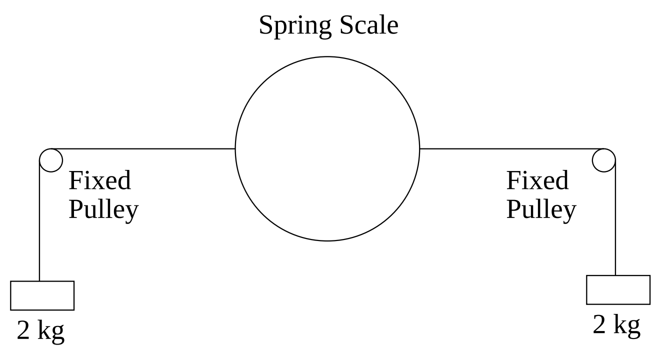
In lecture 9, we completed our initial study of FORCES. Now is a good time to compare your current understanding of forces with the ideas about forces that you brought into the course. In particular, you should think about the DEMO that I call "2 Weights or 1". In the final set-up of this demo, we ended up with the following configuration



For this case, about 90% of students failed to predict that the reading on the spring scale would be about 20 N. Based on my current experience, the cause of this misunderstanding is most likely faulty ideas about forces that students bring with them into the course. In all three cases that we did (the other two can be described as (1) holding the scale and 1 weight vertically, and (2) on the right side, instead of hanging the 2nd weight, tying the string to the pulley mount), the string was being pulled from both ends, just as it is in this case. But somehow, in the other two cases, students don't believe that the pulling force at the end without the hanging weight is a real force. Forces really are interactions between objects. If this DEMO makes sense to you now, and if you can explain it to a friend who knows nothing about physics, then you have made progress in your understanding of forces.

We now continue our study of DYNAMICS by looking at the subject from another perspective, the ENERGY perspective. This new perspective will be useful for solving a class of problems that are difficult to solve with the FORCE perspective (if we try to use forces with those problems, integral calculus is required), and it has important implications for all of science and for all of physics, including physics (such as Quantum Mechanics, the kinematics and dynamics of atomic particles) that goes beyond the Newtonian physics that we are studying now.

We begin with a long, but very important definition, the definition of work done by a constant force. "Work" is one of those words that has a precise meaning in science which may or may not agree with the ideas about that word that you bring to the course. So be sure to learn this DEF well.

DEF The WORK (W) DONE on an object BY A CONSTANT FORCE ($\mathbf{F}$) during a displacement ($\Delta\mathbf{r}$) is given by

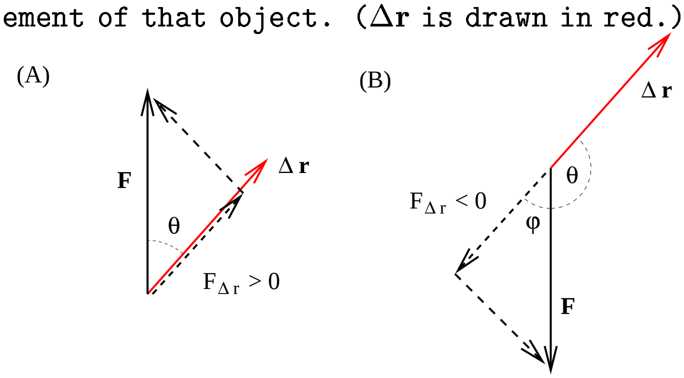
$$W_F \equiv (F_{\Delta r})\Delta r \qquad \text{units are N}\cdot\text{m} \equiv \text{Joule (J)}$$

where W_F indicates the work done BY the force $\mathbf{F}$,

$F_{\Delta r}$ is the component of $\mathbf{F}$ in the direction of the displacement, and

Δr is the magnitude of the displacment of the object.

To understand the meaning of $F_{\Delta r}$, look at the following two examples. As you consider them, remember that $\mathbf{F}$ is one of the forces acting on some object, and $\Delta\mathbf{r}$ is the displacement of that object. ($\Delta\mathbf{r}$ is drawn in red.)



