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## EXPERIMENT 6: VIBRATIONS

Introduction: In this lab, you will test the assertion that the angular motion of a pendulum, if the maximum angle of swing is small enough, is in fact the same motion as that of a mass on a spring, namely the motion that we call Simple Harmonic Motion. The theory that produces this assertion involves Newton's Laws of Motion plus calculus, and that same theory, applied to a simple pendulum (i.e. a ball on a string) asserts that, for small-angle oscillations, the pendulum period squared should be proportional to the length of the pendulum. You will test this assertion in Part B of this lab.

## Procedure

## Part A: Compare Mass-Spring to Pendulum

1. You should find a white spring, hanging vertically, with a one-kilogram mass attached to the end, and a motion sensor connected to a computer. Adjust the position of the motion sensor to get good data, and then use the motion sensor to measure the position of the bottom of the mass while hanging, with no motion, in its equlibrium position (you can click on $\Sigma$ in the toolbar of the Capstone graph to get the measurement to the nearest tenth of a millimeter). Record this measurement in Data Table 6.1. Also, double-check that your motion sensor is set to record 20 positions per second.
2. Now set the mass-spring into motion; pull the mass down by four or five cm and release it from rest. Watch the motion for a few cycles to make sure that it is purely vertical. To record the motion, try to start the motion sensor when the mass it at the very bottom of its motion. You should get data for about four complete cycles. Look through all the data and find the maximum and minimum values of recorded position; enter these values in Data Table 6.1. Use these max and min values, along with your measurement of equilibrium position, to estimate the amplitude of the motion; this should entail taking the average of two values, one for above the midpoint and one for below the midpoint. Enter this amplitude, to three significant figures, in Data Table 6.1, and show your calculation in the white space to the left of that Table.
3. Use the data to estimate the period of the motion to three significant figures; to get the estimate, find the time for two or three cycles and divide that time by the number of cycles used. Enter this period in Data Table 6.1 and show your calculation in the white space to the left of that Table.
4. Examine your data and select a particular complete cycle, from max value to max value, which, after being scaled, will be plotted carefully on graph paper. You will want the first point in the cycle to be as close as possible to the max value. Enter the times and positions for your selected cycle in the "raw" columns of Data Table 6.1.
time position

$$
\begin{aligned}
\text { period } & = \\
\text { eq } \operatorname{pos} & = \\
\max & = \\
\min & = \\
\operatorname{amp} & =
\end{aligned}
$$

| raw <br> (s) | adjusted <br> (s) | scaled | $\begin{aligned} & \text { raw } \\ & (\mathrm{cm}) \end{aligned}$ | adjusted <br> (cm) | scaled |
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5. To create the "adjusted" time column, subtract your first selected time from all the other time values (so that you first adjusted time is zero). To create the adjusted position column, subtract the equilibrium position from each of recorded positions; as a result the positions below equilibrium will be negative in the adjusted position column.
6. To create the "scaled" time column, divide the adjusted times by the period; the scaled times should thus go from zero to one. To create the scaled position column, divide the adjusted positions by the amplitude; your scaled positions should thus go from about one to negative one and back to one.
7. You have been provided with a special eight-block by ten-block piece of graph paper. Use that graph paper to plot scaled position versus scaled time, with the scaled time on the ten-block axis of that graph paper. Your plotted points should use all of the available space on the graph paper.
8. You should find a rod-and-mass pendulum connected to an angle sensor. Make sure that the slideable mass is as low as possible on the rod and that the screw holding the mass in position is reasonably tight. Begin with the rod hanging straight down, and with no motion. You will want to turn on the angle sensor and let it run for between one to two seconds; then, with the angle sensor running, deflect the rod by less than ten degrees to set it into motion while the data collection completes. You might want to practice this procedure once or twice before your actual data run. Given the programming of our angle sensors by the manufacturer, this procedure will ensure that our measured equlibrium position is at zero degrees.
9. You should have about four complete cycles (or more) of the pendulum motion in your angle-sensor data. Look through all the data and find the maximum and minimum values of recorded angular position; enter these values in Data Table 6.2. Use these max and min values, along with your measurement of equilibrium position (which should be at zero degrees), to estimate the amplitude of the motion; this should entail taking the average of two values, one for an angle in the positive direction and one for an angle in the negative direction. Enter this amplitude, to three significant figures, in Data Table 6.2, and show your calculation in the white space to the left of that Table.
10. Use the data to estimate the period of the pendulum motion to three significant figures; to get the estimate, find the time for two or three cycles and divide that time by the number of cycles used. Enter this period in Data Table 6.2 and show your calculation in the white space to the left of that Table.

