
60	3.50
70	3.84
80	4.08
90	4.25
100	4.56
110	4.54
120	4.66
130	4.74
140	4.86
150	4.76
160	4.95
170	4.83
180	5.01
190	4.85
200	5.06