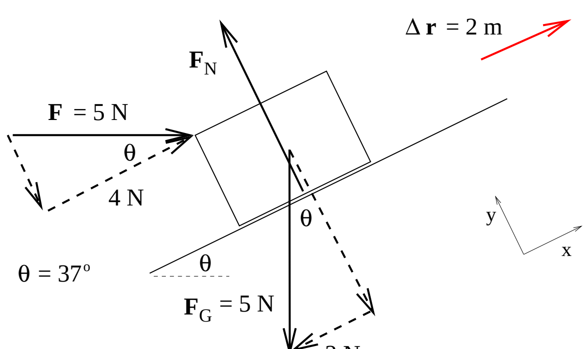
In (A), the component of $\mathbf{F}$ in the direction of $\Delta\mathbf{r}$ (i.e. $F_{\Delta r}$) points in the direction of $\Delta\mathbf{r}$, and is thus positive. In (B), $F_{\Delta r}$ points opposite to the direction of $\Delta\mathbf{r}$ and is thus negative. In both (A) and (B), the angle between

$\mathbf{F}$ and $\Delta\mathbf{r}$ is labeled θ (remember that to get the angle between two vectors, you always must draw them starting from the same place). In (A), it is clear that $F_{\Delta r} = F \cdot \cos\theta$. In (B), it is not as easy to see, but again $F_{\Delta r} = F \cdot \cos\theta$, since $F_{\Delta r} = -F \cdot \cos\phi = F \cdot \cos\theta$. (If you do not immediately recognize that, for θ and ϕ as drawn in (B), $\cos\theta = -\cos\phi$, then you need to brush up on your Unit-Circle Trigonometry. See for example "THE UNIT CIRCLE" at http://www.columbiaview.net/LECTS/TRIG.) Since $F_{\Delta r}$ always turns out to be equal to $F \cdot \cos\theta$, the DEF of work done by a constant force may also be written as

$$W_F \equiv (F_{\Delta r})\Delta r \equiv (F \cdot \cos\theta)\Delta r$$

where θ is the angle between $\mathbf{F}$ and $\Delta\mathbf{r}$. While Δr is a positive-only scalar, and $F_{\Delta r}$ is a component and thus a 1D vector, W_F is a signed scalar (NOT a 1D vector). In other words, W_F can be positive or negative, but the plus or minus sign does not indicate direction. Instead, it indicates that there is more or less of something (More or less of what? That is coming on the next page). From the above example (B), we see that WHENEVER THE FORCE COMPONENT IS OPPOSITE IN DIRECTION TO THE DISPLACEMENT, THE WORK DONE IS NEGATIVE, thus resulting in less of the mysterious something.

Here's a simple example. Use a horizontal force to push a block up a frictionless slope. The FBD for the block is drawn below. We will calculate the work done by each of the three forces during a 2.0 m displacement up the slope. The displacement vector is drawn in red.



I have chosen numbers to make the problem easy, a 5.0 N block, a 5.0 N pushing force, and a slope of θ = 37°. The KEY IDEA is that only the component of the force in the direction of the displacement makes any difference. Note that I did not bother to find the components perpendicular to the displacement!

pushing force $\quad W_F \equiv (F_{\Delta r})\Delta r = (+4.0 \text{ N})(2.0 \text{ m}) = +8.0 \text{ J}$

gravity $\quad W_G \equiv (F_{G,\Delta r})\Delta r = (-3.0 \text{ N})(2.0 \text{ m}) = -6.0 \text{ J}$

normal force $\quad W_{F_N} \equiv (F_{N,\Delta r})\Delta r = (0.0 \text{ N})(2.0 \text{ m}) = 0.0 \text{ J}$

The normal force does not have a component in the direction of $\Delta \mathbf{r}$, and so the work done by the normal force is zero. In fact, ANY FORCE WHICH IS PERPENDICULAR TO THE DISPLACEMENT NEVER DOES ANY WORK; this is the property of work which makes it such a useful idea, as you will see in lecture.

So what is the mysterious something of which there is more or less when work is positive or negative? The answer is KINETIC ENERGY.

DEF The KINETIC ENERGY (KE) of an object of mass m moving with speed v is

$$KE \equiv \tfrac{1}{2}mv^2 \qquad \text{units are } \text{kg} \cdot \text{m}^2/\text{s}^2 = \text{N} \cdot \text{m} \equiv \text{ J}$$

KE is a positive-only scalar.

