

At the end of lecture 11, I showed that Newton's Second Law for a system of particles is

$$\sum \mathbf{F}_{ext} = (M_{sys})\mathbf{a}_{CM},$$

If $\mathbf{a}_{CM} = 0$, then the system of particles is in equilibrium, i.e. we have Newton's First Law for a system of particles, which can be written:

$$\sum \mathbf{F}_{ext} = 0 \iff \mathbf{v}_{CM} \text{ is constant,}$$

where the symbol \iff means "if and only if". Newton's 1st Law for a system of particles can be applied whenever $\sum \mathbf{F}_{ext} = 0$, and is a powerful tool for finding unknown motions of the individual particles in the system. IT IS A VECTOR LAW, so it can be applied for any component of the system's motion. It is used so frequently that it is given a special name, but to understand that name, we first need a new DEF:

DEF The (TRANSLATIONAL) MOMENTUM (\mathbf{p}) of a particle of mass m moving with velocity \mathbf{v} is

$$\mathbf{p} \equiv m\mathbf{v} \quad \text{units are kg} \cdot \text{m/s}$$

The word "translational" just means that the object is moving in x, y, z space, and is usually understood. Later we will define an object's "rotational" or "angular" momentum; the word "translational" is sometimes used so that the two types of momentum are not confused.

The total momentum of a system of objects is then just the sum of the momenta of each particle in the system, i.e.

$$\mathbf{p}_{sys} \equiv \sum_i \mathbf{p}_i = \sum_i m_i \mathbf{v}_i$$

This last equation gives us the connection between momentum and the velocity of the center of mass, \mathbf{v}_{CM} . Recall that

$$\mathbf{v}_{CM} = \frac{\sum_i m_i \mathbf{v}_i}{M_{sys}}$$

The numerator is just the total system momentum \mathbf{p}_{sys} . Therefore

$\mathbf{p}_{sys} = M_{sys} \mathbf{v}_{CM}$, and if \mathbf{v}_{CM} is constant, then \mathbf{p}_{sys} is also constant.

Therefore, Newton's 1st Law for a system of particles is often called

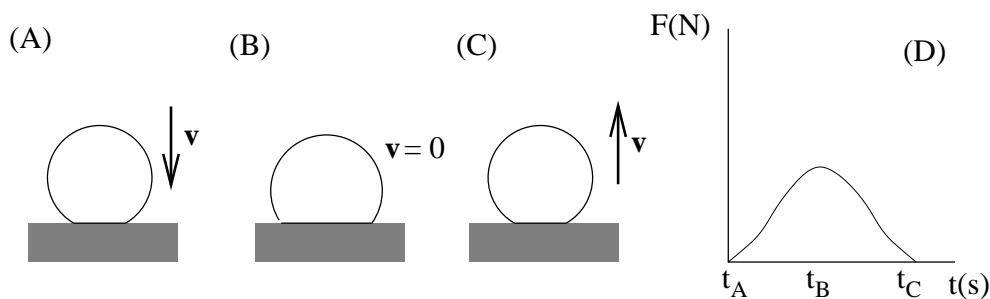
The PRINCIPLE OF CONSERVATION OF (TRANSLATIONAL) MOMENTUM.

For any system of N particles, if $\sum \mathbf{F}_{ext}$ acting on the system is zero, then the total momentum of the system, $\sum_{i=1}^N \mathbf{p}_i$, is constant. In particular, if the system is closed, so that there can be no external forces acting on the system, then the total system momentum is constant. Therefore, if the Universe is a closed system, then the total momentum of the Universe is constant.

This is the second great conservation principle of classical physics, after the principle of conservation of energy. You should recognize it as just a restatement of Newton's 1st Law for a system of particles. It is the foundation of the MOMENTUM perspective on DYNAMICS, and is useful for solving problems about systems of particles for which the sum of external forces is zero, or for which the sum of external forces is very small compared to internal forces. An example of the latter case would be a collision between a block hanging by a string from the ceiling and a bullet fired at the block. The bullet and the block make up a two-object system. The sum of external forces is zero for the block, since the tension in the string cancels its weight; but $\sum \mathbf{F}_{ext}$ is not zero for the bullet because there is no vertical force to cancel its weight. BUT, the internal forces between the bullet and block are much much larger than the weight of the bullet, so any answers that we get from applying conservation of momentum should be good to at least three sig figs.

In lecture, we will do several examples using the conservation of momentum (Cofp). But notice that Cofp cannot tell us anything about the internal forces (for example the contact force between the bullet and block in my example from above), since Cofp always uses a system of particles and the internal forces don't affect the motion of the system. So if we want to know something about the internal forces, we must return to Newton's 2nd Law for the individual particles. Our problem is that these internal forces are not usually constant, and so the accelerations of the individual particles are also not constant. How do we deal with these non-constant forces and accelerations? Before I answer this question, I will give an simple example to make sure that you understand the physical situation we are describing.

