Before beginning this required reading for lecture 11, I suggest you try Self-Assessment Test 6.1, available at the "C&J 6th Ed Web Site" link on our course web page. This will double-check your understanding of the concepts we covered in lecture 10.

At the end of lecture 10, we learned that the MECHANICAL ENERGY of an object is constant whenever only the force of gravity is doing work on the object; this idea is called the CONSERVATION OF MECHANICAL ENERGY. We must always remember that, while it is common to talk about the mechanical energy "of an object", strictly speaking we can only have potential energy for a system of two (or more) objects. We are allowed to talk about the potential energy "of an object" only because the stationary Earth is usually understood to be the second object in the system.

But can we talk about energy conservation for an object (or system of objects) if forces other that gravity are doing work? The answer turns out to be yes, but we have to talk about the conservation of TOTAL ENERGY, not just mechanical energy. I will begin lecture 11 by discussing this PRINCIPLE OF THE CONSERVATION OF (TOTAL) ENERGY, and how it relates to the work done by forces other than gravity. We will begin to think of WORK as a transformation of energy from one form to another, or as a transfer of energy from one object to another.

Our final topic in the ENERGY perspective on DYNAMICS will then be the subject of POWER. In science, it is important to distinguish the word "power" from the word "energy". Here is the definition of average power:

DEF The AVERAGE POWER ($\bar{P}$) associated with any force $\mathbf{F}$ which does an amount of work W_F during elapsed time Δt is

$$\bar{P}_F \equiv \frac{W_F}{\Delta t} \qquad \text{units are J/s} \equiv \text{Watts (W)}$$

Power, like work, is a signed scalar. In words rather than symbols, power is the RATE OF DOING WORK. Average power is just the average rate at which work is done. So energy is not power, and power is not energy; the concepts are related, but the units of the two quantities are different.

A useful shortcut in calculating power is revealed with the following simple derivation:

DEF of average power	$\bar{P}_F \equiv \dfrac{W_F}{\Delta t}$
use DEF of work, $W_F \equiv (F_{\Delta r})\Delta r$	$= \dfrac{(F_{\Delta r})\Delta r}{\Delta t}$
use DEF of average velocity, $\bar{\mathbf{v}} \equiv \dfrac{\Delta \mathbf{r}}{\Delta t}$	$= (F_{\Delta r})\bar{v}$
and since dir. of $\Delta \mathbf{r}$ is also dir. of $\bar{\mathbf{v}}$	$= (F_{\bar{v}})\bar{v}$

We have derived this result for the average power, and thus used the average velocity, but it is true for instantaneous power and velocity as well, i.e.

$$P_F(t) = (F_v)v(t) \tag{1}$$

This equation tells us that the instantaneous power associated with any force $\mathbf{F}$ (i.e. the rate at which $\mathbf{F}$ is doing work on the object in question at some time t) can be gotten by multiplying the component of $\mathbf{F}$ in the direction of the object's velocity times the instantaneous speed of the object. Eq. (1) will appear on your equation sheet.

Here is a simple example with power. Get a full can of pop (or something similar - your numbers may change a little). It contains 0.355 L or 355 cm^3 of what is essentially water. Water has a density of 1.0 $\mathrm{g/cm}^3$, so you have 0.355 kg of water (ignore the mass of the can). So the weight of this can of pop is mg = (0.355 kg)(9.8 $\mathrm{m/s}^2$) or about 3.5 N. If you lift this can straight up for a distance of 1.0 m (either at constant speed, or starting and ending with the same speed, so that $\Delta KE = 0$) then you do +3.5 J of work (the net work done on the can of pop is of course zero). If you do this work in 1.0 s, then your average power output during the lift is +3.5 J/s or +3.5 W. If you do the lift at constant speed, then you can also get the same result by using Eq. (1); the velocity of the can would be 1.0 m/s UP and the force on the can by your hand would be 3.5 N UP, so the power would be (+3.5 N)(1.0 m/s) = +3.5 N·m/s = +3.5 W. Human efficiency is typically in the 10-20% range. If your efficiency is 10%, then during the lift you are consuming energy (i.e. burning sugars to operate your muscles) at a rate of 35 W, a little over half (58%) of the rate of energy consumption by a 60 W light bulb. (By the way, our calculation does not include the baseline energy consumption rate of the human body -- the power your body uses just to keep breathing and pumping blood, etc. This "basal metabolic rate" is about 77 W even when you are sleeping.) How many Calories does your action consume? The Calorie (Cal) is a unit of energy used by nutritionists; 1 Calorie = 4186 J. If you consumed energy at a rate of 35 W for 1.0 s, you would thus have used 35 J of energy, which requires the burning of about 0.008 Cal.

Another commonly used unit of power is horsepower (hp); 1 hp = 746 W. Students who work with engines may like to compare human power outputs to typical engine power outputs. For our lift, the power output of 3.5 W is only about 0.005 hp. On the other hand, a 60 kg person running up a 3.0 m high flight of stairs in 1.0 s has an average power output of 1.76 kW or about 2.4 hp. You should check your understanding of power by confirming my calculation.