Note that only the speed v appears in the DEF of KE, not the velocity $\mathbf{v}$, so KE has nothing to do with direction. In our example, the pushing force does positive work on the block and tends to increase the KE of the block. The force of gravity does negative work on the block and tends to decrease the KE of the block. The normal force has no effect on the KE of the block.

The exact relation between work and kinetic energy is given by the WORK-KINETIC ENERGY THEOREM. Before I give that theorem, we need another definition.

DEF The NET WORK (ΣW) done on an object is simply the total work done on that object by all of the forces acting on that object. Synonyms are the TOTAL WORK and the WORK DONE BY THE NET FORCE.

In our example above, ΣW = +8.0 J + (-6.0 J) = +2.0 J. We could also have gotten the net work by finding the work done by the net force, i.e.

$$\text{net work} \qquad \Sigma W \equiv (\Sigma F_{\Delta r})\Delta r = (+1.0 \text{ N})(2.0 \text{ m}) = +2.0 \text{ J}$$

Now we are ready for the WORK-KINETIC ENERGY (WK) THEOREM, which I give in symbols.

$$\Sigma W = \Delta KE$$

In this theorem, ΔKE is just $KE_f - KE_i$, as you would expect. This theorem is the fundamental rule for the ENERGY perspective on dynamics, and can be used to solve almost all problems you will encounter in this section. We will prove this theorem in lecture, but for now let's just try it out on our example.

Assume the block started from rest (so that KE_i = 0). The mass of the block is (F_G/g) = (5.0 N/(9.8 m/s^2)) = 0.510 kg, and the net force on the block is 1.0 N UP THE SLOPE. So the magnitude of the acceleration of the block turns out to be ($|\Sigma \mathbf{F}|/m$) = (1.0 N /0.510 kg) = 1.96 m/s^2 and $\mathbf{a}$ is UP THE SLOPE. After 2.0 m, the block will have a speed of

$$v_f = \sqrt{v_i^2 + 2a\Delta x} = \sqrt{2 \cdot 1.96 \text{ m/s}^2 \cdot 2.0 \text{ m}} = 2.80 \text{ m/s}$$

So $KE_f \equiv \frac{1}{2}mv_f^2$ = $\frac{1}{2}$(0.510 kg)(2.80 m/s)^2 = 2.0 J. So the WK Theorem is successful, since the net work is +2.0 J and $KE_f - KE_i$ is also +2.0 J.

Note that our book writes the WK Theorem as W = ΔKE. In my opinion, this is a very bad choice, as students tend to forget that the work on the left-hand side (LHS) is always the NET WORK. I strongly recommend that you always write the WK Theorem in the form ΣW = ΔKE.

We have learned about one kind of energy, kinetic energy, which is the energy an object possesses by virtue of its motion. A system of two (or more) objects may also possess energy by virtue of their position relative to one another. This "energy of position" may be called the interaction energy, but is more commonly called the POTENTIAL ENERGY, since it is a kind of stored energy. The classic example of potential energy, one that you probably learned about in high school, uses as the two objects the Earth and any mass near the surface of the Earth. I will take a baseball as the second object. When the baseball is resting on the surface of the Earth, it is as close to the center of the Earth as it can get (unless we take it down in a pit, a mine, etc.) When the baseball is lifted above the surface of the Earth (i.e. separated from the surface of the Earth), the Earth-baseball system then possesses a stored (or potential) energy. If I then release the baseball, that stored energy is released, and the baseball and Earth accelerate towards one another due to their gravitational attraction (of course the Earth's acceleration is miniscule, since its mass is $\sim 10^{24}$ times bigger than the mass of the baseball). The potential energy of the Earth-baseball system is gradually converted into more and more kinetic energy as the baseball goes faster and faster towards its collision with the Earth. This is the basic idea of potential energy -- we now get down to details by making a formal definition:

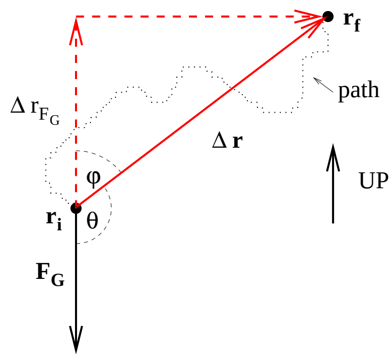
DEF The CHANGE IN GRAVITATIONAL POTENTIAL ENERGY (ΔPE_G) for an object moving between two locations $\mathbf{r}_i$ and $\mathbf{r}_f$ is

$$\Delta PE_{G,\mathbf{r}_i \to \mathbf{r}_f} \equiv -W_G \qquad \text{(units are Joules)}$$

where W_G is the work done by gravity on the object as it moves from $\mathbf{r}_i$ to $\mathbf{r}_f$. ΔPE_G is a signed scalar. The $\mathbf{r}_i \to \mathbf{r}_f$ subscript is usually omitted.