Consider a vertical collision between a tennis ball and the floor (i.e. the Earth). When the tennis ball first makes contact with the floor, the contact force is small (figure (A) below). In the middle of the collision, the ball is severely distorted and the contact force is large (figure (B) below). And as the ball is just about to leave the floor, the contact force is again small (figure (C) below). The graph of the magnitude of the contact force versus time might look like the one drawn in figure (D).



So it doesn't make sense to ask, "What is the force on the ball by the floor?" because that force isn't constant. However, we can ask, "What is the average force on the ball by the floor?". We will learn how to answer this latter

question. In this case, the ball and the Earth form the two-object system. For that two-object system, there are essentially no external forces (unless we want to complicate matters by considering gravitational forces from the Sun and the Moon), since both the mutual gravitational attraction and the contact forces are internal forces. So momentum will be conserved for the ball-Earth system, but that doesn't help us to find the average force on the ball by the floor. We need to consider the ball by itself and apply the 2nd Law. Contact forces of this type, that are sometimes large and typically last for a short time, are often called IMPULSIVE forces. To discuss them, we need one more definition:

DEF The IMPULSE (\mathbf{J}) due to an average net force ($\Sigma\bar{\mathbf{F}}$) acting for time Δt is

$$\mathbf{J} \equiv (\Sigma\bar{\mathbf{F}})\Delta t \quad \text{units are N}\cdot\text{s}$$

Impulse is a vector quantity. The units are $\text{N}\cdot\text{s} = (\text{kg}\cdot\text{m}/\text{s}^2)\cdot\text{s} = \text{kg}\cdot\text{m}/\text{s}$, and are thus the same units as for momentum. But it is traditional to use $\text{N}\cdot\text{s}$ for impulse and $\text{kg}\cdot\text{m}/\text{s}$ for momentum.

Now we will use Newton's 2nd Law for a particle of mass m experiencing an impulsive net force and apply the DEF of impulse.

2nd Law	$\Sigma\mathbf{F} = m\mathbf{a}$
take the average $\Sigma\mathbf{F}$ and \mathbf{a}	$\Sigma\bar{\mathbf{F}} = m\bar{\mathbf{a}}$
use DEF of $\bar{\mathbf{a}} \equiv \frac{\Delta\mathbf{v}}{\Delta t}$	$\Sigma\bar{\mathbf{F}} = m\frac{\Delta\mathbf{v}}{\Delta t}$
use DEF of $\mathbf{p} \equiv m\mathbf{v}$	$\Sigma\bar{\mathbf{F}} = \frac{\Delta\mathbf{p}}{\Delta t}$
multiply both sides by Δt	$\Sigma\bar{\mathbf{F}}\Delta t = \Delta\mathbf{p}$
thus, by the DEF of \mathbf{J}	$\mathbf{J} = \Delta\mathbf{p}$

So our result is that the impulse vector has the same magnitude and direction as the CHANGE IN MOMENTUM vector. This will appear on your equation sheet.

So, to find the average net force acting on an object while it is experiencing an impulsive contact force, all that we need to do is to take the change in momentum of the object and divide by the estimated time during which the contact occurs (we might get a very good estimate for this time by making a video of the collision and replaying it in slow motion). This operation is expressed directly by step 4 in our work above.

By the way, the term "impulse", as used here, has the same meaning as in the popular TV series Star Trek, when they speak of "impulse engines". Impulse engines are just rockets. They operate by throwing hot gas backward so that the "Enterprise" goes forward. The impulsive contact force that accelerates the Enterprise is the contact force ON the Enterprise BY the gas as the gas is thrown away. I will do a demonstration of an impulse engine in lecture; and, we will do a numerical example in which we find an impulsive force.

The ideas of impulse and Cofp are most often used in the analysis of collisions during which the impulsive forces are large in comparison to any external forces. Collisions are classified on the basis of how the total KE of the system changes during the collision. Learn the following simple DEFs:

DEFs A collision during which the total KE decreases is an INELASTIC collision. If the total KE remains constant during the collision, the collision is PERFECTLY ELASTIC (or just ELASTIC). An EXPLOSION is a collision during which the total KE increases due to some internal energy conversion. The maximum KE is "lost" when the objects stick together after the collision, a PERFECTLY (or completely) INELASTIC collision.

Of course, no energy is ever really "lost", it is only converted into some other form. As we do examples of collisions during lecture, we will classify them and discuss any energy transformations that occur.