The topic of POWER concludes our introduction to the ENERGY perspective on DYNAMICS; the energy perspective is very useful and we will revisit it many times during the semester. Next we would like to learn how to handle the dynamics of general systems of particles (or objects). We have already dealt with a few systems of objects, but always the objects were moving as a unit -

- they were always tied together by a rope, or being pushed along in a stack without slipping, etc. What if the objects in our system are not moving as a unit (for example, a system of two balls that are thrown at one another and collide, or the system consisting of all of the air molecules in a room)? Does it make any sense to talk about the position of the system, or the velocity of the system, or the acceleration of the system? The answer turns out to be yes. We begin by defining the position of the system, also known as (AKA) the position of the center-of-mass of the system.

DEF The POSITION of the CENTER OF MASS ($\mathbf{r}_{CM}$) of a system of N particles (AKA the position of the system) is

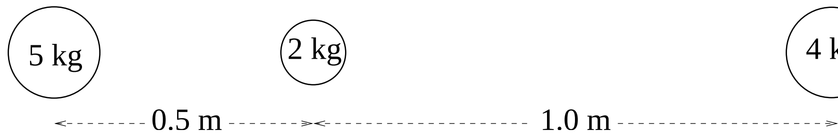
$$\mathbf{r}_{CM} \equiv \frac{\sum_i m_i \mathbf{r}_i}{M_{sys}} \qquad \text{with } \sum_i \equiv \sum_{i=1}^{N}$$

where m_i is the mass of the i^{th} particle

$\mathbf{r}_i$ is the position of the i^{th} particle, and

$M_{sys} = \sum_i m_i$.

This DEF may look complex, but it is actually rather simple. Look at a simple 1D example. We have 3 balls located along a line, as shown below. The 2.0 kg ball is 0.5 m to the right of the 5.0 kg ball, and the 4.0 kg ball is 1.0 m to the right of the 2.0 kg ball. Our task is to find the location of the center of mass of this system of three balls.



We apply the DEF of the position of the center of mass. In order to do so, we need to choose the location where $x = 0$; I choose $x = 0$ at the 5.0 kg ball, then the 2.0 kg ball is at 0.5 m, and the 1.5 kg ball is at 1.5 m. Now apply the DEF, remembering to change over to 1D vector notation.

$$\begin{aligned} x_{CM} &\equiv \frac{\sum_i m_i x_i}{M_{sys}} \\ &= \frac{(5.0\text{ kg})(0\text{ m}) + (2.0\text{ kg})(+0.5\text{ m}) + (4.0\text{ kg})(+1.5\text{ m})}{5.0\text{ kg} + 2.0\text{ kg} + 4.0\text{ kg}} \\ &= +0.636\text{ m} \end{aligned}$$

So the center of mass of this system is located 0.636 m to the right of the 5.0 kg mass (or 0.136 m to the right of the 2.0 kg mass).

Now that we know how to find the position of the system, what can we mean by the velocity of the system? This is easy to get from our DEF of the position of the system simply by imagining some displacement $\Delta\mathbf{r}_{CM}$ of the system and then dividing by the time Δt during which the displacement takes place. In symbols,

DEF of average velocity for CM	$\bar{\mathbf{v}}_{CM} \equiv \frac{\Delta\mathbf{r}_{CM}}{\Delta t}$
DEF of r_{CM}	$= \frac{\sum_i m_i \Delta\mathbf{r}_i}{M_{sys}} \div \Delta t$
algebra — use the distributive property	$= \frac{\sum_i m_i \frac{\Delta\mathbf{r}_i}{\Delta t}}{M_{sys}}$
DEF of average velocity for particle i	$= \frac{\sum_i m_i \bar{\mathbf{v}}_i}{M_{sys}}$

We did this derivation for the average velocity of the CM, but it is true for the instantaneous CM velocity as well, i.e. $\mathbf{v}_{CM} = (\sum_i m_i\mathbf{v}_i)/M_{sys}$. And we can repeat the whole process over again with a $\Delta\mathbf{v}_{CM}$ to get the acceleration of the center of mass, which will be

$$\mathbf{a}_{CM} = \frac{\sum_i m_i \mathbf{a}_i}{M_{sys}}.$$

So now we have a DEF for the position of any system of particles. And we use a similar expression to get the velocity of the system or the acceleration of the system. In each case, we do a sum over all the particles within which we multiply the mass of each particle times its relevant kinematic variable (either its position, or its velocity, or its acceleration), and then divide the sum by the mass of the entire system. But why is this the right DEF for the position of the system? I will show in lecture that, with this DEF for the position of the system, Newton's 2nd Law for the system is just

$$\sum \mathbf{F}_{ext} = (M_{sys})\mathbf{a}_{CM},$$

i.e. the sum of the external forces acting on the system equals the mass of the system times the acceleration of the center of mass of the system. Once we have done that proof, we can see that the DEF of $\mathbf{r}_{CM}$ must be right, because the $\mathbf{a}_{CM}$ turns out to be correct for the acceleration of the system.