Note that ONLY THE CHANGE IN POTENTIAL ENERGY IS DEFINED. This is a good thing because it gives us useful flexibility in solving problems -- I will give examples in lecture. But why is the SYSTEM OF OBJECTS not mentioned in the DEF? This form of the DEF assumes that the second object is a massive object (such as the Earth) which is nearby and which doesn't move; the positions $\mathbf{r}_i$ and $\mathbf{r}_f$ are measured with respect to that massive stationary object. But always remember that the potential energy is a property of a system of two (or more) objects.

Finally, let's use this DEF to calculate the ΔPE_G of an object moving near the surface of the Earth. Imagine an object of weight mg that is moved from $\mathbf{r}_i$ to $\mathbf{r}_f$ along the wiggly path shown below. Note that, in my drawing, UP is towards the top of the page. The displacement vector is drawn in red.



Since we have no numbers, our task is to calculate W_G using symbols; ΔPE_G will then be the negative of W_G. This time I will use $W_F \equiv (F \cdot \cos\theta)\Delta r$, but I will show you a useful trick. Since the order of multiplication doesn't matter, I will group the $\cos\theta$ with Δr, i.e. $W_F \equiv F(\cos\theta \cdot \Delta r) = F(-\cos\phi \cdot \Delta r)$. That way, instead of finding the components of $\mathbf{F}_G$, I can find the components of $\Delta\mathbf{r}$. In other words, there is yet another way to think about the DEF of work,

$$W_F \equiv (F_{\Delta r})\Delta r \equiv F(\Delta r_F)$$

We can either take the component of $\mathbf{F}$ in the direction of $\Delta\mathbf{r}$ times the magnitude of $\Delta\mathbf{r}$, or instead we can take the magnitude of $\mathbf{F}$ times the component of $\Delta\mathbf{r}$ in the direction of $\mathbf{F}$. This time, because the force of gravity always points DOWN, the $F(\Delta r_F)$ form is more convenient. From the drawing, it is easy to see that if Δr_{F_G} is UP, then W_G is negative. Conversely, if Δr_{F_G} were DOWN, then W_G would be positive. The result we get is usually written in this way:

$$W_G \equiv F_G(\Delta r_{F_G}) = mg(-\Delta h)$$

where Δh, the change in height, is defined to be a 1D vector with UP positive. So if Δh is UP then W_G < 0, and if Δh is DOWN then W_G > 0. We now have ΔPE_G:

$$\Delta PE_G \equiv -W_G = mg\Delta h \tag{1}$$

with Δh defined as above. If Δr_{F_G} is UP, then ΔPE_G > 0; if Δr_{F_G} is DOWN, then ΔPE_G < 0. This equation will appear on your equation sheet.

It is not uncommon to use equation (1) as the DEF of the change in gravitational potential energy; our book does something like this. But this is a bad choice. Eq. (1) is only true in a region over which g is constant. Suppose we wanted to know the ΔPE_G for a satellite of mass m as it is moved between two circular Earth (mass = M_E) orbits of different radius r_i and r_f, with r_f > r_i. The answer $mg(r_f - r_i)$ WOULD NOT BE CORRECT. The right answer is $GM_E m\frac{(r_f - r_i)}{r_i r_f}$, which is gotten by using the correct DEF of ΔPE_G, namely $\Delta PE_G \equiv -W_G$, and calculus. You don't have to do PE problems of this type, but you should be aware that ΔPE_G = $mg\Delta h$ is not universally true.

We need one more simple definition:

DEF The MECHANICAL ENERGY (E) of an object is given by $E \equiv KE + PE$.

Mechanical energy turns out to be a very useful quantity, because it is constant IF ONLY GRAVITY IS DOING WORK. But there is a problem with this DEF, because as yet we have only defined ΔPE_G; PE_G is not defined! We will take care of this in lecture, and we will do examples in which E is constant.