| Data Table 6.2 Pendulum <br> eriod $=$ $\qquad$ | time |  |  | angle |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | raw <br> (s) | adjusted <br> (s) | scaled | $\begin{gathered} \text { raw } \\ (\mathrm{deg}) \end{gathered}$ | $\begin{gathered} \text { adjusted } \\ \text { (deg) } \end{gathered}$ | scaled |
|  |  |  |  |  |  |  |
| eq ang = |  |  |  |  |  |  |
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11. Examine your pendulum data and select a particular complete cycle, from max positive value to max positive value, for the purpose of scaling and then plotting. You will want the first point in the cycle to be as close as possible to the max angle. Enter the times and angles for your selected cycle in the raw columns of Data Table 6.2.
12. To create the adjusted time and position columns, and also the scaled time and position columns, use the same methods as were used for the mass-spring table. For the angular position, the equibrium position should be zero degrees, so there should be no difference between the raw and adjusted position columns.
13. Using a different color pencil or pen, plot the scaled position versus scaled time for the pendulum motion on the same piece of graph paper as was used for the mass-spring.

## Part B: Period-Squared Versus Pendulum Length

1. For Part B, use a simple pendulum consisting of a metal ball attached to a single string held at the top by a clamp. You will be asked to measure the period for each of five selected lengths, from about $20-25 \mathrm{~cm}$ to as much as 150 cm . You will need to measure each selected length, and record that measurement in Data Table 6.3; measure each length from the bottom of the clamp to the center of the metal ball.

Data Table 6.3

|  | Length 1 | Length 2 | Length 3 | Length 4 | Length 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\ell(\mathrm{cm})$ |  |  |  |  |  |
| $t_{1}(\mathrm{~s})$ |  |  |  |  |  |
| $t_{2}(\mathrm{~s})$ |  |  |  |  |  |
| $t_{3}(\mathrm{~s})$ |  |  |  |  |  |
| $t_{a v g}(\mathrm{~s})$ |  |  |  |  |  |
| $T(\mathrm{~s})$ |  |  |  |  |  |
| $T^{2}\left(\mathrm{~s}^{2}\right)$ |  |  |  |  |  |

2. For each selected length, set your pendulum into motion with an amplitude of less than ten degrees (a protractor is provided). Watch the motion for a few cycles to make sure that it is purely back-and-forth in a single plane. To measure the period, you will use the provided stopwatch to measure the time for ten complete cycles, and then eventually divide by ten. Making one count per complete cycle, count 3-2-1-go, starting the stopwatch on "go", and then count up to ten, stopping the stopwatch on "ten". Repeat this three times, entering your three measured times in Data Table 6.3 $\left(t_{1}, t_{2}\right.$, and $\left.t_{3}\right)$; then take the average of your three measurements $\left(t_{\text {avg }}\right)$ and finally divide by ten to get your measured period $(T)$ for that chosen length. The last row in Data Table 6.3 allows you to enter the measured value of the period squared.
3. Plot period-squared versus length and draw a best fit line through your data points.

## Results

1. Does your plotted data for the pendulum have the same shape as your plotted data for the mass-spring? It is possible that the your two datasets do indicate the same shape but that one is shifted slightly in time compared to the other. If that is the case for your two datasets, then measure the time-shift (as a fraction of a period) at three different locations and take the average. Indicate on your graph which three locations were used for your time-shift measurements. In the space below, explain your measured time shift (if any) and show your calculations. If you do find a non-zero time shift, can you still agree that the two graphs have the same shape?
2. Looking at your graph of period-squared versus length, does it confirm that periodsquared is proportional to pendulum length for small-angle oscillations?
3. The same theory that predicts that period-squared should be proportional to length for a simple pendulum also predicts that the slope of such a graph should have a value of $4 \pi^{2}$ divided by the local value of $g$ (for us, that is $980 \mathrm{~cm} / \mathrm{s}^{2}$ ). Find the slope of your best-fit line; show your calculation of slope here and on your graph.
4. Compare your value of slope with the value of $4 \pi^{2}$ divided by $g$; what is the percent error between your value and the accepted value of $4 \pi^{2}$ divided by $g$ ?

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